MA153 Lesson 39

LESSON 39 - FTC and Line Integrals

3 November, 2008

Outline

1. Admin
2. Block IV - Chapter 16
3. Last Class
   - Line Integral I
   - Line Integral II
   - Homework Help
4. FTC and Line Integrals
   - Course Guide
   - Definitions and Derivations
   - Homework Help
5. Look Forward

Admin

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Definitions and Derivations

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2. $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

Definitions and Derivations

1. $\int_C f(x,y)ds$
2. $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
3. Similar arguments help us define the line integral as the following:

$$\int_C f(x,y)ds = \int_a^b f(x(t),y(t))\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

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2. Do a similar process, but instead:

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3. We may also have to remember the equation of a line segment:

$$\overrightarrow{r}(t) = (1-t)\overrightarrow{r_0} + t\overrightarrow{r_1}$$

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1. Page 1039 - Start with a plane curve C given by the parametric equations
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$$\int_a^b f(x(t), y(t), z(t))\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}dt$$
3. $$\int_a^b f(\overrightarrow{r}(t))|\overrightarrow{r}'(t)|dt$$

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2. We suppose that $\overrightarrow{D} = \overrightarrow{PQ}$ the displacement vector, and that $\overrightarrow{F} = P\widehat{i} + Q\widehat{j} + R\widehat{k}$.
3. Now work can be described by:

$$W = \int_C \overrightarrow{F}(x, y, z) \cdot \overrightarrow{T}(x, y, z) ds = \int_C \overrightarrow{F} \cdot \overrightarrow{T} ds$$

Line Integrals of Vector Fields

1. Also

$$\int_C \overrightarrow{F} \cdot d\overrightarrow{r} = \int_a^b \overrightarrow{F}(\overrightarrow{r}(t)) \cdot \overrightarrow{r}'(t)dt = \int_C \overrightarrow{F} \cdot \overrightarrow{T}\, ds$$

2. So

$$\int_C \overrightarrow{F} \cdot d\overrightarrow{r} = \int_C Pdx + Qdy + Rdz \quad \text{where} \quad \overrightarrow{F} = P\widehat{i} + Q\widehat{h} + R\widehat{k}$$

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Course Guide

FTC and Line Integrals - 16.3

1. Understand the Fundamental Theorem for Line Integrals.
2. Understand independence of path and its equivalent statements.
3. Understand the definitions of a simple curve and a simply-connected region.
4. HOMEWORK PROBLEMS: 4, 7, 14, 29

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FTC

1. Page 1046 - Theorem 2
2. Page 1048 - Theorem 3
3. Page 1048 - Theorem 4
4. Page 1049 - Theorem 5
5. Page 1050 - Theorem 6
6. Page 1052 - Conservation of Energy

Board Work

1. Problem 2, Page 1053
2. Problem 11, Page 1053

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Homework Help

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Green's Theorem - 16.4

1. Understand how Green's Theorem relates line integrals and double integrals.
2. Understand the extension of Green's Theorem to domains with holes.
3. Understand the importance of Green's Theorem in solving difficult line integrals or difficult area integration problems.
4. Use Green's Theorem to find the area of a planar region.
5. HOMEWORK PROBLEMS: 3, 6

Questions?

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