MA205 Lesson 7

The Fundamental Theorem of Calculus I

Tuesday, August 28, 2007

Outline

1. ADMIN
2. From Lesson 6
   - The Definite Integral
3. Antiderivatives
   - Antiderivatives You Need To Know
4. Fundamental Theorem of Calculus I
   - Fundamental Theorem of Calculus I
5. Fundamental Theorem of Calculus II
   - Fundamental Theorem of Calculus II
6. More FTC I and II
   - Look at for tomorrow

ADMIN NOTES

- **Pick Up Your Course Guide Today**
- **Homework Solutions are in the Math Study Clinic, Thayer 226**

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Definition of the Definite Integral

- $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \lim_{n\to\infty} [f(x_1^*)\Delta x + f(x_2^*)\Delta x + ... + f(x_n^*)\Delta x]$
- If f is a continuous function defined for $a \leq x \leq b$, we divide the interval $[a, b]$ into n subintervals of equal width $\Delta x = (b - a)/n$. We let $x_0(= a), x_1, x_2, ..., x_n(= b)$ be the endpoints of these subintervals, so x_i^* lies in the ith subinterval $[x_{i-1}, x_i]$. Then the **definite integral of f from a to b** is:

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

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  where C is an arbitrary constant.
- For example if $f(x) = x^2$ it's antiderivative is $F(x) = \frac{x^3}{3} + C$.

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Antiderivatives You Need To Know

- $cf(x)$ antiderivative is $cF(x)$.
- $f(x) + g(x)$ antiderivative is $F(x) + G(x)$.
- $x^n (n \neq -1)$ antiderivative is $\frac{x^{n+1}}{n+1}$.
- $1/x$ antiderivative is $\ln|x|$.
- e^x antiderivative is e^x.
- $\cos x$ antiderivative is $\sin x$.
- $\sin x$ antiderivative is $-\cos x$.

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Fundamental Theorem of Calculus I

- If f is continuous on $[a, b]$, then the function g defined by

$$g(x) = \int_a^x f(t)dt \quad a \leq x \leq b$$

is continuous on $[a, b]$ and differentiable on (a, b), and $g'(x) = f(x)$.

Fundamental Theorem of Calculus I

- For Example: Find the derivative of the function

$$g(x) = \int_0^x \sqrt{1 + t^2}\,dt$$

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- Since $f(t) = \sqrt{1+t^2}$ is continuous, FTC 1, states that

$$g'(x) = \sqrt{1+x^2}$$

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Fundamental Theorem of Calculus II

- If f is continuous on $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Where F is any antiderivative of f, that is, a function such that $F' = f$.

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FTC I and II

- Look at Stewart, Chap 4, Section 10, 353-358
- Stewart Chap 5, Section 3, 394-401

Questions