Wreckless Abandon[1]

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One measure of teaching effectiveness is "teacher liberation." In many evaluations it is teacher enthusiasm that marks student interest and if the teacher feels uninhibited, energized, and inspired, then so too will the students. A second measure of teaching effectiveness is how emboldened the students are to explore, to learn on their own. We offer some illustrations of how using technology to teach with "wreckless abandon" can enhance both the teacher's and the students' measures of effective teaching and effective learning. We refer to the Old School here as without technology and the New School with technology.

Long ago I was given a gift in my teaching career – a VAX Workstation and Maple computer algebra software. This was a precious gift and I was on my own to explore how to use it in my teaching. It could be abused, e.g., assign students "by-hand" work and use Maple to check the answers. I was determined to do more creative things with the software, to go places I had not been able to visit, but I was not quite ready to proceed with "wreckless abandon."

One of the first things I did was to reverse the game on assembling information on intercepts, derivatives, slopes, etc. to gain information on a function in order to render a plot of the function. I simply used Maple to plot the function FIRST and then we "studied" it like one would study a creature. We analyzed it with derivatives to try to explain why it went the way it did. We sought its peaks, its troughs. We had the object in front of us and we were strengthening our understanding of the calculus notions by this scrutiny. This is a different approach than using the calculus information to build a graph, the latter could now easily be done by Maple, and so this freed us to study the function with the power of Maple and ask deeper questions about the function and its properties.

For example, we could plot and see the first and second derivative instantly along with a plot of the function itself and thus match up properties of derivatives with behaviors of plots instantly. We could ask students to identify plots of $f(x)$, $f'(x)$, and $f''(x)$; indeed we had students make up quizzes for each other doing just that, namely - for a plot of $f(x)$, $f'(x)$, and $f''(x)$, identify each. Old school could not do all this because the technology was not there. However, some strengths of the old school were inquiry, students' taking ownership for learning, and constructivism in learning. Technology enabled all of these nicely.

[1] We note that rather than use Reckless Abandon we prefer Wreckless Abandon, because that is the hope -- teaching with reckless abandon, letting technology carry the day and pick up the pieces of tedium called algebra and symbol manipulation, but without "wrecks."

A weakness in the old school was the universal expectation of flawless algebraic skills, of attention to the minute details of symbol manipulation, and of the development of a set of clever tricks to reduce problems or equations to some special form so another trick could be applied. Now with computers performing the tricks, the student is free to move beyond manipulation and recalled tricks to see the big picture without the hindrance of algebra errors, lack of insight into the domain of tricks, and pages of hand written calculations.

Students routinely question their work more with the new school of technology, for to find an error is then but to make a simple change in a term and re-execute the calculations of a page or more. While in the old school, before technology, to find an error meant redoing all the hand calculations at best, or throwing that write-up out and starting anew. Most students did not redo their work if they found an error. Most students did not even go looking for an error because they understood the tremendous consequences – better to just hand it in and hope for the best. New school technology changes all that for the better.

Fourier Series

We give two examples of how the new school thinking with technology actually enabled us to teach with wreckless abandon. The first is Fourier series. For a number of years now we have introduced students to Fourier series, by not introducing them to Fourier series, but rather having them discover how to approximate a function with a "bunch of sinusoidal generators." They never fail to discover the theory, to make it their own, and to enjoy it. It is the technology that permits this discovery and the reinforcing feedback to support the students as they conjecture and move ahead.

We introduce the idea of using sine functions to approximate functions, usually over the interval [-π, π]. We have been very successful in getting students to discover Fourier series – we never use that name, we stick to the idea of building signals with signal generators. We use signal generators $g_n(x) = \sin(n\,\pi\,x)$, $n = 1, 2, 3, 4, \ldots$ to generate signals $f(x)$ --- specifically odd functions in the interval [-π, π] --- as this is a reasonable task for new school students equipped with technology, but not as notationally confusing as the full general Fourier series.

We ask the students to find constants a_n such that the sum

$$\sum_{n=1}^{m} a_n \cdot \sin(n\pi x)$$

is a good approximation of $f(x)$. I.e.

$$f_{approx}(x) \approx \sum_{n=1}^{m} a_n \cdot \sin(n\pi x)$$

Students discover the least square criteria – they do so EVERY time on their own and then they set out to try to minimize the least square error. Here l = π.

$$\int_{-1}^{1} \left(f(x) - \sum_{n=1}^{\infty} a_n \cdot \sin(n\pi x) \right)^2 dx$$

They first see if they can do this for a finite approximating sum of sine functions, $n = 2$.

$$\int_{-1}^{1}\left(f(x) - \sum_{n=1}^{2} a_n \cdot \sin(n\pi x) \right)^2 dx$$

We use a nice function, say $f(x) = x$.

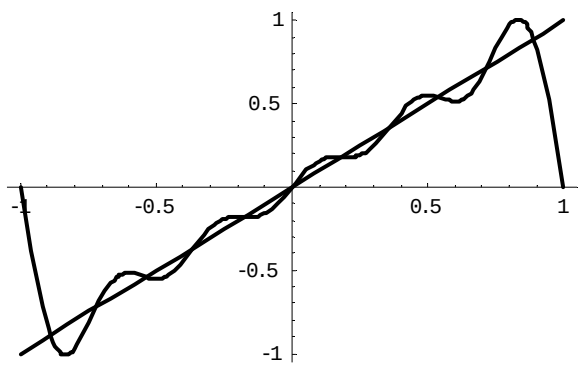
This sum of squares is a function of two variables, $a = a_1$ and $b = a_2$, and we can minimize it using calculus, computing the partial derivatives with respect to a and b and setting these derivatives equal to 0 to determine the a and b which make $S(a,b)$ minimum.

$$S(a,b) = \int_{-1}^{1} \left(x - a \cdot \sin(\pi x) - b \cdot \sin(2\pi x)\right)^2 dx$$

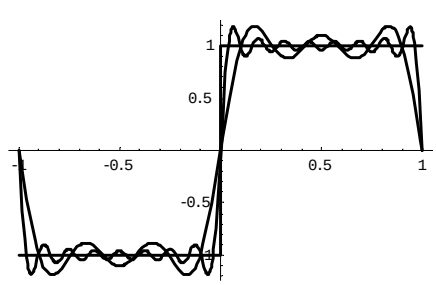
Then we do this for three terms $S(a,b,c)$, and four terms, $S(a,b,c,d)$. We observe a pattern and predict what the a_n's will be at the same time we confirm our bettering of the approximations with plots. This is an example of immediate feedback with new school technology.

Now we generalize with 2, 3, 4, etc. terms using general odd functions $f(x)$ to obtain the a_n's.

We confirm our strategy with plots for $f(x) = x$; $n = 5$ here.



Here is a plot of $n = 3$ and $n = 5$ term series over a step function.



As you can see from this stream of consciousness development the students can discover the concept of a Fourier series with the new school of technology tools of differentiation and plotting in a computer algebra system.

Ecological modeling –more and more realistic efforts.

In most modeling efforts students are encouraged to build a classical predator prey-model of the following form:

$$N'(t) = r * N(t) - a * N(t) * P(t) \quad \text{and} \quad P'(t) = - c * P(t) + b * N(t) * P(t) \ .$$

We encourage students to build such a model for $N'(t) =$_____ and $P'(t) =$ _____ stating reasonable assumptions about a closed system predation model with $N(t)$, the biomass of prey, and $P(t)$, the biomass of predators at time t, and to immediately plot the numerical solutions to the system of nonlinear differential equations with a reasoned set of parameters. For example, consider the relationship between a (predation notion) and b (consumption notion). Thus we see $b < a$, i.e. conservation is in effect. The emphasis is on assumptions, on units, on terms, and on interactions between variables, NOT on algebra or calculus solution strategies. It is time for something new in a mathematics class – play! We study the system, its flow, its stability, its range, etc. We see how these are altered by changing parameters, one at a time as a good scientist does in a controlled experiment. We do a reality check as we go, storing up the good points and looking out for the bad ones.

We proceed to find inadequacies in the model that make it unrealistic and attempt to patch it up with an improved model. Students point out the "foolishness" of the $r * N(t)$ term in $N'(t)$ as it represents exponential or unlimited growth so we attempt to remedy that with the introduction of a carrying capacity and notions from the logistic equation. We continue to keep an eye on the reality check aspect of modeling by examining the consequences – usually with the new school technology. Again we are always checking units, looking at plots.

$$N'(t) = r * N(t)*(K - N(t))/K - a * N(t) * P(t) \quad \text{and} \quad P'(t) = - c * P(t) + b * N(t) * P(t)$$

One of the last modifications is to examine the predation term $a * N(t) * P(t)$ by thinking of the predator ($P(t)$) as the student and the prey ($N(t)$) as pizza. If $P(t) = 1$, i.e. YOU and $a = 0.1$, say, then if $N(t)$ pizzas show up at your door you eat $a * N(t) * P(t) = 0.1*N(t)*1$ pizzas per hour --- say t is in hours (i.e. one-tenth of ALL the pizzas that show up at your door per unit time!), in particular if $N(t) = 1$ you eat 0.1 pizza per hour; if $N(t) = 100$ you eat 10 pizzas per hour; if $N(t) = 1000$ you eat 100 pizzas per hour; etc. Not so!!! You become full, indeed, you become stuffed. There is a satiation going on here and you can only eat so much per hour so this predation term, $a * N(t) * P(t)$, and the corresponding assimilation term, $b* N(t) * P(t)$, have to be modified to reflect some limiting factor, i.e., as $N(t)$ gets larger and larger we can eat only a maximum amount per unit time. We actually devote a whole class period to this one term, with students attempting to model satiation with lots of conjectures, either qualitatively by hand or by implementing them on our technology.

Students come to grips with the fact that they now struggle to offer up a term or modification to the predation term to reflect satiation. They usually can arrive at something like this:

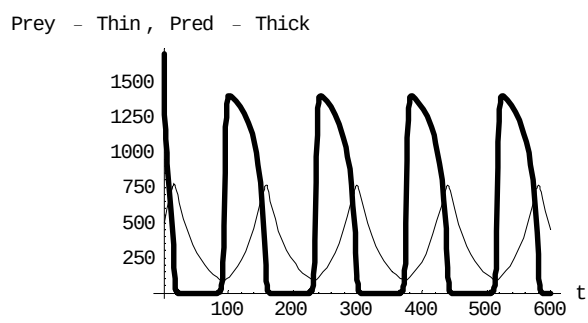
$$N'(t) = r * N(t)*(K - N(t))/K - a * N(t) * P(t) /(L + N(t))$$

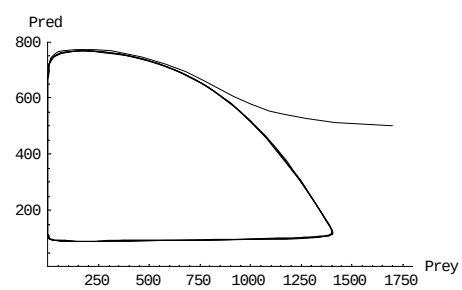
$$P'(t) = - c * P(t) + b* N(t) * P(t) /(L + N(t))$$

where L is the level of the prey at which the predator feeds at one half its maximum feeding rate, that maximum level being $b*P(t)$ and when $N(t) = L$ we have a feeding rate of

$$b* N(t) * P(t) /(L + N(t)) = b* L * P(t) /(L + L) = b/2*P(t).$$

Below we offer a plot of the result of the students' modeling, just one of many plots made on the road to success, only because our new school technology lets us explore, to play "what if" games, and to try out our theories easily.





Conclusion

We have demonstrated several instances of how the new school with technology is richer than the old school of pencil and paper, of how the students explore in this new world, and of how we teach to a new school of technology-enabled students. Liberate yourself from the tedium of by hand algebra and symbol manipulation, take risks, play, improvise, build models, do "what if" games, experiment, let your students lead, and enjoy the new school approach with technology.