MA153 Lesson 37

LESSON 37 - Line Integrals I

29 RockTober, 2008

Outline

1. Admin
   - WPR III
2. Block IV - Chapter 16
3. Last Class
   - Vector Fields
   - Homework Help
4. Line Integral I
   - Course Guide
   - Definitions and Derivations
   - Homework Help
5. Look Forward

Admin

1. This week - Line Integrals I and II Today and Thursday, Drop on Friday

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1. This week - Line Integrals I and II Today and Thursday, Drop on Friday
2. Next Week - Line Integrals III Monday, Greens Theorem Tuesday and Wednesday, Drop Thursday, Guest Lecture Friday

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WPR III

1. WPR III - Course Stats
2. Class Stats

Block IV - Chapter 16

1. Vector Fields - 16.1: Functions that assign vectors to points in space.

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1. Vector Fields - 16.1: Functions that assign vectors to points in space.
2. Line Integrals - 16.2: Integrating over a curve.

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6. Parametric Surfaces and their Area - 16.6:
7. Surface Integrals - 16.7:

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Vector Fields

1. Definition 1 - Page 1028 -
$F(x, y) = P(x, y)\,\widehat{i} + Q(x, y)\,\widehat{j} = \langle P(x, y), Q(x, y)\rangle$

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2. Definition 2 - Page 1028 -
$F(x, y, z) = P(x, y, z)\,\widehat{i} + Q(x, y, z)\,\widehat{j} + R(x, y, z)\,\widehat{k} = \langle P(x, y, z), Q(x, y, z), R(x, y, z)\rangle$

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3. Gradient Fields - Page 1031 -
$\nabla f(x,y,z) = f_x(x,y,z)\,\widehat{i} + f_y(x,y,z)\,\widehat{j} + f_z(x,y,z)\,\widehat{k}$

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Homework Help

Cool? Line Integrals

Wikipedia

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Course Guide

Line Integrals I - 16.2

1. Understand the meaning of the line integral of a scalar function $f(x, y)$ along a curve C.
2. Understand the meaning of $\int Pdx + Qdy$ along a curve C.
3. Understand the relationship between line integrals and arc length.
4. Determine the line integral with respect to arc length of a multi-variable function.
5. Determine the line integral with respect to the variables of a multi-variable function.
6. Determine the line integral of a vector field.
7. Understand how line integrals relate to work.
8. HOMEWORK PROBLEMS: 1, 3, 10, 17

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Definitions and Derivations

1. 1 - Page 1034 - Start with a plane curve C given by the parametric equations

$$x = x(t) \quad y = y(t) \quad a \leq t \leq b$$

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2. Definition 2 - As long as f is defined on a smooth curve C given by Equations 1, then the **line integral of f along C** is given by:

$$\int_C f(x, y) ds$$

Definitions and Derivations

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2. $\int_C f(x, y) ds$
3. We learned that the length - arc length of C is

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

Definitions and Derivations

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Definitions and Derivations

1. $\int_C f(x,y)ds$
2. $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
3. Similar arguments help us define the line integral as the following:

$$\int_C f(x,y)ds = \int_a^b f(x(t),y(t))\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

Board Work

1. Problem 2, Page 1043

More Definitions and Derivations

1. What if we want to parameterize with something other than t?

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2. Do a similar process, but instead:

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3. We may also have to remember the equation of a line segment:

$$\overrightarrow{r}(t) = (1-t)\overrightarrow{r_0} + t\overrightarrow{r_1}$$

Board Work

1. Problem 4, Page 1043
2. Problem 6, Page 1043
3. Problem 18, Page 1044

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Homework Help

Look Forward

Line Integrals II - 16.2

1. Understand the meaning of the line integral of a scalar function $f(x, y)$ along a curve C.
2. Understand the meaning of $\int Pdx + Qdy$ along a curve C.
3. Understand the relationship between line integrals and arc length.
4. Determine the line integral with respect to arc length of a multi-variable function.
5. Determine the line integral with respect to the variables of a multi-variable function.
6. Determine the line integral of a vector field.
7. Understand how line integrals relate to work.
8. HOMEWORK PROBLEMS: 21, 29, 33, 43

Questions?

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