MA153 Lesson 41

LESSON 41 - Green's Theorem II

5 November, 2008

Outline

Admin

1. This Week - Green's Theorem, Drop Thursday (Surgery Day), Guest Lecture
(Info on the MA153 Web Site: Friday, November 7, Dean's Hour: Mahan Hall Auditorium, the movie theater)

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3. Quiz - You will have 10 minutes.

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9. Divergence Theorem - 16.9:

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Definitions and Derivations

1. $$\int_C f(x,y)ds$$
2. $$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$
3. Similar arguments help us define the line integral as the following:

$$\int_C f(x,y)ds = \int_a^b f(x(t),y(t))\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

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3. We may also have to remember the equation of a line segment:

$$\overrightarrow{r}(t) = (1-t)\overrightarrow{r_0} + t\overrightarrow{r_1}$$

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$$\int_a^b f(x(t), y(t), z(t)) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt$$

3. $$\int_a^b f(\overrightarrow{r}(t)) |\overrightarrow{r}'(t)| dt$$

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2. We suppose that $\overrightarrow{D} = \overrightarrow{PQ}$ the displacement vector, and that $\overrightarrow{F} = P\widehat{i} + Q\widehat{j} + R\widehat{k}$.
3. Now work can be described by:

$$W = \int_C \overrightarrow{F}(x, y, z) \cdot \overrightarrow{T}(x, y, z) ds = \int_C \overrightarrow{F} \cdot \overrightarrow{T} ds$$

Line Integrals of Vector Fields

1. Also
$$\int_C \overrightarrow{F} \cdot d\overrightarrow{r} = \int_a^b \overrightarrow{F}(\overrightarrow{r}(t)) \cdot \overrightarrow{r}'(t)dt = \int_C \overrightarrow{F} \cdot \overrightarrow{T}\, ds$$

2. So
$$\int_C \overrightarrow{F} \cdot d\overrightarrow{r} = \int_C Pdx + Qdy + Rdz \quad \text{where} \quad \overrightarrow{F} = P\widehat{i} + Q\widehat{h} + R\widehat{k}$$

Outline

FTC

1. $\int_C \nabla f \cdot d\vec{r} = f(x_2, y_2) - f(x_1, y_1)$
2. $\int_C \vec{F} \cdot d\vec{r}$ Path Independent if and only if $\int_C \vec{F} \cdot d\vec{r} = 0$
3. $\vec{F}$ is a conservative vector field on D if $\int_C \vec{F} \cdot d\vec{r}$ is path independent.
4. If $\vec{F}(x, y) = P(x, y)\hat{i} + Q(x, y)\hat{j}$ is a conservative field then:
$$\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$$
5. Page 1050 - Theorem 6

Outline

Green's Theorem

1. $$\int_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

Outline

Homework Help

Outline

Course Guide

Green's Theorem - 16.4

1. Understand how Green's Theorem relates line integrals and double integrals.
2. Understand the extension of Green's Theorem to domains with holes.
3. Understand the importance of Green's Theorem in solving difficult line integrals or difficult area integration problems.
4. Use Green's Theorem to find the area of a planar region.
5. HOMEWORK PROBLEMS: 8, 13

Outline

Green's Theorem II

1. $$A = \oint_C x\,dy = -\oint_C y\,dx = \frac{1}{2}\oint_C x\,dy - y\,dx$$

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2. Holes and Green's Theorem

Board Work

1. Problem 22, Page 1061
2. Problem 5, Page 1060

Outline

Homework Help

Look Forward

Curl and Divergence - 16.5

1. Understand the significance of the vector differential operator (∇).
2. Determine the curl of a vector field.
3. Understand the physical interpretation of curl.
4. Determine the divergence of a vector field.
5. Understand the physical interpretation of divergence.
6. Use curl and divergence to re-write Greens Theorem in vector forms.
7. HOMEWORK PROBLEMS: 2, 4, 15, 16

Questions?

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