FIRST YEAR CALCULUS STUDENTS* AS IN-CLASS CONSULTANTS†

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ABSTRACT: We demonstrate how we use traditional text book calculus problems to involve first year calculus students as consultants. Students are asked to solve a "real world" problem after interviewing a "guest" client - a printer. Students must go through the initial steps of interviewing client, extracting information, ascertaining the problem, formulating the problem, solving the problem, and writing up two "solutions" - one is a step by step solution for the client and the second is a technical support document for the "senior consultant". This description can serve as a model for further student involvement with fresh approaches to applications of the calculus.

KEYWORDS: Calculus application, problem formulation, problem solving, consulting, writing.

INTRODUCTION

Over the past few years, as teachers of college level mathematics, we have been intrigued with a number of ideas related to applications in our classrooms. Often the application is a forced one, with short lived applicable values, rather like a short extension of the immediate topic under study at the moment. We try to bring these applications into the classroom in the traditional way, lecture. Or we give an extra assignment handout using the problem to stimulate student *excitement*.

Occasionally we even go to a new format, say using UMAP modules.[1] Or we may bring material to the class which we have garnered from some other source, perhaps a keen article by a colleague on an idea that worked in her classroom, or a problem from an advanced modeling book which we believe our students can get their teeth into after suitable modification.

In fact, some of us, in an attempt to get some relevance into our courses, and excitement and development into our own careers, teach a whole course in modeling with a good book such as Bender's modeling book [1] or we base such a course on provocative problems. Some even try to do real world modeling in our courses by lining up outside sources of problems for students to tackle. The latter is very demanding.

In what follows we propose to share a source of interesting applications *and approaches* to these applications which are *readily available* to the instructor and very appropriate to the calculus level course. Simply put, this is the fresh development of uses of standard calculus text book problems.

INITIAL PROJECT APPROACH

We teach in an engineering school and we believe engineers will be paid (and paid well!) for solving problems. Of course the first task in solving a problem is to find the problem amidst the confusion of the situation and to extract the information from the party who perceives a problem, but is not sure what it is. The second task is to formulate the problem, and finally, the third task is to solve

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[1]UMAP Modules are produced by COMAP, Inc. 271 Lincoln St., Suite 4, Lexington MA 02173 USA.

To: Junior Consultant

From: Senior Consultant - Brian J. Winkel

Re: New Client - Print Shop

You are to meet with our new client's representative at 9:00 AM (today). You are to inquire of the client the precise nature of his planned project, including the details and the specific data. You are to help him formulate his problem and to offer a solution to his problem. You may ask him any questions related to the project or get him to ask question, but under no circumstances can you expect him to solve the problem. That is your job and you are to report your results per the instructions below.

Moreover, you are to offer him a general approach to similar projects he may be contemplating.

By Monday morning you are to have on my desk two reports. The first is to be a clear solution to the client's problem, telling him just what he must do to solve his problem. This is to be submitted to the client as our product. The mathematical analysis and details which support this solution are to be on a separate, but attached report, for our internal files only. The second report is as the first, except that this should deal with his general problem.

We fully expect you will represent the firm with your best efforts and we look forward to receiving your reports on Monday. We shall be away from our desk for the duration, so you are on your own.

Figure 1. Letter left on the desks of calculus class at 8:55 AM of the morning of the 9:00 AM interview to be conducted.

the problem using the skills available. It is the first two aspects of problem solving which we have a great deal of difficulty covering in calculus (if indeed we take the time to discuss these at all), namely probing to discover the nature of the problem and formulating the problem in tractable mathematics.

We have found that a fresh approach to traditional calculus text book problems can address the need for problem formulation and problem solving. We use an interview process, with the students in the role of consultants and the instructor in the role of client. Figure 1 is a copy of a letter we give to our calculus students after introducing them to the usual max-min problems in a Calculus course. We have used this particular exercise in Calculus at the end of the max-min sections and we have used it with only notions of function and graph and no in-class coverage of max-min approaches. In the latter situation students tend to solve the problem by listing all the possibilities or drawing the optimizing function, rather than use the calculus as a tool. *Usually* the students have several days of coverage on maximizing and minimizing a function of one variable over an interval and they are quite confident in their abilities to find and evaluate the function at the usual critical points. We present the introductory volume maximization problems from the text and we discuss the formulation of max-min problems from the presented word problems in the text.

This consulting project begins on Friday and is completed by the following Monday. We put the students in the role of a junior consultant who must meet with a client printer and solve his problem. This activity forces the students to discover the nature and structure of the printer's problem. Then the students, on their own or in consultation among themselves, formulate the problem in mathematical terms and solve it. Not only are they to solve the formulated problem, but they then must offer up instructions suitable for the printer to follow to get his problem solved. Furthermore, they must render a technical report to the Senior Consultant (the instructor) defending their solution to the setup problem for the printer, with a defense of the general solution.

On the day of the project we place copies of the letter - one for each student/junior consultant - (see Figure 1) on the front desk 5 minutes before class starts with a note on the blackboard that they should take a copy and read it. When the class begins we enter the class in the role of the printer. The class interviews the printer for 1/2 hour. This interview process could last longer, but it usually only takes about 1/2 hour. At the end of the interview the printer leaves and the instructor returns without mentioning one word about the project. In fact all questions concerning the printer's problem and the nature of the report are refused! We usually devote the remainder of the class to going over homework, and answering questions about the "normal" work of the course. We do not introduce any new material other than the consulting project on this day.

On the following Monday *all* of the students hand in their reports. They take the project seriously. Some type up formal reports to both the printer and the Senior Consultant, while others write out instructions and mathematical solutions in the terse manner associated with the usual fare offered for homework papers. By Wednesday we return the evaluated reports and we discuss the evaluations of both reports, from the technical point of view of the mathematical model they offered and the techniques they used. We also discuss the appropriateness of these reports for the respective audience, i.e. the printer and the Senior Consultant. We assign 300 total homework points to the project in an environment which usually sees about 7-10 homework problems (worth 10 points each) due daily. Hence this assignment is worth about 3-4 night's of homework. The reports are not given equal credit. The report on the solution of the specific problem for the printer is given 160 points while the general solution report for the Senior Consultant is given 140 points. This is to insure that even the weakest student can get a good chunk of points. Most students get both parts done well and we usually only pick away some points for presentation and small details.

We do not tell the students the point values before the assignment; however, the printer emphasizes the importance of a solution to the problem and suggests that in his previous work with this Senior Consultant he found him willing to reward good work with appropriate praise and unwilling to tolerate poor or sloppy work.

THE PROBLEM PROBED IN CLASS

What is the printer's problem? Before answering that, we should like to give the flavor of just what the students (and the instructor) go through in the interview. Let us describe what happens in class. We enter as the printer (occasionally we borrow our printshop personnel's *inky* printer's apron!) and say, "Thank you for coming to help me with my problem." And then silence follows and continues! The instructor inside the printer has doubts about the nature of this whole experiment! But up comes the first question. "What is your problem?" This is a brave calculus student's way of saying, "What is it you are trying to do?"

So we describe the problem. We have to make embossed napkins for Princess Diana and Prince Charles of Great Britain. Stop. Silence and more silence. "How many napkins do you have to make?" is the next question. "610,000 for the Prince and 500,000 for the Princess, for the upcoming social season. We are not sure why the difference in numbers, perhaps the prince does more entertaining." We throw in this extra prattle whenever we can, for it represents the extraneous information a client would offer and the human frailty of chatting, but not informing. Silence - but not for so long now. "Well, how do you make these?" comes the next question. "We have to purchase engraved plates for the napkins. We set up the press with the plates on them and we run the napkins off." Some key words are emerging now. In the minds of the students, ideas are going around, "Purchase plates (how many?)..." and "...hope you can advise us in this regard." Bright student is thinking, "My gosh, there is the variable, x = number of plates." Not so bright student is pondering, "I wonder how many plates he really wants?" Poor or slow student now wakes up in the back row, opens up his notebook, and realizes the Professor (or at least this printer) is serious and asks, "How many napkins did you say you have to have?" We give out the information again. We make no remarks about inattentiveness.

"Well how is this printing done?" they ask the printer. "We have a press and we lay out the engraved plates and pass the paper through the press to print them and thence to a cutter to cut, fold and stack the printed napkins." Now the questions start to get very specific. "How much does the paper cost?" "It comes out to about 1 penny per napkin for paper, and [a volunteered piece of information] it does not matter how we cut it or fold it, for these costs are all the same." Now they are getting to the heart of the matter. They ask questions about how many plates can be placed on

$$\text{Total costs} = (\text{cost of plates}) + (\text{cost of machine operations}) + (\text{cost napkins})$$

$$\begin{aligned}\text{Total cost} &= (\#\text{plates})(\$/\text{plate}) \\ &\quad + \frac{\text{impressions}}{\#\,\text{plates} \times \text{imp/min} \times 60\,\text{min/hr}} \times (\$/\text{hr}) \\ &\quad + (\#\,\text{napkins})(\$/\text{napkins})\end{aligned}$$

$$\begin{aligned}
\mathrm{C} &= \text{total cost} \\
\mathrm{p} &= \#\ \text{plates} \\
\mathrm{P} &= \$/\text{plate} \\
\mathrm{I} &= \#\ \text{impressions} \\
\mathrm{i} &= \text{imp/min} \\
\mathrm{H} &= \text{amount of \$/hr to run machine} \\
\mathrm{n} &= \#\ \text{napkins} \\
\mathrm{N} &= \text{amount of \$/napkins to purchase paper} \\
\mathrm{q} &= 60\ \text{min/hr}
\end{aligned}$$

$$C(p) = pP + \frac{HI}{piq} + nN.$$

Figure 2. Typical student setup of units
for solution of general printer's problem.

the press at once. A maximum of 8 plates on the press at one time is the response. The printer knows that these napkins are to be 1 foot square, before folding, and he has a 4 foot by 2 foot press. He is prepared to give this data should the question be asked that way. But these junior consultants get right to the point of how many plates can the press hold.

"So how expensive *are* these plates?" they ask. "$100 each and there is no discount for multiple purchases because they are each hand carved." They find out the printer has only one press dedicated to this job. Now they are into expenses in running the machine. Many are on to the fact that we are trying to do this job as cheaply as possible. The press costs $36.00 per hour to run and after asking, they find out that it gets 600 impressions per minute. But the printer has a new press coming soon which will get 800 impressions per minute but costs $42.00 per hour. The printer would like to be able to know how to "set up the job" for the old *and* the new machine. In fact, he says, "It would really help if you could come up with a general rule to apply so that we could determine just the right setups no matter what kind of job we have coming in and so that we could figure out what to do for *any* cost machine we purchase. That would be a real help to us around here."

Now the printer volunteers, "What's really going on here is this. The guys in the front office have to bid on this project - prestige and all that, you know. *We* have to come up with the cheapest way to do *this* job. And I want to know how to do other similar jobs in the cheapest manner. I would really appreciate it if you could tell me how to do these things. I never was any good in math, but if you tell me, step by step, just what I have to do, I can do it. Thank you for coming today and I look forward to receiving your solutions soon. Bye." So ends the printer's interview.

Incidentally, we throw in lots of extraneous information, some of which they may ask about, and some they do not ask about, e.g. potential union problems, square footage of the plant, descriptions of the machines, the fact that this printer is the brother-in-law of the owner and the owner is really anxious about this order and very interested in making lots of money, etc.

STUDENT FORMULATION AND SOLUTION

We do not really know exactly what the individual student mind does with this material. We do know that they do something with it for they write up their formulations and solutions and bring them to class on Monday. The best of the students write a very simple directive to the printer telling him exactly how to set up the press and how many impressions to run.

We tend to stress units or dimensions in our presentations in class and it is nice to see them applied in the general solution techniques. Typical is Figure 2. Here minimum cost will be at a critical point (where $C'(p) = 0$, or $C'(p)$ does not exist) or at interval end points $p = 1$ or $p = 8$.

There follows the usual $0 = C'(p) = P - HI/(p^2 iq)$ and the solution, $p = (HI/piq)^{1/2}$. Now the student would offer the solution for the data set at hand. For the Princess on the present machine $p = 2.47$, and for the Prince $p = 2.24$. But this means checking the values $p = 2$ and $p = 3$ as partial plates are not possible. Upon checking these critical points the student finds the value of $p = 3$ gives minimum costs for the Princess while $p = 2$ gives minimum costs for the Prince.

The students give the description of the setup, the process the printer needs to follow, and they offer total costs for this process. We believe we give the students the opportunity to gather data through inquiry, to work through the problem formulation process, to set up the problem in their own minds at their own pace, to bring to bear the problem solving techniques of the mathematics being taught, and to write the problem solution up for both client and technical supervisor. The students confirm this by their questions in class, their thorough efforts in identifying the problem variables, their problem formulation, their analysis of the problem using the calculus, and their final write ups.

STUDENT REACTION AND SOURCE MATERIAL

We offer some comments from student evaluations at the end of the course in response to a call for reaction to the simple phrase: "Consultant to Printer."

- A good break from regular homework. [Notice the perception that this was a break in the pattern.]
- Great to apply the material learned in class.
- Offered good practical experience in applying mathematics.
- Different, but lifelike. Kind of left out in the cold, though. We hadn't any previous experiences like it.
- Not too bad. I was worried, but it was okay.
- A lot of fun and it used our math skills.

The sources are all around us. All we have to do is look at the extra calculus books on our shelves. Consider a few of the types of problems found in the max-min section of these texts. For example, there is the packing of an orchard with trees with (density dependent) decreasing yield per tree in order to maximize the yield of the orchard. Or take the problem of a trucking firm trying to set a reasonable speed to minimize trip costs for their truckers with hourly driver costs and decreasing mileage efficiency for higher speeds.

CONCLUSION

We encourage you to try this experiment. It will add freshness to your class – be sure to do it unannounced. It will add fun to the course for the students and to your professional life as a teacher. It will generate confidence in the students. It will challenge them and get them to speak out in class

- they have to! But most important, after struggling with problem formulation, it will permit the students to experience the thrill of solving a real problem using the calculus.

REFERENCES

1. Bender, E. A. 1978. *An Introduction to Mathematical Modeling* (New York: John Wiley & Sons).