MA104 Lesson 33

LESSON 33 - The Dot and Cross Product

Monday, 10 March, 2008

Outline

1. Admin
2. Last Class
3. The Dot and Cross Products
   - Vectors So Far
     - Parametric Equations, Intersection, and Collision
     - Three Dimensional Coordinate System
     - Vectors
   - Course Guide
   - Definitions
   - Do Problem Help
4. Look Forward - EQUATIONS OF LINES IN SPACE
   - Course Guide

Admin

1. WPR II Hard Copy of Solution

INTRODUCTION TO VECTOR AND VECTOR OPERATIONS

Questions?

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Parametric Equations, Intersection, and Collision

1. Parametric Equations are equations with the x, y and z components represented as a function of t.
2. Collision and/or intersection.
   1. Set the $x(t)$ components equal to each other and solve for t. You may have more than one t.
   2. Substitute these t's into the y and z components.
   3. If both vectors are equal you have a collision and intersection at these times.
   4. If they are not equal check for intersection.
   5. Replace your t with an r in one of the vectors.
   6. Set the $x(t) = x(r)$ and $y(t) = y(r)$ and solve the system of equations for t and r.
   7. These are your possible times for intersection where t is the time the first particle will go through the intersection point and r is the time for the second particle to go through the intersection point.
   8. Substitute your t and r into the respective vectors and check if you have intersection and where it will be.

Three Dimensional Coordinate System

1. Distance Formula in Three Dimensions: The distance $|P_1P_2|$ between the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

2. Equation of a Sphere: An equation of a sphere with center $C(h, k, l)$ and radius r is

$$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$$

If the center is the origin O, then the equation is

$$x^2 + y^2 + z^2 = r^2$$

Vectors

1. A **vector** has both magnitude and direction.
2. A vector is often represented by an arrow or a directed line segment. The length of the arrow represents the magnitude and the arrow points in the direction of the vector.
3. A vector is represented by a bold ($\mathbf{v}$) or an arrow above it ($\vec{v}$).
4. Suppose a particle or ball moves along a line segment from point A to point B. The **displacement vector** $\mathbf{v}$, has **initial point** A (the tail) and **terminal point** B (the tip) and we write this as $\mathbf{v} = AB$.
5. Another vector $\mathbf{u} = CD$ could have the same length and direction as $\mathbf{v}$ but be in a different position. In this case $\mathbf{u}$ and $\mathbf{v}$ are **equivalent** (or **equal**) and we can write $\mathbf{u} = \mathbf{v}$.
6. The **zero vector**, denoted by $\mathbf{0}$, has length 0. It is the only vector with no specific direction.

Vectors Operations: Addition

1. Vector Addition:
   1. If a particle moves from A to B, displacement $\overrightarrow{AB}$. Then the particle moves from B to C, displacement $\overrightarrow{BC}$. The combined effect is that the particle has moved from A to C, displacement $\overrightarrow{AC}$, is called the sum of $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

   2. The sum of $\mathbf{u} + \mathbf{v}$ is the vector from the initial point of $\mathbf{u}$ to the terminal point of $\mathbf{v}$.
   3. An example mathematically. If the vector $\mathbf{u} = \langle 1, 2, 3 \rangle$ is added to $\mathbf{v} = \langle 2, 3, 4 \rangle$ the resulting vector is the addition of each of the components meaning add the x components, the y components, and the z components to get $\mathbf{u} + \mathbf{v} = \langle 3, 5, 7 \rangle$.

Vectors Operations: Scaler Multiplication and Components

1. Scaler Multiplication:
   1. If c is a scalar and $\mathbf{v}$ is a vector, then the scalar multiple $c\mathbf{v}$ is the vector whose length is $|c|$ times the length of $\mathbf{v}$ and whose direction is the same if $c > 0$ and opposite if $c < 0$. If $c = 0$ or $\mathbf{v} = \mathbf{0}$, then $c\mathbf{v} = 0$.
2. Components - a review:
   1. Vectors have components

$$\mathbf{a} = \langle a_1, a_2, a_3 \rangle$$

   2. A vector from the origin O to the point P where $P(3, 2, 1)$ is called the position vector of the point P.
   3. Given the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$, the vector $\mathbf{a}$ with representation $\overrightarrow{AB}$ is

$$\mathbf{a} = \langle x_2 - x_1, \quad y_2 - y_1, \quad z_2 - z_1 \rangle$$

Vectors Operations: Magnitude, Length, and the Unit Vector

1. Magnitude or Length:
   1. The **magnitude** or **length** of the vector **v** is the length of any of its representations and is denoted by $|v|$ or $||v||$. We use the distance formula to compute the length.

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

   2. The Magnitude is a number, not a vector!
2. The Unit Vector!
   1. A Unit Vector is a vector whose length is 1. For example $\mathbf{i}, \mathbf{j}$, and $\mathbf{k}$ are all unit vectors where $\mathbf{i} = \langle 1, 0, 0 \rangle$. If $\mathbf{a} \neq \mathbf{0}$, then the unit vector that has the same direction as $\mathbf{a}$ is

$$\mathbf{u} = \frac{1}{|\mathbf{a}|}\mathbf{a} = \frac{\mathbf{a}}{|\mathbf{a}|}$$

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Objectives

1. Understand the definition of the dot product and how it can be computed.
2. Understand what it means for two vectors to be orthogonal.
3. Understand what a vector projection is and how it is computed.
4. Apply the dot product to find the work done by a force.
5. Understand the definition of the cross product and what it gives us.
6. Know how to calculate the cross product using Mathematica.

READ

1. Stewart: Section 12.3, pages 779-784 (Skip Direction Angles and Direction Cosines).
2. Stewart: Section 12.4, the first two paragraphs on page 786 and pages 788-789 (Read and understand Theorems 5 and 6, and Corollary 7).
3. Student Notes.

THINK ABOUT

1. Interpret what the dot product of two vectors means.

DO Problems

1. Section 12.3/ 5, 7, 9, 14, 18, 46
2. Section 12.4/ 4, 5, 7, 16, 19

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Definitions, Dot Product

1. Definition 1, Page 779
2. Properties of a Dot Product, Page 779
3. Theorem 3, Page 780
4. Corollary 6, Page 780
5. Page 781
6. Projections, Page 783
7. Cross Product, Page 786

Definitions, Cross Product

1. Cross Product, Page 786
2. Theorem 5, Page 788
3. Theorem 6, Page 788
4. Corollary 7, Page 789

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1. Section 12.3/ 5, 7, 9, 14, 18, 46
2. Section 12.4/ 4, 5, 7, 16, 19

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LESSON 34 - EQUATIONS OF LINES IN SPACE

1. OBJECTIVES:
   1. Understand how to develop the vector and parametric equations of a line in space using a vector.
   2. Understand what two pieces of information are necessary to find the equation of a line in three dimensional space.
2. READ:
   1. Stewart: Section 12.5, pages 794-797 (Stop at Planes).
3. THINK ABOUT:
   1. How is finding the equation of a line in two dimensions similar to finding the equation of a line in three dimensions? How are they different?
4. DO:
   1. DO: Section 12.5/ 3, 4, 13, 17, 18

Questions?

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