MA205

Board Sheet - 30 August 2007, FTC I and II, Lesson 8

Some info to help you understand FTC

Antiderivatives You Need To Know

1. $x^n (n \neq -1)$ antiderivative is $\frac{x^{n+1}}{n+1}$.
2. $1/x$ antiderivative is $\ln|x|$.
3. e^x antiderivative is e^x.
4. $\cos x$ antiderivative is $\sin x$.
5. $\sin x$ antiderivative is $-\cos x$.

Find the antiderivative then check your answer by differentiation

1. $f(x) = 1 - x^3 + 5x^5 - 3x^7$

2. $f(x) = 5x^{\frac{1}{4}} - 7x^{\frac{3}{4}}$

3. $f(x) = 6\sqrt{x} - \sqrt[6]{x}$

4. $f(x) = \frac{10}{x^9}$

FTC I and II

- FTC I: If f is continuous on $[a, b]$, then the function g defined by

$$g(x) = \int_a^x f(t)dt \quad a \leq x \leq b$$

  is continuous on $[a, b]$ and differentiable on (a, b), and $g'(x) = f(x)$.

- FTC II: If f is continuous on $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

  Where F is any antiderivative of f, that is, a function such that $F' = f$.

Do these problems to help in your understanding

1. Find a function f and a number a such that: $6 + \int_a^x \frac{f(t)}{t^2} dt = 2\sqrt{x}$ for all $x \geq 0$.

2. Use FTC to calculate $g'(5)$ given that: $g(x) = \int_0^x (t^2 + 2t + 5)dt$. Then show that your answer is correct by performing the integration and subsequent differentiation and evaluation.

3. Given a particle has acceleration $a(t) = e^{-t} + t^2$ in feet per second squared, and at $t = 3$ the particle is traveling at 21 feet per second, find a velocity function for the particle at any time t.