MA153 Lesson 46

LESSON 46 - Stokes' Theorem

18 November, 2008

Outline

1. Admin
2. Last Class
3. Stokes' Theorem
   - Course Guide
   - Definitions and Derivations
   - Homework Help
4. Look Forward

Birthday Cadet

Who's Birthday is It?

Birthday Cadet

Who's Birthday is It?

1. This cadet is from a city in Florida

Birthday Cadet

Who's Birthday is It?

2

Birthday Cadet

Who's Birthday is It?

3. Yes its Jacksonville, Florida.

Birthday Cadet

1.

Birthday Cadet

2. This cadet is on the swimming team.

Birthday Cadet

3

Birthday Cadet

3.
4. Byron Plapp turned 19 Yesterday, 17 November!

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5. Project Questions?

Block IV - Chapter 16

1. Vector Fields - 16.1: Functions that assign vectors to points in space.
2. Line Integrals - 16.2: Integrating over a curve.
3. Fundamental Theorem for Line Integrals - 16.3: Using FTC for line integrals.
4. Green's Theorem - 16.4:
5. Curl and Divergence - 16.5:
6. Parametric Surfaces and their Area - 16.6:
7. Surface Integrals - 16.7:

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9. Divergence Theorem - 16.9:

Surface Integrals

1. $$\iint_S f(x,y,z)dS = \iint_D f(\vec{r}(u,v))|\vec{r}_u \times \vec{r}_v|dA$$

Surface Integrals

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2. $$\iint_S f(x,y,z)dS = \iint_D f(x,y,g(x,y))\sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}dA$$

Surface Integrals of Vector Fields

1. $$\iint_S \vec{F} \cdot d\vec{S} = \iint_S \vec{F} \cdot \vec{n} dS$$

Surface Integrals of Vector Fields

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Surface Integrals of Vector Fields

1. $$\iint_S \vec{F} \cdot d\vec{S} = \iint_S \vec{F} \cdot \vec{n} dS$$
2. This is called the **flux** of $\vec{F}$ across S.
3. $$\iint_S \vec{F} \cdot d\vec{S} = \iint_S \vec{F} \cdot (\vec{r}_u \times \vec{r}_v) dA$$

Surface Integrals of Vector Fields

1. $$\iint_S \vec{F} \cdot d\vec{S} = \iint_S \left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R \right) dA$$

Homework Help

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Course Guide

Stokes' Theorem - 16.8

1. Understand how Stokes' Theorem relates surface integrals to line integrals around a boundary curve.
2. Be able to apply Stokes' Theorem.
3. HOMEWORK PROBLEMS: 3, 8

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1. Let S be an oriented piecewise-smooth surface that is bounded by a simple, closed, piecewise-smooth boundary curve C with positive orientation. Let $\vec{F}$ be a vector field whose components have continuous partial derivatives on an open region in the real three dimensions that contains S. Then
2. $$\int_C \vec{F} \cdot d\vec{r} = \iint_S \text{curl}\vec{F} \cdot d\vec{S}$$

1. Another way of saying this

$$\iint_S \text{curl}\vec{F} \cdot d\vec{S} = \int_{\partial S} \vec{F} \cdot d\vec{r}$$

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$$\iint_S \text{curl}\vec{F} \cdot d\vec{S} = \int_{\partial S} \vec{F} \cdot d\vec{r}$$

2. If the surface is flat and lies in the xy-plane we get:

$$\int_C \vec{F} \cdot d\vec{r} = \iint_S \text{curl}\vec{F} \cdot d\vec{S} = \iint_S (\text{curl}\vec{F}) \cdot \hat{k} dA$$

The vector form of Green's Theorem.

1. $$\iint_S \text{curl}\vec{F} \cdot d\vec{S} =$$

$$\iint_D \left[-\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\frac{\partial z}{\partial x} - \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\frac{\partial z}{\partial y} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\right] dA$$

1. One more way

$$\iint_{S_1} \text{curl}\vec{F} \cdot d\vec{S} = \int_C \vec{F} \cdot d\vec{r} = \iint_{S_2} \text{curl}\vec{F} \cdot d\vec{S}$$

Board Work

1. Problem 5, page 1097

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Homework Help

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2. Be able to apply Stokes' Theorem.
3. HOMEWORK PROBLEMS: 13, 17

Divergence Theorem

Questions?

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