

MA153 Lesson 40

LESSON 40 - Green's Theorem I

4 November, 2008

Outline

- 1 Admin
- 2 Block IV - Chapter 16
- 3 Last Class
 - Line Integral I-III
 - Line Integral II
 - FTC
 - Homework Help
- 4 Green's Theorem
 - Course Guide
 - Definitions and Derivations
 - Homework Help
- 5 Look Forward

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- 1 This Week - Greens Theorem Tuesday and Wednesday, Drop Thursday (Surgery Day), Guest Lecture (Info on the MA153 Web Site: Friday, November 7, Friday, Dean's Hour: Mahan Hall Auditorium, the movie theater)

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- 7 Project II - Draft

Block IV - Chapter 16

- 1 Vector Fields - 16.1: Functions that assign vectors to points in space.

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- 9 Divergence Theorem - 16.9:

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Definitions and Derivations

1 $\int_C f(x, y) ds$

2 $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$

- 3 Similar arguments help us define the line integral as the following:

$$\int_C f(x, y) ds = \int_a^b f(x(t), y(t)) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

More Definitions and Derivations

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- 3 We may also have to remember the equation of a line segment:

$$\vec{r}(t) = (1 - t)\vec{r}_0 + t\vec{r}_1$$

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Line Integrals in Space

- ① Page 1039 - Start with a plane curve C given by the parametric equations

$$x = x(t) \quad y = y(t) \quad z = z(t) \quad a \leq t \leq b$$

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$$\int_a^b f(x(t), y(t), z(t)) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt$$

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$$\int_a^b f(x(t), y(t), z(t)) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt$$

3

$$\int_a^b f(\vec{r}(t)) |\vec{r}'(t)| dt$$

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$$W = \vec{F} \cdot \vec{D}$$

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$$W = \vec{F} \cdot \vec{D}$$
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- 1 Page 1041 - If we talk about work we remember
 $W = \vec{F} \cdot \vec{D}$
- 2 We suppose that $\vec{D} = \overrightarrow{PQ}$ the displacement vector, and that $\vec{F} = P\hat{i} + Q\hat{j} + R\hat{k}$.
- 3 Now work can be described by:

$$W = \int_C \vec{F}(x, y, z) \cdot \vec{T}(x, y, z) ds = \int_C \vec{F} \cdot \vec{T} ds$$

Line Integrals of Vector Fields

1 Also

$$\int_C \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t) dt = \int_C \vec{F} \cdot \vec{T} ds$$

2 So

$$\int_C \vec{F} \cdot d\vec{r} = \int_C Pdx + Qdy + Rdz \quad \text{where} \quad \vec{F} = P\hat{i} + Q\hat{j} + R\hat{k}$$

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FTC

- 1 $\int_C \nabla f \cdot d\vec{r} = f(x_2, y_2) - f(x_1, y_1)$
- 2 $\int_C \vec{F} \cdot d\vec{r}$ Path Independent if and only if $\int_C \vec{F} \cdot d\vec{r} = 0$
- 3 \vec{F} is a conservative vector field on D if $\int_C \vec{F} \cdot d\vec{r}$ is path independent.
- 4 If $\vec{F}(x, y) = P(x, y)\hat{i} + Q(x, y)\hat{j}$ is a conservative field then:

$$\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$$

- 5 Page 1050 - Theorem 6

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Course Guide

Green's Theorem - 16.4

- 1 Understand how Green's Theorem relates line integrals and double integrals.
- 2 Understand the extension of Green's Theorem to domains with holes.
- 3 Understand the importance of Green's Theorem in solving difficult line integrals or difficult area integration problems.
- 4 Use Green's Theorem to find the area of a planar region.
- 5 **HOMEWORK PROBLEMS: 3, 6**

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Green's Theorem

1

$$\int_C P dx + Q dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

Board Work

- 1 Problem 2, Page 1060
- 2 Problem 5, Page 1060

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