MA153 Lesson 44

LESSON 44 - Parametric Surfaces and their Areas

13 November, 2008

Outline

1. Admin
2. Last Class
3. Parametric Surfaces and their Areas
   - Course Guide
   - Definitions and Derivations
   - Homework Help
4. Look Forward

Admin

1. This Week - Parametric Surfaces, and Surface Integrals.

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5. Homework 8 due next Friday, 21 November!

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6. MA 206 and Physics Validation - I emailed you the info!

Block IV - Chapter 16

1. Vector Fields - 16.1: Functions that assign vectors to points in space.
2. Line Integrals - 16.2: Integrating over a curve.
3. Fundamental Theorem for Line Integrals - 16.3: Using FTC for line integrals.
4. Green's Theorem - 16.4:
5. Curl and Divergence - 16.5:

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9. Divergence Theorem - 16.9:

Curl

1. If $\vec{F} = P\hat{i} + Q\hat{j} + R\hat{k}$ is a vector field in three dimensions and the partial derivatives of P, Q, and R all exist, then the curl of $\vec{F}$ is the vector field in three dimensions defined by:

Curl

2

$$\text{curl}\,\vec{F} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\hat{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\hat{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\hat{k}$$

More about Curl

1. The Curl of a gradient field is **0**. This is because the curl is the cross product of the gradient with the gradient.

$$\text{curl}(\nabla f) = \nabla \times (\nabla f)$$

More about Curl

1. The Curl of a gradient field is **0**. This is because the curl is the cross product of the gradient with the gradient.

$$\text{curl}(\nabla f) = \nabla \times (\nabla f)$$

2. So we have an extension of what we learned in Green's Theorem. If $\vec{F}$ is a vector field defined on all reals in three dimensions whose component functions have continuous partial derivatives and curl $\vec{F} = \mathbf{0}$, then $\vec{F}$ is a conservative vector field.

Divergence

1. If $\vec{F} = P\hat{i} + Q\hat{j} + R\hat{k}$ is a vector field in three dimensions and the partial derivatives of P, Q, and R all exist, then the **divergence of F** is the function of three variables defined by:

Divergence

2. $$\operatorname{div} \vec{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

Divergence

2. $$\text{div}\,\vec{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

3. Similar to the dot product where we get a number, divergence is:
$$\text{div}\,\vec{F} = \nabla \cdot \vec{F}$$

More about Divergence

1. If $\vec{F} = P\hat{i} + Q\hat{j} + R\hat{k}$ is a vector field in the real three dimensions and P, Q, and R have continuous second-order partial derivatives, then

More about Divergence

1. If $\vec{F} = P\hat{i} + Q\hat{j} + R\hat{k}$ is a vector field in the real three dimensions and P, Q, and R have continuous second-order partial derivatives, then
2. $$\text{div curl}\vec{F} = 0$$

Physical description of Divergence

1. Fluid flow again is one of the best examples. Divergence can be seen as the net rate of change of the mass of the fluid flowing from the point P per unit volume. Or the divergence measures the tendency of the fluid to diverge from point P.

Physical description of Divergence

1. Fluid flow again is one of the best examples. Divergence can be seen as the net rate of change of the mass of the fluid flowing from the point P per unit volume. Or the divergence measures the tendency of the fluid to diverge from point P.
2. If div $\vec{F} = 0$ at a point P, the the fluid is said to be incompressible.

1. If we take the divergence of the gradient vector of a function of three variables, we get:

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2. $$\text{div}(\nabla f) = \nabla \cdot (\nabla f) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

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3. This is often called the Laplace operator and looks like:
4. $$\nabla^2 = \nabla \cdot \nabla$$

Vector Form of Green's Theorem

1. $$\oint_C \vec{F} \cdot d\vec{r} = \oint_C P dx + Q dy$$

Vector Form of Green's Theorem

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Vector Form of Green's Theorem

1. $$\oint_C \vec{F} \cdot d\vec{r} = \oint_C Pdx + Qdy$$
2. If we regard $\vec{F}$ as a vector field in three dimensions with the third component 0, we get:
3. $$\text{curl}\vec{F} = \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\hat{k}$$

Vector Form of Green's Theorem

1. So:

$$(\text{curl}\vec{F}) \cdot \hat{k} = \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) \hat{k} \cdot \hat{k} = \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$$

Vector Form of Green's Theorem

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2. We can rewrite the Green's Theorem in vector form:

Vector Form of Green's Theorem

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2. We can rewrite the Green's Theorem in vector form:
3. $$\oint_C \vec{F} \cdot d\vec{r} = \iint_D (\text{curl}\vec{F}) \cdot \hat{k} dA$$

Second Vector Form of Green's Theorem

1. $$\oint_C \vec{F} \cdot \vec{n} ds = \iint_D \text{div}\vec{F}(x, y) dA$$

Homework Help

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Course Guide

Parametric Surfaces and their Areas - 16.6

1. Understand the role of gridlines in studying these surfaces.
2. Understand how the form or symmetry of a surface helps one in choosing the parameterizations.
3. Determine the parametric representation of a surface.
4. Determine the tangent plane to a parametric surface.
5. Understand the role of the area element $|r_u \times r_v|$ for a general parametric surface $r(u, v)$.
6. Determine the area of a parametric surface.
7. HOMEWORK PROBLEMS: 3, 9, 19

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1. Theorem 1 - Page 1070
2. Theorem 2 - Page 1070
3. Theorem 4 - Tangent Planes - Page 1074
4. Theorem 5 - Page 1074
5. Theorem 6 - WOW! - Try to say this three times fast! - Surface Area? - Page 1076
6. Theorem 7 through 9 - Page 1077 - Surface area of a graph.

Board Work

1. Studnet Examples

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Surface Integrals - 16.7

1. Develop an understanding of the surface integral of a scalar functions viewed as and extension of the surface area integral.
2. Develop an understanding of oriented surfaces and positive orientation.
3. Determine the surface integral over a surface.
4. Develop an understanding of flux (surface integrals of vector fields).
5. Use surface integrals to determine the electric flux, net charge, and heat flow across a solid region.
6. HOMEWORK PROBLEMS: 2, 10, 25

Questions?

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