MA153 Lesson 10

LESSON 10 - Motion in Space

9 September, 2008

Outline

1. Admin
2. Last Class
3. Arc-Length
   - Arc Length Is Total Distance
     - Total Distance
   - Last Class Homework Help
4. Motion in Space
   - Derivation of Motion is Space
   - Board Work
   - Homework Help
5. Look Forward

Admin

1. Homework 2 is due 10 September in class - The assignment is posted on the web site

Admin

2. Don't write on the back of the pages, Staple the homework together, Printed cover sheet

Admin

3. Mid Term Exam 1 is on 11 September during normal class time - Bring a formula sheet - handwritten

Admin

4. We Drop Your Lowest WPR score!

Admin

5. Permanent Mathematica Code is out. I will email it to you with directions on updating it.

Admin

6. Lesson 13 - Next Monday, will be on technical writing. You need to read all the documents under the Projects Section in preparation for Monday! This includes the Project Write-Up Guidance, and the Writing Guide. You should also look at the example MA 153 Project and Student Write-Up.

More Admin

1. Dean's Hour, 12 September, This Friday
   The day of our drop
   Guest Speaker in Robinson Auditorium
   I highly encourage you to attend - You will enjoy it!

More Admin

2. WPR I Checklist and my "Bowman Notes"

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Arc Length Is Total Distance

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$= \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2} \ = \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2 + (\frac{dz}{dt})^2}$

Total Distance

1. We know that Total Distance is : $D = R * T$ (constant rate or speed)

Total Distance

2. So $D =$speed$* T$

Total Distance

3. All we have left to do now is multiply by time

Total Distance

4. So $L = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2} dt$

Total Distance

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5. Or $L = \int_a^b |\vec{r}'(t)| dt$

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6. Clever Algebra is needed

Total Distance

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7. NIntegrate

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Homework Help

Questions? - Homework Help
FCE Question 4?

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Derivation of Motion is Space

1. $x(t) = x_0 + (v_0 \cos\alpha)t \qquad y(t) = y_0 + (v_0 \sin\alpha)t - \frac{1}{2}gt^2$

Derivation of Motion is Space

2. Or $\mathbf{r}(t) = \langle x_0 + (v_0 \cos\alpha)t, \quad y_0 + (v_0 \sin\alpha)t - \frac{1}{2}gt^2 \rangle$

Derivation of Motion is Space

2. Or $\mathbf{r}(t) = \langle x_0 + (v_0 \cos\alpha)t, \quad y_0 + (v_0 \sin\alpha)t - \frac{1}{2}gt^2 \rangle$
3. The velocity vector is the derivative of the position vector $\mathbf{v}(t) = \mathbf{r}'(t) = \langle v_0 \cos\alpha, \quad v_0 \sin\alpha - gt \rangle$

Derivation of Motion is Space

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3. The velocity vector is the derivative of the position vector $\mathbf{v}(t) = \mathbf{r}'(t) = \langle v_0 \cos\alpha, \quad v_0 \sin\alpha - gt \rangle$
4. The acceleration vector is the derivative of the velocity vector $\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t) = \langle 0, \quad -g \rangle$

Derivation of Motion is Space

5. Now go backwards using integration

Questions to Ask

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   1. The derivative of the y component $= 0$. So we take the derivative of $y(t)$ and set it equal to zero, $y'(t) = 0$. In our case $y(t) = y_0 + (v_0 \sin\alpha)t - \frac{1}{2}gt^2$ so taking the derivative with respect to t gives $y'(t) = 0 + \sin\alpha - gt$. Setting this equal to 0 and solving for t gives $t = \dfrac{\sin\alpha}{g}$

Questions to Ask

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   2. Now we have t or time when the particle is at the highest point. If we substitute this t back into our position vector we will get the x and y position at that time - giving us the highest y.

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Board Work

1. Look at Stewart, Chapter 13, section 14, Example 5, pg 873 - Mortar Fire
2. If a nine iron lofts a golf ball at an angle of 60°, how fast would you have to swing in order to hit the ball 30 feet?

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Homework Help

Homework Questions?

Look Forward

Review and launchers
Set up equations so you can solve for muzzle velocity and other key elements of projectile motion

Questions?

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