MA153 Lesson 17

LESSON 17 - Tangent Planes

19 September, 2008

Outline

1. Admin
2. Last Class
3. Tangent Planes and Linear Approximations
   - Course Guide
   - Definitions
   - Clairaut's Theorem and Laplace's Equation
   - Homework Help
4. Look Forward

Admin

1. Homework 3 is posted and due 22 September - Next Monday

Admin

2. Project Questions?

Notations For Partial Derivatives

If $z = f(x, y)$ we write

1. $f_x(x, y) = f_x = \dfrac{\partial f}{\partial x} = \dfrac{\partial}{\partial x} f(x, y) = \dfrac{\partial z}{\partial x} = f_1 = \mathbf{D}_1 f = \mathbf{D}_x f$

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2. $f_y(x, y) = f_y = \dfrac{\partial f}{\partial y} = \dfrac{\partial}{\partial y} f(x, y) = \dfrac{\partial z}{\partial y} = f_2 = \mathbf{D}_2 f = \mathbf{D}_y f$

Rules for Finding Partial Derivatives of $z = f(x, y)$

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2. To find f_y, regard x as a constant and differentiate $f(x, y)$ with respect to y.
3. To find the second or higher derivatives keep taking the derivative of the derivative.

Homework Help

Questions? - Homework Help

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Objectives

1. Understand the tangent plane's analogous relationship with the tangent line.
2. Understand the analogous relationship between approximations along the tangent plane and approximations along the tangent line.
3. Determine the tangent plane to a surface.
4. Determine the linear approximation to a function of two variables.
5. Understand what it means for a function to be differentiable in $\mathbb{R}^2$ and $\mathbb{R}^3$.
6. Understand the difference between f being differentiable and the existence of f_x and f_y.
7. Determine the total differential of a function of a function of two variables.
8. HOMEWORK PROBLEMS: 1, 4, 20, 22

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Definitions

1. Equation of a Tangent Plane:

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

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$$y - y_0 = f'(x_0)(x - x_0)$$

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1. Equation of a Tangent Plane:

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

2. Very similar to the equation of a tangent line:

$$y - y_0 = f'(x_0)(x - x_0)$$

3. Also similar to the equation of a plane:

$$\overrightarrow{n} \cdot \langle \overrightarrow{r} - \overrightarrow{r}_0 \rangle = 0$$

Board Work

1. Problem 5, Page 899

Linear Approximations

1. $$f(x,y) \approx f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

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2. Board Work - Problem 16, Page 899

Differentials

1. $$dz = f_x(x,y)dx + f_y(x,y)dy = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

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2. $$dz = f_x(x,y)(x-a) + f_y(x,y)(y-b)$$

Projectile Motion

1. Projectile Motion - Max Range

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2. The equation for range with respect to initial velocity and angle is:

Projectile Motion

1. Projectile Motion - Max Range
2. The equation for range with respect to initial velocity and angle is:
3. $$r(v_0, \theta) = \frac{2v_0^2 \cos\theta \sin\theta}{g}$$

Board Work

1. find the first partial derivative of $f(x, y) = x^4y^3 + 8x^2y$
2. find the first partial derivative of $f(x, y, z, t) = \frac{xy^2}{t+2z}$
3. find all second partial derivatives of $f(x, y) = x^3y^5 + 2x^4y$

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Clairaut's Theorem

$$f_{xy}(a, b) = f_{yx}(a, b)$$

Laplace's Equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

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Homework Help

Look Forward

The Chain Rule - 14.5

1. Use the chain rule to differentiate a function of several variables.
2. Use tree diagrams to represent the chain rule.
3. HOMEWORK PROBLEMS: 1, 5, 11, 23, 42

Questions?

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