

MA153

Board Sheet - 16 October 2007, Polar Coordinates

Polar Coordinates

1. $r^2 = x^2 + y^2$
2. $x = r \cos \theta$
3. $y = r \sin \theta$
4.
$$\iint_R f(x, y) dA = \int_{\alpha}^{\beta} \int_a^b f(r \cos \theta, r \sin \theta) r dr d\theta$$

Polar Coordinates Algorithm

1. If possible create a 3D plot of the surface over the region
 - Study diagram to determine if this is positive, negative, or mixed.
 - Establish a very rough idea of the volume of the space
2. Draw and Label the Region
3. Determine the limits of integration
 - Max and Min radial limits
 - Max and Min angular limits
4. Convert the integrand to an equivalent polar expression
5. Set up the iterated integral
6. Compute/Evaluate the inside integral
7. Compute/Evaluate the outside integral
8. Look back, does it make sense?

The Big Brew Question

1. Major Bowman is going to brew a batch of beer and has some calculations to make to decide if his equipment and supplies are adequate. Major Bowman has a yeast culture in a nutrient package and wants to know what the volume of the package is to determine if the yeast will have enough nutrients. The package looks like a round bag sealed on each end and all blown up inside from the yeast activity. The bag's top layer can be described as $z = 8 - x^2 - y^2$ and the bottom layer as $z = x^2 + y^2$. Graph the bag and determine the volume of the bag. Units are in inches.
2. Major Bowman pours the culture of yeast precisely in the middle of a square fermenting vessel with each side of length 4 feet. The yeast culture was placed at the point $(0,0)$, and yeast multiplies and disperses in such a manner that the concentration at any point (x,y) in the vessel is given by $C(x,y) = 1000000(300 - 3x^2 - 3y^2)$, where $C(x,y)$ is the number of yeast cells per square foot of surface of beer (x,y) in the vessel. Find the total concentration of yeast and the average concentration of yeast on the surface of the beer and plot the concentration as density or contour over the surface of the fermenter.
3. Major Bowman brews his beer near the laundry room and Cindy Bowman, Major Bowman's wife, does laundry every Monday. The Monday after brewing beer Cindy accidentally dropped baking soda on the surface of the square fermenting vessel. Fortunately she only dropped a small amount on the surface of the beer and the yeast only died where she dropped the baking soda. The baking soda fell in a pattern following the equations $y = 2 - \frac{1}{2}x^2$, $(-2 \leq x \leq 2)$ on the top half of the fermenter surface and $y = -2 - x$, $(-2 \leq x < 0)$ and $y = x - 2$, $(0 \leq x \leq 2)$ on the bottom half of the fermenter surface. Find the total concentration of yeast if the yeast disperses as before and was placed in the middle of the fermenter. Again give a plot of the concentration as density or contour over the surface of the fermenter.
4. The next step in the brewing process is to transfer the beer from the square fermenter to a cylindrical fermenter called a carboy whose surface can be described by $x^2 + y^2 \leq (\frac{10.5}{2})^2$. When this is done the baking soda is removed so the yeast covers the entire surface and has turned most of the sugar in the wort into alcohol. However, the yeast needs another week to complete the process. The concentration of yeast cells is now given by $C(x,y) = 1000000(300 - 5x^2 - 5y^2)$. Find the total concentration of yeast and plot it as a density or contour plot.

