

# MA153

Board Sheet - 17 October 2008, Center of Mass

## Step by Step COM

1. Draw and label the region and look at the density function  $\rho(x, y)$

2. Compute Mass where  $m = \iint_D \rho(x, y) dA$

3. Compute the Moments for x and y where

$$M_x = \iint_D y\rho(x, y) dA \quad M_y = \iint_D x\rho(x, y) dA$$

4. Finally compute  $\bar{x}$  and  $\bar{y}$  which are the exact coordinates of the center of mass.

$$\bar{x} = \frac{M_y}{m} = \frac{1}{m} \iint_D x\rho(x, y) dA, \quad \bar{y} = \frac{M_x}{m} = \frac{1}{m} \iint_D y\rho(x, y) dA$$

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rho[x_, y_] := y
c[x_] := 0
d[x_] := E^x
a[y_] := 0
b[y_] := 1
mass = Integrate[rho[x, y], {x, c[y], d[x]}, {y, a[y], b[y]}]
xbar = 1/mass Integrate[x * rho[x, y], {x, c[y], d[x]}, {y, a[y], b[y]}]
ybar = 1/mass Integrate[y * rho[x, y], {x, c[y], d[x]}, {y, a[y], b[y]}]
Dens = DensityPlot[1 / (1 + rho[x, y]), {x, a[y], b[y]},
{y, c[x], d[x]}, Mesh -> 10, ColorFunction -> "BlueGreenYellow"];
COMDens = ListPlot[{{xbar, ybar}}, PlotStyle -> {PointSize[Large], Red}] ;
Show[Dens, COMDens]

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