MA153 Lesson 54

LESSON 54 - Power Series

8 December, 2008

Outline

Admin

1. This Week - Functions using Power, Series Taylor Series and Reviews

Admin

1. This Week - Functions using Power, Series Taylor Series and Reviews
2. Next Week - TEE

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2. Next Week - TEE
3. Homework 9 - Section 11.8 - 11.10 and Calc Labs
   Due Thursday 11 December

Admin

4. WPR IV Re-Submit Homework : Due Today 10 December

Admin

1. End of Course Survey on CIS

Admin

2. TEE INFO

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    1. Time: 0735-1105, Tuesday, 16 December

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    4. 5 - 8.5x11 sheets of handwritten notes

Block IV - Chapter 11

1. Sequences - 11.1:

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7. Representations of Functions of Power Series - 11.9:
8. Taylor / Maclaurin Series - 11.10

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Sequence Notation and Limits

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if we can make the terms a_n as close to L as we like by taking n sufficiently large. If $\lim_{n\to\infty} a_n$ exits, we say the sequence **converges** (or is **convergent**). Otherwise, we say the sequence **diverges** (or is **divergent**).

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if for every $\varepsilon > 0$ there is a corresponding integer N such that

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$$\text{if} \quad n > N \quad \text{then} \quad |a_n - L| < \varepsilon$$

Definitions

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3. Review the Limit Laws for Sequences on page 678.

Squeeze Theorem

1. If $a_n \leq b_n \leq c_n$ for $n \geq n_0$ and $\lim_{n \to \infty} a_n = \lim_{n \to \infty} c_n = L$, then $\lim_{n \to \infty} b_n = L$.

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$$\lim_{n\to\infty} f(a_n) = f(L)$$

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2. $\lim_{n\to\infty} r^n = \begin{cases} 0 & \text{if } -1 < r < 1 \\ 1 & \text{if } r = 1 \end{cases}$

Increasing, Decreasing, and Monotonic

1. A sequence $\{a_n\}$ is called **increasing** if $a_n < a_{n+1}$ for all $n \geq 1$, that is, $a_1 < a_2 < a_3 < ....$ It is called **decreasing** if $a_n > a_{n+1}$ for all $n \geq 1$.
   It is called **monotonic** if it is either increasing or decreasing.

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If it is bounded above and below, then $\{a_n\}$ is a **bounded sequence.**
2. **Monotonic Sequence Theorem**: Every bounded, monotonic sequence is convergent.

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2. which is called an **infinite series** (or just a series) and is denoted, for short by the symbol

$$\sum_{n=1}^{\infty} a_n \quad \text{or} \quad \sum a_n$$

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$$a_1 + a_2 + a_3 + ... + a_n + ... = s \quad \text{or} \quad \sum_{n=1}^{\infty} a_n = s$$

The number s is called the **sum** of the series. Otherwise, the series is called **divergent**.

The geometric series

The geometric series

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + ...$$

is convergent if $|r| < 1$ and its sum is

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r} \quad |r| < 1$$

If $|r| \geq 1$, the geometric series is divergent.

The p series

The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$ and divergent if $p \leq 1$.

The Test for Divergence

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2. The Test for Divergence: If $\lim_{n\to\infty} a_n$ does not exist or if $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

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where c is a finite number and $c > 0$, then either both series converge or both diverge.

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then the series is convergent

Outline

1. Admin
2. Last Classes
   - Sequences and Series
   - The Comparison Tests
   - Alternating Series
   - Absolute Convergence and The Ratio Test and Root Tests
   - Power Series
3. Representations of Functions as Power Series
   - Definitions and Notation
   - Differentiation and Integration of Power Series
4. Look Forward

Absolute Convergence

1. A series $\sum a_n$ is called **absolutely convergent** if the series of absolute values $\sum |a_n|$ is convergent.
2. A series $\sum a_n$ is called **conditionally convergent** if it is convergent but not absolutely.
3. If a series $\sum a_n$ is absolutely convergent, then it is convergent.

The Ratio Test

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3. If $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = 1$, then the Ratio Test is inconclusive.

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$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots + x^n + \ldots$$

This series converges when $-1 < x < 1$ and diverges when $|x| \geq 1$

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2. $$\sum_{n=0}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \ldots$$
3. is called a **power series in (x - a)** or a **power series centered at a** or a **power series about a**

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    4. The number R in case (iii) is called the **radius of convergence** of the power series. The radius of convergence $R = 0$ in case (i) and $R = \infty$ in case (ii).
    5. The **interval of convergence** of a power series is the interval that consists of all values of x for which converges.

Course Guide

Representations of Functions as Power Series - 11.9

1. Determine a power series representation of a function.
2. Differentiate and integrate a given power series to obtain a new power series with the same radius of convergence.
3. Modify given power series to represent different functions.
4. HOMEWORK PROBLEMS: 3, 13, 20

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2. We change our point of view and regard the above equation as expressing the function $f(x) = \dfrac{1}{(1-x)}$ as a sum of a power series.

Book Example

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5. This is a geometric series, it converges when $|-x^2| < 1$, which means, $x^2 < 1$, or $|x| < 1$. So the interval of convergence is $(-1, 1)$.

Outline

Term by Term differentiation and integration

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- The radii of convergence of the power series in the above equations is R.

Example Problem

Example Problem

Homework Help

homework help

Look Forward

Taylor and Maclaurin Series - 11.10

1. Understand the definition of a Taylor Series.
2. Know the Maclaurin series for; $\sin x, \cos x, e^x$.
3. HOMEWORK PROBLEMS: 5, 13, 30, 56

Questions?

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