MA153 Lesson 38

LESSON 38 - Line Integrals II

30 RockTober, 2008

Outline

Admin

1. This week - Drop on Friday

Admin

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2. Next Week - Line Integrals III Monday, Greens Theorem Tuesday and Wednesday, Drop Thursday (Surgery Day), Guest Lecture Friday

Admin

3. Andrew Rodriguez has been selected as the National Football Foundation's 2008 East Region High School Athlete of the Year.

Block IV - Chapter 16

1. Vector Fields - 16.1: Functions that assign vectors to points in space.

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9. Divergence Theorem - 16.9:

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1. $\int_C f(x,y)ds$
2. $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
3. Similar arguments help us define the line integral as the following:

$$\int_C f(x,y)ds = \int_a^b f(x(t),y(t))\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

More Definitions and Derivations

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3. We may also have to remember the equation of a line segment:

$$\vec{r}(t) = (1-t)\vec{r_0} + t\vec{r_1}$$

Outline

Homework Help

Cool? Line Integrals

Animation

Outline

Course Guide

Line Integrals II - 16.2

1. Understand the meaning of the line integral of a scalar function $f(x, y)$ along a curve C.
2. Understand the meaning of $\int Pdx + Qdy$ along a curve C.
3. Understand the relationship between line integrals and arc length.
4. Determine the line integral with respect to arc length of a multi-variable function.
5. Determine the line integral with respect to the variables of a multi-variable function.
6. Determine the line integral of a vector field.
7. Understand how line integrals relate to work.
8. HOMEWORK PROBLEMS: 21, 29, 33, 43

Outline

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   $$\int_a^b f(x(t),y(t),z(t))\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\,dt$$
3. $$\int_a^b f(\vec{r}(t))|\vec{r}\,'(t)|dt$$

Board Work

1. Problem 16, Page 1043
2. Problem 18, Page 1044

Line Integrals of Vector Fields

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1. Page 1041 - If we talk about work we remember $W = \vec{F} \cdot \vec{D}$
2. We suppose that $\vec{D} = \overrightarrow{PQ}$ the displacement vector, and that $\vec{F} = P\hat{i} + Q\hat{j} + R\hat{k}$.
3. Now work can be described by:

$$W = \int_C \vec{F}(x, y, z) \cdot \vec{T}(x, y, z) ds = \int_C \vec{F} \cdot \vec{T}\, ds$$

Line Integrals of Vector Fields

1. Also

$$\int_C \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t)dt = \int_C \vec{F} \cdot \vec{T}\, ds$$

2. So

$$\int_C \vec{F} \cdot d\vec{r} = \int_C P dx + Q dy + R dz \quad \text{where} \quad \vec{F} = P\hat{i} + Q\hat{h} + R\hat{k}$$

Board Work

1. Problem 19, Page 1044

Outline

Homework Help

Look Forward

FTC and Line Integrals - 16.3

1. Understand the Fundamental Theorem for Line Integrals.
2. Understand independence of path and its equivalent statements.
3. Understand the definitions of a simple curve and a simply-connected region.
4. HOMEWORK PROBLEMS: 4, 7, 14, 29

Questions?

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