MA153 Lesson 23

LESSON 23 - Optimization and Lagrange

30 September, 2008

Outline

1. Admin
2. Last Class
   - Min and Max
   - Optimization
3. Optimization and Lagrange Multipliers
   - Course Guide
   - Steps for LaGrange Multipliers
   - Board Work
   - Examples
   - Homework Help
4. Look Forward

Birthday Cadet

Who's Birthday is It?

Birthday Cadet

Who's Birthday is It?

1. This cadet is from a city in New Jersey

Birthday Cadet

Who's Birthday is It?

2

Birthday Cadet

Who's Birthday is It?

3. Yes its Ridgewood New Jersey

Birthday Cadet

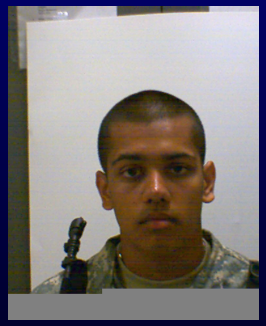
1.

Birthday Cadet

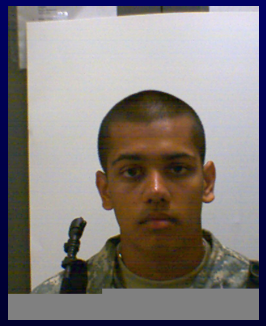
2. This cadet learned French while he was a French school in Canada

Birthday Cadet

3



Birthday Cadet

3. 
4. Avinash Chaurasia is 18 on Friday, 3 October!

Admin

1. Project Questions?

Admin

2. This week - IT Lab Wednesday and Thursday - Project Comp Day Drop Friday
   Next Week - PSL Monday, WPR Tuesday

Admin

3. Turn in Homework 4 before the end of your normal class period on Friday 3 October

Admin

4. Still a few students who have not come by for initial AI

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Critical Points, Second Derivative Test, Min and Max

1. **Step 1** Find partial derivatives and set equal to zero - solve the system of equations to find all critical points.

$$\nabla f(a,b) = \langle f_x, f_y \rangle = \langle 0, 0 \rangle$$

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   1. If f has a local maximum or minimum at (a,b) and the first-order partial derivatives of f exist there, then $f_x(a,b) = 0$ and $f_y(a,b) = 0$.

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   2. So we get critical points any where all the partial derivatives equal zero. Because the rate of change is zero.

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   3. So we find the second partial derivatives and the mixed partial. We then need to test $D(x,y)$ at each of our critical points.

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      2. If $D(x,y) > 0$ and $f_{xx}(x,y) < 0$ we have a maximum.
      3. If $D(x,y) < 0$ we have a possible saddle point. As long as $D \neq 0$ we have a saddle point.

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   $\nabla f(a,b) = \langle f_x, f_y \rangle = \langle 0, 0 \rangle$
2. **Step 2** Classify Critical Points using $D = f_{xx}f_{yy} - (f_{xy})^2$

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   1. Now we want to get the maximum and minimum values the function achieves on the boundary.

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3. **Step 3** Check the function on the boundary.
4. **Step 4** Compare our Values on the boundary to our values at the critical points
   1. Find the values of the function at the critical points.
   2. Compare the function values at the critical points to those on the boundary. The biggest is your global max, the smallest your global minimum.

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Optimization

These steps are similar to many other models we have done. If we want a box without a top to have the maximum volume while using only 12 inches of cardboard we do the following:

1. **Step 1** Find the Objective Equation - The equation we want to maximize or minimize. $V = xyz$.

Optimization

These steps are similar to many other models we have done. If we want a box without a top to have the maximum volume while using only 12 inches of cardboard we do the following:

1. **Step 1** Find the Objective Equation - The equation we want to maximize or minimize. $V = xyz$.
2. **Step 2** Find the Constraint Equation - Our limitations. Our Surface Area has to satisfy $2xz + 2yz + xy = 12$.

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These steps are similar to many other models we have done. If we want a box without a top to have the maximum volume while using only 12 inches of cardboard we do the following:

1. **Step 1** Find the Objective Equation - The equation we want to maximize or minimize. $V = xyz$.
2. **Step 2** Find the Constraint Equation - Our limitations. Our Surface Area has to satisfy $2xz + 2yz + xy = 12$.
3. **Step 3** Use the Constraint Equation to reduce our Objective Equation to an equation of only two variables. $z = \frac{12-xy}{2(x+y)}$

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4. **Step 4** Find the Critical Points of the new Objective Equation by taking the partial derivatives and setting them equal to zero then solving the system of equations. Our only positive critical point is $(2, 2)$
5. **Step 5** Use the Second Derivative Test to determine the classification of the critical points. In our case we get $D > 0$ and $f_{xx} < 0$ proving we have a maximum.

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5. **Step 5** Use the Second Derivative Test to determine the classification of the critical points. In our case we get $D > 0$ and $f_{xx} < 0$ proving we have a maximum.
6. **Step 6** Test the Critical Points in the function to determine the Max or Min you are seeking. In our case we get $f(2, 2) = 4$

Homework Help

Questions? - Homework Help

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Objectives

Lagrange Multipliers - 14.8

1. Understand the geometric justification for the method of Lagrange multipliers.
2. Use the method of Lagrange multipliers to determine the absolute maximum and minimum values of a function of two variables subject to one constraint.
3. Use the method of Lagrange multipliers to determine the absolute maximum and minimum values of a function of three variables subject to two constraints.
4. Model and analyze problems using Lagrange multipliers.
5. HOMEWORK PROBLEMS: 1, 3, 10

Motivation for Lagrange Multipliers

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Steps for LaGrange Multipliers

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Steps for LaGrange Multipliers

1. Find the Objective Equation - The equation we want to maximize or minimize. $V = xyz$.
2. Find the Constraint Equation - Our limitations. Our Surface Area has to satisfy $2xz + 2yz + xy = 12$.
3. Find the gradient of both the Objective and Constraint Equation $\nabla Ob, \quad \nabla Co$ by taking the partial derivatives and putting them in vector notation. $\nabla Ob(x, y, z) = \langle yz, xz, xy \rangle$, $\nabla Co(x, y, z) = \langle 2z + y, 2z + x, 2x + 2y \rangle$

Steps for LaGrange Multipliers

1. Find the Objective Equation - The equation we want to maximize or minimize. $V = xyz$.
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4. Set $\nabla Ob(x, y, z) = \lambda Co(x, y, z)$. $\langle yz, xz, xy \rangle = \lambda \langle 2z + y, 2z + x, 2x + 2y \rangle$

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5. Solve the equations $\nabla Ob(x, y, z) = \lambda Co(x, y, z)$ and $Co(x, y, z) = k$ for all values of x, y, z, λ by solving the system of four equations and four unknowns.

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6. Evaluate Ob at all the points (x, y, z). The largest of these values is the max; the smallest is the minimum.

MA 153 Block II Review

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Board Work

1. Problem 6, Page 940 -

$$f(x,y) = e^{xy}; \quad x^3 + y^3 = 16$$

2. Problem 11, Page 940 -

$$f(x,y) = x^2 + y^2 + z^2; \quad x^4 + y^4 + z^4 = 1$$

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Mathematica Examples

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Homework Help

Look Forward

More Lagrange Multipliers - 14.8

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3. Use the method of Lagrange multipliers to determine the absolute maximum and minimum values of a function of three variables subject to two constraints.
4. Model and analyze problems using Lagrange multipliers.
5. HOMEWORK PROBLEMS: 20, 25, 36

Questions?

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