MA153 Lesson 19

LESSON 19 - Directional Derivatives and the Gradient Vector

23 September, 2008

Outline

1. Admin
2. Last Class
   - Chain Rule
3. Directional Derivatives and the Gradient Vector
   - Course Guide
   - Definitions
   - Examples
   - Homework Help
4. Look Forward

Admin

1. Project Questions?

Admin

2. Guest Lecture - Friday, 26 September, Dean's Hour, Washington Hall 5006

Admin

3. Next week - IT Lab Wednesday and Thursday - Project Comp Day Drop Friday

Admin

4. Guess what today is!

Admin

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5. White Chocolate Day!

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Chain Rule

1. $$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

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3. $$\frac{\partial u}{\partial t_i} = \frac{\partial u}{\partial x_1}\frac{\partial x_1}{\partial t_i} + \frac{\partial u}{\partial x_2}\frac{\partial x_2}{\partial t_i} + \ldots + \frac{\partial u}{\partial x_n}\frac{\partial x_n}{\partial t_i}$$

Chain Rule

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4. $$\frac{dy}{dx} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}} = -\frac{F_x}{F_y}$$

Homework Help

Questions? - Homework Help

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Objectives

Directional Derivatives and the Gradient Vector I - 14.6

1. Understand the analytic formulations and geometric meaning of the directional derivative.
2. Determine the directional derivative of a function of two or more variables.
3. Understand the geometric meanings of the gradient vector: A normal vector to a surface, the direction of greatest change, a perpendicular vector to contour curves and surfaces, a vector with length equal to the maximum value of the directional derivative.
4. Determine the gradient vector of a function of two or more variables.
5. Determine the tangent plane to a level surface.
6. HOMEWORK PROBLEMS: 6, 8, 11

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Definitions

1. Definition 2: The directional derivative of f at (x_0, y_0) in the direction of a unit vector $\mathbf{u} = \langle a, b \rangle$ is

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2. $$D_u f(x_0, y_0) = \lim_{h \to 0} \frac{f(x_0 + ha, y_0 + hb) - f(x_0, y_0)}{h}$$

Definitions

1. Definition 3: If f is a differentiable function of x and y, then f has a directional derivative in the direction of any unit vector $\mathbf{u} = \langle a, b \rangle$ and

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2. $$D_u f(x, y) = f_x(x, y)a + f_y(x, y)b$$

Definitions

1. Definition 3: If f is a differentiable function of x and y, then f has a directional derivative in the direction of any unit vector $\mathbf{u} = \langle a, b \rangle$ and
2. $$D_u f(x, y) = f_x(x, y)a + f_y(x, y)b$$
3. Board Work - Problem 4, Page 920

Definitions

1. Definition 8: If f is a function of two variables x and y, then the **gradient** of f is the vector function ∇f defined by

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2. $$\nabla f(x, y) = \langle f_x(x, y), f_y(x, y) \rangle = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j}$$

Definitions

3. So we can re-write our Directional Derivative as:

$$D_u f(x, y) = \nabla f(x, y) \cdot \overrightarrow{u}$$

Definitions

4. This of course can be extended into functions of three variables

Definitions

5. Board Work - Problem 10, Page 920

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Mathematica Examples

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Homework Help

Look Forward

Directional Derivatives and the Gradient Vector II - 14.6

1. Understand the analytic formulations and geometric meaning of the directional derivative.
2. Determine the directional derivative of a function of two or more variables.
3. Understand the geometric meanings of the gradient vector: A normal vector to a surface, the direction of greatest change, a perpendicular vector to contour curves and surfaces, a vector with length equal to the maximum value of the directional derivative.
4. Determine the gradient vector of a function of two or more variables.
5. Determine the tangent plane to a level surface.
6. HOMEWORK PROBLEMS: 23,30,36

Questions?

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