MA153 Lesson 2

LESSON 2 - Vectors and the Dot Product

Monday, 25 August, 2008

Outline

1. Admin
2. Last Class
   - Do Problem Help
3. INTRODUCTION TO VECTOR AND VECTOR OPERATIONS
   - Course Guide
   - Vectors
4. Dot Product
   - Course Guide
   - Definitions
   - Dot Product
   - Projections
5. Look Forward

Admin

1. Schedule an interview - Use outlook!

Admin

2. Homework 1 is due 2 September in class - The assignment is posted on the web site

3D

Questions?

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Section 12.1/ 2, 5, 8, 10, 19, 33, 39

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Objectives

Vectors (12.2)

1. Determine the vector AB..... between two points.
2. Determine the length (magnitude) of a vector.
3. Determine the resultant of vector addition and scalar multiplication.
4. Understand scalar multiplication and how it relates to parallel vectors.
5. Understand the properties of vectors.
6. Determine the unit vector in a given direction.
7. Determine the resultant force of several forces.

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Vectors

1. Vectors have Magnitude and Direction

Vectors

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2. Vectors are annotated a few ways:
   1. $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ the letter can be bold (hard to see on the chalk board or in hand writing)
   2. $\overrightarrow{a} = \langle a_1, a_2, a_3 \rangle$ an arrow over the letter (much easier for board and paper work)

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3. Vector Addition

Vectors

1. Vectors have Magnitude and Direction

3. Vector Addition
4. Scaler Multiplication

Vectors

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4. Scaler Multiplication
5. Components of Vectors

Vectors

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5. Components of Vectors
6. The vector $\overrightarrow{a} = \overrightarrow{AB}$

Vectors

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6. The vector $\overrightarrow{a} = \overrightarrow{AB}$
7. Length or Magnitude of a vector

Vectors

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7. Length or Magnitude of a vector
8. Properties of Vectors

Unit Vectors

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2. Unit vectors are annotated by hand with a "hat" - $\hat{u}$
3. Common unit vectors are $\hat{i}, \hat{j}, \hat{k}$ describing the standard basis vector. Any vector multiplied by one of these unit vectors will result a vector pointing in the positive x, y, z direction.

Unit Vectors

1. A unit vector has direction but its magnitude is always 1
2. Unit vectors are annotated by hand with a "hat" - $\hat{u}$

4. So vectors can also be described as $\overrightarrow{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

Unit Vectors

1. A unit vector has direction but its magnitude is always 1
2. Unit vectors are annotated by hand with a "hat" - $\hat{u}$

5. A unit vector can be found by $\overrightarrow{u} = \dfrac{\overrightarrow{a}}{|\overrightarrow{a}|}$

Board Work and Homework Help

Do problem 12.2.26 page 777 from the book
Homework Help

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Objectives

Dot Product (Stewart 12.3)

1. Understand the definition of the dot product.
2. Determine the dot product of two vectors using both the algebraic and geometric formulations of the dot product.
3. Determine the angle between two vectors.
4. Understand how the sign of $\overrightarrow{a} \cdot \overrightarrow{b}$ relates to the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$.
5. Understand how the dot product relates to work.
6. Understand the concept of orthogonality.
7. Determine the scalar projection of $\overrightarrow{b}$ onto $\overrightarrow{a}$.
8. Determine the vector projection of $\overrightarrow{b}$ onto $\overrightarrow{a}$.

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Definitions, Dot Product

1. Definition 1, Page 779
2. Properties of a Dot Product, Page 779
3. Theorem 3, Page 780
4. Corollary 6, Page 780
5. Page 781
6. Projections, Page 783

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Dot Product

1. Can we multiply two vectors? Of course we can, we get the Dot Product.
2. If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then the **dot product** of $\mathbf{a}$ and $\mathbf{b}$ is the number $\mathbf{a} \cdot \mathbf{b}$ given by

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

3. So we multiply corresponding components and add. The result is a number not a vector. It is a real number known as a scaler.
4. The five properties of the Dot Product are given in your text on page 779
5. Dot product could be seen as a vendor at a football game.

Dot Product Angle

1. The dot product $\mathbf{a} \cdot \mathbf{b}$ can be given a geometric interpretation in terms of the **angle θ between a and b**.
2. If θ is the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$, then

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$$

3. If θ is the angle between the nonzero vectors $\mathbf{a}$ and $\mathbf{b}$, then

$$\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

4. Two vectors $\mathbf{a}$ and $\mathbf{b}$ are orthogonal if and only if $\mathbf{a} \cdot \mathbf{b} = 0$.

Board Work

1. Do 12.3.14
2. Do 12.3.20

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Projections

1. Projections can be seen as the shadow of a vector onto another.
2. Scalar projection of $\mathbf{b}$ onto $\mathbf{a}$: is a number $\text{comp}_a\mathbf{b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}$
3. Vector projection of $\mathbf{b}$ onto $\mathbf{a}$: is a vector
$$\text{proj}_a\mathbf{b} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}\right)\frac{\mathbf{a}}{|\mathbf{a}|} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\,\mathbf{a}$$
4. To find work $W = \mathbf{F}\cdot\mathbf{D} = |\mathbf{F}||\mathbf{D}|\cos\theta$

Board Work

1. Do 12.3.46

Look Forward

Cross Product (Stewart 12.4)

1. Understand the definition of the cross product.
2. Determine the cross product of two vectors.
3. Understand that the resultant vector of the cross product is orthogonal to the two given vectors.
4. Understand how the cross product relates to the area of the parallelogram determined by the two given vectors.
5. Understand the right-hand rule.
6. Understand how the cross product relates to torque.
7. Understand the definition of the scalar triple product.
8. Understand how the scalar triple product relates to volume.

Questions?

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