MA153 Lesson 33

LESSON 33 - Triple Integrals - Spherical Coordinates

23 RockTober, 2008

Outline

1. Admin
2. Last Class
   - Triple Integrals - Cylindrical Coordinates
   - Cylindrical Coordinates Algorithm
   - Homework Help
3. Triple Integrals - Spherical Coordinates
   - Course Guide
   - Converting from and to Spherical Coordinates
   - Integrating with Spherical Coordinates
   - Spherical Coordinates Algorithm
   - Board Work
   - Homework Help
4. Look Forward

Birthday Cadet

Who's Birthday is It?

Birthday Cadet

Who's Birthday is It?

1. This cadet is from Belgrade

Birthday Cadet

Who's Birthday is It?

2

Birthday Cadet

Who's Birthday is It?

3. Yes its Belgrade, Serbia

Birthday Cadet

1.

Birthday Cadet

2. This Cadet has probably shot more different types of weapons than other cadets in this class.

Birthday Cadet

3

Birthday Cadet

3.
4. Branko Kovacevic is 19 Today, 23 October!

Admin

1. This week - Triple Integrals and a Review
   Next Week - WPR III on Monday, 27 October, in class.
   Tuesday WE HAVE CLASS, not a drop, Vector Fields.
   Wednesday - Line Integrals I. Thursday - Line Integrals II.
   Friday, Halloween, we have a drop, I travel with the wrestling team.

Admin

2. Homework 6 is due this Friday, 24 October, in class

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Triple Integrals - Cylindrical Coordinates

1. $x = r\cos\theta \quad y = r\sin\theta \quad z = z$

Triple Integrals - Cylindrical Coordinates

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2. $r^2 = x^2 + y^2 \quad \tan\theta = \frac{y}{x} \quad z = z$

Triple Integrals - Cylindrical Coordinates

1. $x = r\cos\theta \quad y = r\sin\theta \quad z = z$
2. $r^2 = x^2 + y^2 \quad \tan\theta = \frac{y}{x} \quad z = z$
3. $\int\int\int_E f(x,y,z)dV =$
$$\int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} \int_{u_1(r\cos\theta, r\sin\theta)}^{u_2(r\cos\theta, r\sin\theta)} f(r\cos\theta, r\sin\theta, z) r \; dz \; dr \; d\theta$$

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Cylindrical Coordinates

1. If possible create a 3D plot of the region then label it.

Cylindrical Coordinates

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2. Determine the limits of integration

Cylindrical Coordinates

1. If possible create a 3D plot of the region then label it.
2. Determine the limits of integration
   - Max and Min z values. Where
     $z_1 = u_1(x, y) = u_1(r\cos\theta, r\sin\theta)$
     $z_2 = u_2(x, y) = u_2(r\cos\theta, r\sin\theta)$

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   - Max and Min radial limits.
     Where $r_1 = h_1(\theta), \quad r_2 = h_2(\theta)$

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     $z_2 = u_2(x, y) = u_2(r\cos\theta, r\sin\theta)$
   - Max and Min radial limits.
     Where $r_1 = h_1(\theta), \quad r_2 = h_2(\theta)$
   - Max and Min angular limits

Cylindrical Coordinates

1. If possible create a 3D plot of the region then label it.
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3. Convert the integrand to an equivalent cylindrical expression

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4. Set up the triple integral - where $dV = r\ dz\ dr\ d\theta$

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5. Compute/Evaluate the integrals from inside out.

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Homework Help

Questions

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Course Guide

Triple Integrals in Spherical Coordinates - 15.8

1. Understand the geometry of the spherical coordinate system.
2. Understand the basic shapes of solids in spherical coordinates.
3. Understand that some triple integrals are simpler to compute in spherical coordinates.
4. Convert from rectangular to spherical coordinates in a triple integral.
5. HOMEWORK PROBLEMS: 1, 10, 17

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Converting from and to Spherical Coordinates

1. $x = \rho \sin\phi \cos\theta \quad y = \rho \sin\phi \sin\theta \quad z = \rho \cos\phi$

Converting from and to Spherical Coordinates

1. $x = \rho \sin\phi \cos\theta \quad y = \rho \sin\phi \sin\theta \quad z = \rho \cos\phi$
2. The distance formula gives us:

$$\rho^2 = x^2 + y^2 + z^2$$

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Integrating with Spherical Coordinates

1. $$\int\int\int_E f(x,y,z)dV = \int_c^d \int_\alpha^\beta \int_a^b f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\rho^2\sin\phi\ d\rho\ d\theta\ d\phi$$

Integrating with Spherical Coordinates

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2. Where E is a spherical wedge given by:

$$E = \{(\rho,\theta,\phi)|a \leq \rho \leq b, \alpha \leq \theta \leq \beta, c \leq \phi \leq d\}$$

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   - Max and Min θ angle values. The angle of rotation in the *xy* plane.

Spherical Coordinates Algorithm

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2. Determine the limits of integration
   - Max and Min ρ values. Similar to the radius.
   - Max and Min θ angle values. The angle of rotation in the *xy* plane.
   - Max and Min ϕ angle values. The angle of rotation from the positive *z* axis.

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Board Work

1. Problem 2, Page 1010

Board Work

2. Problem 3, Page 1010

Board Work

3. Problem 9, Page 1010

Board Work

4. Problem 22, Page 1010

Board Work

5. Problem 18, Page 1010

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Look Forward

Review - Look at the WPR III checklist and my Bowman Notes
Come with questions

Questions?

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