MA153

Block III Review Sheet

Multiple Integrals - Stewart Chapter 15

1. Iterated Integrals over Rectangular Regions
   (a) Create a 3D plot of the surface over the region
   (b) Draw and label the region
   (c) Choose the order of integration
   (d) Compute/Evaluate the "inside" integral
   (e) Compute/Evaluate the "outside" integral
   (f) Does the answer make sense?

2. Iterated Integrals over General Regions
   (a) Sketch the region
   (b) Label the boundaries
   (c) Solve for and label the intersections
   (d) Compute/Evaluate the "inside" integral
   (e) Compute/Evaluate the "outside" integral
   (f) If possible, check answer in Mathematica

3. Polar Coordinates
   (a) $r^2 = x^2 + y^2$
   (b) $x = r\cos\theta$
   (c) $y = r\sin\theta$
   (d) $\iint\limits_R f(x,y)dA = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta)rdrd\theta$

4. Polar Coordinates Algorithm
   (a) If possible create a 3D plot of the surface over the region
       - Study diagram to determine if this is positive, negative, or mixed.
       - Establish a very rough idea of the volume of the space
   (b) Draw and Label the Region
   (c) Determine the limits of integration
       - Max and Min radial limits

- Max and Min angular limits

(d) Convert the integrand to an equivalent polar expression

(e) Set up the iterated integral

(f) Compute/Evaluate the inside integral

(g) Compute/Evaluate the outside integral

(h) Look back, does it make sense?

5. Center of Mass

(a) Draw and label the region and look at the density function $\rho(x, y)$

(b) Compute Mass where $m = \iint\limits_D \rho(x, y)dA$

(c) Compute the Moments for x and y where

$$M_x = \iint\limits_D y\rho(x, y)dA \quad M_y = \iint\limits_D x\rho(x, y)dA$$

(d) Finally compute $\overline{x}$ and $\overline{y}$ which are the exact coordinates of the center of mass.

$$\overline{x} = \frac{M_y}{m} = \frac{1}{m}\iint\limits_D x\rho(x, y)dA, \quad \overline{y} = \frac{M_x}{m} = \frac{1}{m}\iint\limits_D y\rho(x, y)dA$$

6. Triple Integrals

(a) Cubic Regions - $\int\int\int_B f(x, y, z)dV = \int_r^s \int_c^d \int_a^b f(x, y, z)dx\ dy\ dz$

(b) General Regions - $\int\int\int_E f(x, y, z)dV = \int_a^b \int_{g_1(x)}^{g_2(x)} \int_{u_1(x,y)}^{u_2(x,y)} f(x, y, z)dz\ dy\ dx$

7. Triple Integrals Cylindrical Coordinates

(a) $x = r\cos\theta \quad y = r\sin\theta \quad z = z$

(b) $r^2 = x^2 + y^2 \quad \tan\theta = \frac{y}{x} \quad z = z$

(c) $\int\int\int_E f(x, y, z)dV = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} \int_{u_1(r\cos\theta, r\sin\theta)}^{u_2(r\cos\theta, r\sin\theta)} f(r\cos\theta, r\sin\theta, z)r\ dz\ dr\ d\theta$

8. Cylindrical Algorithm

(a) If possible create a 3D plot of the region then label it.

(b) Determine the limits of integration

- Max and Min z values. Where $z_1 = u_1(x, y) = u_1(r\cos\theta, r\sin\theta)$ $z_2 = u_2(x, y) = u_2(r\cos\theta, r\sin\theta)$

  - Max and Min radial limits.
    Where $r_1 = h_1(\theta), \quad r_2 = h_2(\theta)$
  - Max and Min angular limits

(c) Convert the integrand to an equivalent cylindrical expression

(d) Set up the triple integral - where $dV = r\,dz\,dr\,d\theta$

(e) Compute/Evaluate the integrals from inside out.

(f) Look back, does it make sense?

9. Triple Integrals Spherical Coordinates

   (a) $x = \rho \sin\phi \cos\theta \quad y = \rho \sin\phi \sin\theta \quad z = \rho \cos\phi$

   (b) The distance formula gives us:

   $$\rho^2 = x^2 + y^2 + z^2$$

   (c) $\displaystyle \int\int\int_E f(x,y,z)dV = \int_c^d \int_\alpha^\beta \int_a^b f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\rho^2 \sin\phi\, d\rho\, d\theta\, d\phi$

   (d) Where E is a spherical wedge given by:

   $$E = \{(\rho,\theta,\phi) | a \le \rho \le b, \alpha \le \theta \le \beta, c \le \phi \le d\}$$

10. Spherical Coordinates Algorithm

    (a) If possible create a 3D plot of the region then label it.

    (b) Determine the limits of integration
      - Max and Min ρ values. Similar to the radius.
      - Max and Min θ angle values. The angle of rotation in the xy plane.
      - Max and Min ϕ angle values. The angle of rotation from the positive z axis.

    (c) Convert the integrand to an equivalent spherical expression

    (d) Set up the triple integral - where $\rho^2 \sin\phi\, d\rho\, d\theta\, d\phi$

    (e) Compute/Evaluate the integrals from inside out.

    (f) Look back, does it make sense?