MA153 Lesson 56

LESSON 56 - Taylor Series

11 December, 2008

Outline

1. Admin
2. Last Classes
   - Sequences and Series
   - Testing Series for convergence
   - Power Series
   - Representations of Functions as Power Series
3. Taylor Series
4. Block V - Review
5. Look Forward

Birthday Cadet

Who's Birthday is It?

Birthday Cadet

Who's Birthday is It?

1. This cadet is from a city in Texas

Birthday Cadet

Who's Birthday is It?

2

Birthday Cadet

Who's Birthday is It?

3. Yes its Houston, Texas.

Birthday Cadet

1.

Birthday Cadet

2. This cadet is part of the reconditioning team and the Staff and Ushers program.

Birthday Cadet

3

Birthday Cadet

3.
4. David Durr turns 20, Tomorrow 12 December!

Birthday Cadet

Who's Birthday is It?

Birthday Cadet

Who's Birthday is It?

1. This cadet is from a city in Pennsylvania.

Birthday Cadet

Who's Birthday is It?

2

Birthday Cadet

Who's Birthday is It?

3. Yes its Benton, Pennsylvania.

Birthday Cadet

1.

Birthday Cadet

2. This cadet is part of the Band (Spirit Support Group).

Birthday Cadet

3

Birthday Cadet

3.
4. Paul Anderson turns 18, on 18 December!

Birthday Cadet

Who's Birthday is It?

Birthday Cadet

Who's Birthday is It?

1. This cadet is from a city in New York.

Birthday Cadet

Who's Birthday is It?

2

Birthday Cadet

Who's Birthday is It?

3. Yes its Fairport, New York.

Birthday Cadet

1.

Birthday Cadet

2. This cadet is part of the Soccer Team.

Birthday Cadet

3

Birthday Cadet

3.
4. Teddy Taggart turns 19, on 19 December!

Birthday Cadet

Who's Birthday is It?

Birthday Cadet

Who's Birthday is It?

1. This cadet is from a city in New Hampshire.

Birthday Cadet

Who's Birthday is It?

2

Birthday Cadet

Who's Birthday is It?

3. Yes its Bedford, New Hampshire.

Birthday Cadet

1.

Birthday Cadet

2. This cadet is part the paintball club, snow sport instructor club, and the French Language club.

Birthday Cadet

3

Birthday Cadet

3.
4. Adam Fulling turns 19, on 22 December!

Admin

1. This Week - Taylor Series and Reviews

n WPR Regrade: Make sure you can get the right answer - I did not include solutions.

Admin

1. This Week - Taylor Series and Reviews
2. Next Week - TEE

n WPR Regrade: Make sure you can get the right answer - I did not include solutions.

Admin

1. This Week - Taylor Series and Reviews
2. Next Week - TEE
3. Homework 9 - Section 11.8 - 11.10 and Calc Labs
   Due Friday 12 December

   n WPR Regrade: Make sure you can get the right answer
   - I did not include solutions.

Admin

4. On WPR Regrade: Make sure you can get the right answer - I did not include solutions.

Admin

1. End of Course Survey on CIS due by 19 December.

Admin

2. TEE INFO

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   1. Time: 0735-1105, Tuesday, 16 December

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      1. B3 - BH304
      2. **C3 - BH322 - Change to previous**
      3. D3 - BH306

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   4. 5 - 8.5x11 sheets of handwritten notes
3. Study Session Saturday Morning, 0830-1100, TH 342, LTC Outing.

Block IV - Chapter 11

1. Sequences - 11.1:

Block IV - Chapter 11

1. Sequences - 11.1:
2. Series - 11.2:

Block IV - Chapter 11

1. Sequences - 11.1:
2. Series - 11.2:
3. Comparison Tests - 11.4:

Block IV - Chapter 11

1. Sequences - 11.1:
2. Series - 11.2:
3. Comparison Tests - 11.4:
4. Alternating Series - 11.5:

Block IV - Chapter 11

1. Sequences - 11.1:
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5. Absolute Convergence and the Ratio and Root Tests - 11.6:

Block IV - Chapter 11

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6. Power Series - 11.8:

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7. Representations of Functions of Power Series - 11.9:

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5. Absolute Convergence and the Ratio and Root Tests - 11.6:
6. Power Series - 11.8:
7. Representations of Functions of Power Series - 11.9:
8. Taylor / Maclaurin Series - 11.10

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Sequence Notation and Limits

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if we can make the terms a_n as close to L as we like by taking n sufficiently large. If $\lim_{n\to\infty} a_n$ exits, we say the sequence **converges** (or is **convergent**). Otherwise, we say the sequence **diverges** (or is **divergent**).

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if for every $\varepsilon > 0$ there is a corresponding integer N such that

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if for every $\varepsilon > 0$ there is a corresponding integer N such that

$$\text{if} \quad n > N \quad \text{then} \quad |a_n - L| < \varepsilon$$

Definitions

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3. Review the Limit Laws for Sequences on page 678.

Squeeze Theorem

1. If $a_n \leq b_n \leq c_n$ for $n \geq n_0$ and $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L$, then $\lim_{n\to\infty} b_n = L$.

Functions of sequences

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$$\lim_{n\to\infty} f(a_n) = f(L)$$

Convergence of Sequences

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2. $\lim_{n\to\infty} r^n = \begin{cases} 0 & \text{if } -1 < r < 1 \\ 1 & \text{if } r = 1 \end{cases}$

Increasing, Decreasing, and Monotonic

1. A sequence $\{a_n\}$ is called **increasing** if $a_n < a_{n+1}$ for all $n \geq 1$, that is, $a_1 < a_2 < a_3 < \ldots$. It is called **decreasing** if $a_n > a_{n+1}$ for all $n \geq 1$.
   It is called **monotonic** if it is either increasing or decreasing.

Bounded Sequences

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$$a_n \leq M \qquad \text{for all } n \geq 1$$

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If it is bounded above and below, then $\{a_n\}$ is a **bounded sequence**.

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$$m \leq a_n \qquad \text{for all } n \geq 1$$
If it is bounded above and below, then $\{a_n\}$ is a **bounded sequence**.
2. **Monotonic Sequence Theorem**: Every bounded, monotonic sequence is convergent.

Notation of Series

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2. which is called an **infinite series** (or just a series) and is denoted, for short by the symbol

$$\sum_{n=1}^{\infty} a_n \quad \text{or} \quad \sum a_n$$

Limits and Convergence

1. Given a series $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + ...$, let s_n denote its nth partial sum:

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If the sequence $\{s_n\}$ is convergent and $\lim_{n\to\infty} s_n = s$ exists as a real number, then the series $\sum a_n$ is called **convergent** and we write

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$$a_1 + a_2 + a_3 + ... + a_n + ... = s \quad \text{or} \quad \sum_{n=1}^{\infty} a_n = s$$

The number s is called the **sum** of the series. Otherwise, the series is called **divergent**.

The geometric series

The geometric series

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \ldots$$

is convergent if $|r| < 1$ and its sum is

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r} \quad |r| < 1$$

If $|r| \geq 1$, the geometric series is divergent.

The p series

The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$ and divergent if $p \leq 1$.

The Test for Divergence

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2. The Test for Divergence: If $\lim_{n\to\infty} a_n$ does not exist or if $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

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   - If $\sum b_n$ is convergent and $a_n \leq b_n$ for all n, then $\sum a_n$ is also convergent.
   - If $\sum b_n$ is divergent and $a_n \geq b_n$ for all n, then $\sum a_n$ is also divergent.

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$$\lim_{n\to\infty} \frac{a_n}{b_n} = c$$

where c is a finite number and $c > 0$, then either both series converge or both diverge.

Alternating Series

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$$\sum_{n=1}^{\infty}(-1)^{n-1}b_n = b_1 - b_2 + b_3 - b_4 + b_5 - b_6 + \ldots \quad b_n > 0$$

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$$b_{n+1} \leq b_n \quad \text{for all } n$$

$$\lim_{n\to\infty} b_n = 0$$

then the series is convergent

Absolute Convergence

1. A series $\sum a_n$ is called **absolutely convergent** if the series of absolute values $\sum |a_n|$ is convergent.
2. A series $\sum a_n$ is called **conditionally convergent** if it is convergent but not absolutely.
3. If a series $\sum a_n$ is absolutely convergent, then it is convergent.

The Ratio Test

1. If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent (and therefore convergent).

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3. If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$, then the Ratio Test is inconclusive.

The Root Test

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Power Series

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2. $$\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \ldots$$
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3. If $c_n = 1$ for all n the power series becomes the geometric series
$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots + x^n + \ldots$$
This series converges when $-1 < x < 1$ and diverges when $|x| \geq 1$

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General form of Power Series

1. The general form of the power series can be given as:
2. $$\sum_{n=0}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \ldots$$
3. is called a **power series in (x - a)** or a **power series centered at a** or a **power series about a**

Theorem of Power Series

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   3. There is a positive number R such that the series converges if $|x-a| < R$ and diverges if $|x-a| > R$.
   4. The number R in case (iii) is called the **radius of convergence** of the power series. The radius of convergence $R = 0$ in case (i) and $R = \infty$ in case (ii).

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   2. The series conversions for all x.
   3. There is a positive number R such that the series converges if $|x-a| < R$ and diverges if $|x-a| > R$.
   4. The number R in case (iii) is called the **radius of convergence** of the power series. The radius of convergence $R = 0$ in case (i) and $R = \infty$ in case (ii).
   5. The **interval of convergence** of a power series is the interval that consists of all values of x for which converges.

Outline

1. Admin
2. Last Classes
   - Sequences and Series
   - Testing Series for convergence
   - Power Series
   - Representations of Functions as Power Series
3. Taylor Series
4. Block V - Review
5. Look Forward

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2. We change our point of view and regard the above equation as expressing the function $f(x) = \frac{1}{(1-x)}$ as a sum of a power series.

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which is differentiable on the interval $(a-R, a+R)$ and:

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- The radii of convergence of the power series in the above equations is R.

Course Guide

Taylor and Maclaurin Series - 11.10

1. Understand the definition of a Taylor Series.
2. Know the Maclaurin series for; $\sin x, \cos x, e^x$.
3. HOMEWORK PROBLEMS: 5, 13, 30, 56

Definitions

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2. $$f(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \ldots \quad |x-a| < R$$
3. Through a series of taking derivatives when $x = a$ we get the following:

Definitions

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2. $$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$
3. $$= f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \ldots$$

Special Taylor Series

1. $$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad \text{for all } x$$
2. If we put $x = 1$ we get:
3. $$e = \sum_{n=0}^{\infty} \frac{1}{n!} = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots$$

Special Taylor Series

1. $$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$$
$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \qquad \text{for all } x$$

Special Taylor Series

1. $$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$
$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \qquad \text{for all } x$$

Homework Help

homework help

Know How to do the following

1. Test a sequence for convergence
2. Test a series for convergence
3. Find the radius and interval of convergence for a power series
4. Find terms of a Taylor Series

Look Forward

Block I - IV Review

1. Block I - Vectors and Geometry of Space
2. Block II - Multi Variable Functions and Partial Derivatives.
3. Block III - Multiple Integrals
4. Block IV - Vector Calculus

Questions?

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