MA153 Lesson 3

LESSON 3 - Dot Product and Cross Product

Tuesday, 26 August, 2008

Outline

1. Admin
2. Last Class
   - Do Problem Help
3. Dot Product
   - Course Guide
   - Definitions
   - Dot Product
   - Projections
4. Cross Product
   - Course Guide
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   - Cross Product
5. Look Forward

Admin

1. Schedule an interview - Use outlook! - Populate your calendar.

Admin

2. Homework 1 is due 2 September in class - The assignment is posted on the web site

Admin

3. No Class on Friday! We have a drop! Don't show up to this class!

Vectors

Questions?

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Do Problem Help

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Objectives

Dot Product (Stewart 12.3)

1. Understand the definition of the dot product.
2. Determine the dot product of two vectors using both the algebraic and geometric formulations of the dot product.
3. Determine the angle between two vectors.
4. Understand how the sign of $\overrightarrow{a} \cdot \overrightarrow{b}$ relates to the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$.
5. Understand how the dot product relates to work.
6. Understand the concept of orthogonality.
7. Determine the scalar projection of $\overrightarrow{b}$ onto $\overrightarrow{a}$.
8. Determine the vector projection of $\overrightarrow{b}$ onto $\overrightarrow{a}$.

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Definitions, Dot Product

1. Definition 1, Page 779
2. Properties of a Dot Product, Page 779
3. Theorem 3, Page 780
4. Corollary 6, Page 780
5. Page 781
6. Projections, Page 783

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Dot Product

1. Can we multiply two vectors? Of course we can, we get the Dot Product.
2. If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then the **dot product** of $\mathbf{a}$ and $\mathbf{b}$ is the number $\mathbf{a} \cdot \mathbf{b}$ given by

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

3. So we multiply corresponding components and add. The result is a number not a vector. It is a real number known as a scaler.
4. The five properties of the Dot Product are given in your text on page 779
5. Dot product could be seen as a vendor at a football game.

Dot Product Angle

1. The dot product $\mathbf{a} \cdot \mathbf{b}$ can be given a geometric interpretation in terms of the **angle θ between a and b**.
2. If θ is the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$, then

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$$

3. If θ is the angle between the nonzero vectors $\mathbf{a}$ and $\mathbf{b}$, then

$$\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

4. Two vectors $\mathbf{a}$ and $\mathbf{b}$ are orthogonal if and only if $\mathbf{a} \cdot \mathbf{b} = 0$.

Board Work

1. Do 12.3.14
2. Do 12.3.20

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Projections

1. Projections can be seen as the shadow of a vector onto another.
2. Scalar projection of $\mathbf{b}$ onto $\mathbf{a}$: is a number $\text{comp}_a\mathbf{b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}$
3. Vector projection of $\mathbf{b}$ onto $\mathbf{a}$: is a vector
$$\text{proj}_a\mathbf{b} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}\right)\frac{\mathbf{a}}{|\mathbf{a}|} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\,\mathbf{a}$$
4. To find work $W = \mathbf{F}\cdot\mathbf{D} = |\mathbf{F}||\mathbf{D}|\cos\theta$

Board Work

1. Do 12.3.46
2. Page 785, Problem 38
3. Page 785, Problem 40

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Objectives

Cross Product (Stewart 12.4)

1. Understand the definition of the cross product.
2. Determine the cross product of two vectors.
3. Understand that the resultant vector of the cross product is orthogonal to the two given vectors.
4. Understand how the cross product relates to the area of the parallelogram determined by the two given vectors.
5. Understand the right-hand rule.
6. Understand how the cross product relates to torque.
7. Understand the definition of the scalar triple product.
8. Understand how the scalar triple product relates to volume.

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Definitions, Cross Product

1. Cross Product, Page 786
2. Theorem 5, Page 788
3. Theorem 6, Page 788
4. Corollary 7, Page 789

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Cross Product

1. The **cross product** $\mathbf{a} \times \mathbf{b}$ is a vector and only works when the vector are three-dimensional.
2. The vector $\mathbf{a} \times \mathbf{b}$ is perpendicular or orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.
3. If θ is the angle between $\mathbf{a}$ and $\mathbf{b}$, then

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin \theta$$

4. Two nonzero vectors $\mathbf{a}$ and $\mathbf{b}$ are parallel if and only if

$$\mathbf{a} \times \mathbf{b} = \mathbf{0}$$

5. The length of the cross product $\mathbf{a} \times \mathbf{b}$ is equal to the area of the parallelogram determined by $\mathbf{a}$ and $\mathbf{b}$.

Look Forward

Equations of Lines and Planes (Stewart 12.5)

1. Use vector, scalar, and symmetric equations to express a line.
2. Use vectors to parameterize a line segment between two points.
3. Use vector, scalar, and linear equations to express a plane.
4. Determine the angle between two intersecting planes.
5. Determine the point of intersection of a line and a plane.
6. Determine the equation for the line of intersection of two planes.
7. Determine the distance from a point to a plane.
8. Determine the distance between two parallel planes.

Questions?

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