MA153 Lesson 54

LESSON 54 - Power Series

8 December, 2008

Outline

1. Admin
2. Last Class
   - Sequences
   - Series
   - The Comparison Tests
   - Alternating Series
   - Absolute Convergence
   - The Ratio Test
   - The Root Test
3. Power Series
   - Definitions and Notation
4. Look Forward

Admin

1. This Week - Power Series Taylor Series and Reviews

Admin

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2. Next Week - TEE

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2. Next Week - TEE
3. Homework 9 - Through Section 11.6
   Due Today 8 December

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2. Next Week - TEE
3. Homework 9 - Through Section 11.6
   Due Today 8 December
4. Homework 9 - Section 11.8 - 11.10 and Calc Labs
   Due Thursday 11 December

Admin

5. WPR IV Re-Submit Homework
   Due Tomorrow 9 December, but can be turned in early

Admin

6. Lowest WPR grade has been dropped, check your grade for correctness!

Admin

7. Guess what today is!

Admin

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8. Chocolate Brownie Day!

Admin

1. End of Course Survey on CIS by 8 December.

Admin

2. TEE INFO

Admin

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   1. Time: 0735-1105, Tuesday, 16 December

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      1. B3 - BH304
      2. C3 - BH305
      3. D3 - BH306

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   4. 5 - 8.5x11 sheets of handwritten notes

Block IV - Chapter 11

1. Sequences - 11.1:

Block IV - Chapter 11

1. Sequences - 11.1:
2. Series - 11.2:

Block IV - Chapter 11

1. Sequences - 11.1:
2. Series - 11.2:
3. Comparison Tests - 11.4:

Block IV - Chapter 11

1. Sequences - 11.1:
2. Series - 11.2:
3. Comparison Tests - 11.4:
4. Alternating Series - 11.5:

Block IV - Chapter 11

1. Sequences - 11.1:
2. Series - 11.2:
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5. Absolute Convergence and the Ratio and Root Tests - 11.6:

Block IV - Chapter 11

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6. Power Series - 11.8:

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7. Representations of Functions of Power Series - 11.9:

Block IV - Chapter 11

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6. Power Series - 11.8:
7. Representations of Functions of Power Series - 11.9:
8. Taylor / Maclaurin Series - 11.10

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Sequence Notation and Limits

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if we can make the terms a_n as close to L as we like by taking n sufficiently large. If $\lim_{n\to\infty} a_n$ exits, we say the sequence **converges** (or is **convergent**). Otherwise, we say the sequence **diverges** (or is **divergent**).

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if for every $\varepsilon > 0$ there is a corresponding integer N such that

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if for every $\varepsilon > 0$ there is a corresponding integer N such that

$$\text{if} \quad n > N \quad \text{then} \quad |a_n - L| < \varepsilon$$

Definitions

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$$\text{if} \quad n > N \quad \text{then} \quad a_n > M$$

3. Review the Limit Laws for Sequences on page 678.

Squeeze Theorem

1. If $a_n \leq b_n \leq c_n$ for $n \geq n_0$ and $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L$, then $\lim_{n\to\infty} b_n = L$.

Functions of sequences

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$$\lim_{n\to\infty} f(a_n) = f(L)$$

Convergence of Sequences

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2. $\lim_{n\to\infty} r^n = \begin{cases} 0 & \text{if } -1 < r < 1 \\ 1 & \text{if } r = 1 \end{cases}$

Increasing, Decreasing, and Monotonic

1. A sequence $\{a_n\}$ is called **increasing** if $a_n < a_{n+1}$ for all $n \geq 1$, that is, $a_1 < a_2 < a_3 < \ldots$. It is called **decreasing** if $a_n > a_{n+1}$ for all $n \geq 1$.
   It is called **monotonic** if it is either increasing or decreasing.

Bounded Sequences

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$$m \leq a_n \qquad \text{for all } n \geq 1$$
If it is bounded above and below, then $\{a_n\}$ is a **bounded sequence**.
2. **Monotonic Sequence Theorem**: Every bounded, monotonic sequence is convergent.

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Notation

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$$a_1 + a_2 + a_3 + \ldots + a_n + \ldots$$

2. which is called an **infinite series** (or just a series) and is denoted, for short by the symbol

$$\sum_{n=1}^{\infty} a_n \quad \text{or} \quad \sum a_n$$

Limits and Convergence

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If the sequence $\{s_n\}$ is convergent and $\lim_{n\to\infty} s_n = s$ exists as a real number, then the series $\sum a_n$ is called **convergent** and we write

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$$a_1 + a_2 + a_3 + ... + a_n + ... = s \quad \text{or} \quad \sum_{n=1}^{\infty} a_n = s$$

The number s is called the **sum** of the series. Otherwise, the series is called **divergent**.

The geometric series

The geometric series

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \ldots$$

is convergent if $|r| < 1$ and its sum is

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r} \quad |r| < 1$$

If $|r| \geq 1$, the geometric series is divergent.

The p series

The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$ and divergent if $p \leq 1$.

The Test for Divergence

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2. The Test for Divergence: If $\lim_{n\to\infty} a_n$ does not exist or if $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

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$$\lim_{n\to\infty} \frac{a_n}{b_n} = c$$

where c is a finite number and $c > 0$, then either both series converge or both diverge.

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then the series is convergent

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Absolute Convergence

1. A series $\sum a_n$ is called **absolutely convergent** if the series of absolute values $\sum |a_n|$ is convergent.
2. A series $\sum a_n$ is called **conditionally convergent** if it is convergent but not absolutely.
3. If a series $\sum a_n$ is absolutely convergent, then it is convergent.

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The Ratio Test

1. If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent (and therefore convergent).

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3. If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$, then the Ratio Test is inconclusive.

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The Root Test

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Course Guide

Power Series - 11.8

1. Understand the definition of a power series.
2. Understand the importance of the Ratio Test in determining the radius and interval of convergence of a power series.
3. HOMEWORK PROBLEMS: 3, 10, 13

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$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots + x^n + \ldots$$
This series converges when $-1 < x < 1$ and diverges when $|x| \geq 1$

General form of Power Series

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General form of Power Series

1. The general form of the power series can be given as:
2. $$\sum_{n=0}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \ldots$$
3. is called a **power series in (x - a)** or a **power series centered at a** or a **power series about a**

Theorem of Power Series

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   3. There is a positive number R such that the series converges if $|x-a| < R$ and diverges if $|x-a| > R$.
   4. The number R in case (iii) is called the **radius of convergence** of the power series. The radius of convergence $R = 0$ in case (i) and $R = \infty$ in case (ii).
   5. The **interval of convergence** of a power series is the interval that consists of all values of x for which converges.

Board Work

Find the radius of convergence and interval of convergence of the following series:

1. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n^3}$
2. $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$
3. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2 x^n}{2^n}$

Look Forward

Representations of Functions as Power Series - 11.9

1. Determine a power series representation of a function.
2. Differentiate and integrate a given power series to obtain a new power series with the same radius of convergence.
3. Modify given power series to represent different functions.
4. HOMEWORK PROBLEMS: 3, 13, 20

Questions?

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