MA153 Lesson 20

LESSON 20 - Directional Derivatives and the Gradient Vector II

24 September, 2008

Outline

1. Admin
2. Last Class
   - Directional Derivatives and the Gradient Vector
3. Directional Derivatives and the Gradient Vector II
   - Course Guide
   - Definitions
   - Examples
   - Homework Help
4. Look Forward

Admin

1. Project Questions?

Admin

2. Guest Lecture - Friday, 26 September, Dean's Hour, Washington Hall 5006

Admin

3. Next week - IT Lab Wednesday and Thursday - Project Comp Day Drop Friday

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Directional Derivatives and the Gradient Vector

1. $$D_u f(x_0, y_0) = \lim_{h \to 0} \frac{f(x_0 + ha, y_0 + hb) - f(x_0, y_0)}{h}$$

Directional Derivatives and the Gradient Vector

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2. $$D_u f(x, y) = f_x(x, y)a + f_y(x, y)b$$

3. $$\nabla f(x, y) = \langle f_x(x, y), f_y(x, y) \rangle = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j}$$

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4. So we can re-write our Directional Derivative as:
$$D_u f(x, y) = \nabla f(x, y) \cdot \overrightarrow{u}$$

Homework Help

Questions? - Homework Help

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Objectives

Directional Derivatives and the Gradient Vector II - 14.6

1. Understand the analytic formulations and geometric meaning of the directional derivative.
2. Determine the directional derivative of a function of two or more variables.
3. Understand the geometric meanings of the gradient vector: A normal vector to a surface, the direction of greatest change, a perpendicular vector to contour curves and surfaces, a vector with length equal to the maximum value of the directional derivative.
4. Determine the gradient vector of a function of two or more variables.
5. Determine the tangent plane to a level surface.
6. HOMEWORK PROBLEMS: 23,30,36

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Definitions

1. Definition 2: The directional derivative of f at (x_0, y_0) in the direction of a unit vector $\mathbf{u} = \langle a, b \rangle$ is

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2. $$D_u f(x_0, y_0) = \lim_{h \to 0} \frac{f(x_0 + ha, y_0 + hb) - f(x_0, y_0)}{h}$$

Definitions

1. Suppose f is a differentiable function of two or three variables. The maximum value of the directional derivative $D_u f(\overrightarrow{x})$ is $|\nabla f(\overrightarrow{x})|$ and it occurs when $\overrightarrow{u}$ has the same direction as the gradient vector $\nabla f(\overrightarrow{x})$.

Definitions

1. Tangent Planes to Level Surfaces

Definitions

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$$F_x(x_0, y_0, z_0)(x - x_0) + F_y(x_0, y_0, z_0)(y - y_0) + F_z(x_0, y_0, z_0)(z - z_0) = 0$$

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Mathematica Examples

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Homework Help

Look Forward

Maximum and Minimum - 14.7

1. Understand the contrast between single variable optimization problems (relatively few) and multi-variable optimization (many possible solutions).
2. Determine the critical points of a function.
3. Classify the critical points of a function using the Second Derivative Test.
4. Understand the difference between local and absolute extreme points.
5. Determine the absolute maximum and minimum values of a function of two variables.
6. HOMEWORK PROBLEMS: 3, 6, 39, 44

Questions?

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