

MA153 Lesson 9

LESSON 9 - Arc-Length and Curvature

8 September, 2008

Outline

- 1 Admin
- 2 Last Class
- 3 Arc-Length
 - Arc Length I?
- 4 Arc Length II, TOTAL DISTANCE
 - Arc Length Is Total Distance
 - Total Distance
 - Predator the Mathematician
 - Board Work
- 5 Motion in Space

Admin

- 1 Homework 2 is due 10 September in class - The assignment is posted on the web site

Admin

- 2 Homework 1 - Return - Don't write on the back of the pages, Staple the homework together, Printed cover sheet

Admin

- ③ Mid Term Exam 1 is on 11 September during normal class time - Bring a formula sheet - handwritten

Admin

④ FCE Homework - Due today

Derivatives and Integrals of Vector Functions

Questions? - Homework Help

Objectives

Arc-Length and Curvature: (Stewart 13.4)

- 1 Become familiar with smoothness.
- 2 Determine the arc length of a space curve.
- 3 Understand the independence of arc length and parametrization.
- 4 Understand the geometric definition of curvature.
- 5 Determine the curvature of a space curve.
- 6 Determine the normal and binormal vectors of a space curve.

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$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

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or

Arc Length Refresher; What was Arc Length Again

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$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

or

2

$$L = \int_c^d \sqrt{1 + [g'(y)]^2} dy$$

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 $= \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}$

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- 4 So $L = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2} dt$
- 5 Or $L = \int_a^b |\vec{r}'(t)| dt$

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Predator was a Mathematician?

Video: You be the judge!

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The Dreaded Thayer Board Work

- 1 Graph the following parametric curve and find its exact length over the specified interval. Use Technology.

$$x = e^t - t, y = 4e^{\frac{t}{2}}, -8 \leq t \leq 3$$

- 2 Without technology find the exact length of the curve over the specified interval.

$$\vec{r}(t) = \langle 2 \sin t, 5t, 2 \cos t \rangle, -10 \leq t \leq 10$$

Motion in Space

- 1 Look at Stewart, Chapter 13, section 14, Example 5, pg 873 - Mortar Fire
- 2 If a nine iron lofts a golf ball at an angle of 60° , how fast would you have to swing in order to hit the ball 30 feet?

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