Some Algorithms for the Attenuation of Broadband Noise

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Overview

- Noise cancellation with noise reference
  - MSE criterion-Wiener filtering
  - Adaptive filtering
- LMS algorithm
  - Performance limitations
- Adaptive Prediction Filters
  - Block, Normalized

Noise Sources

M113, HMMV

- Tones from engine harmonics
- Broad-band components

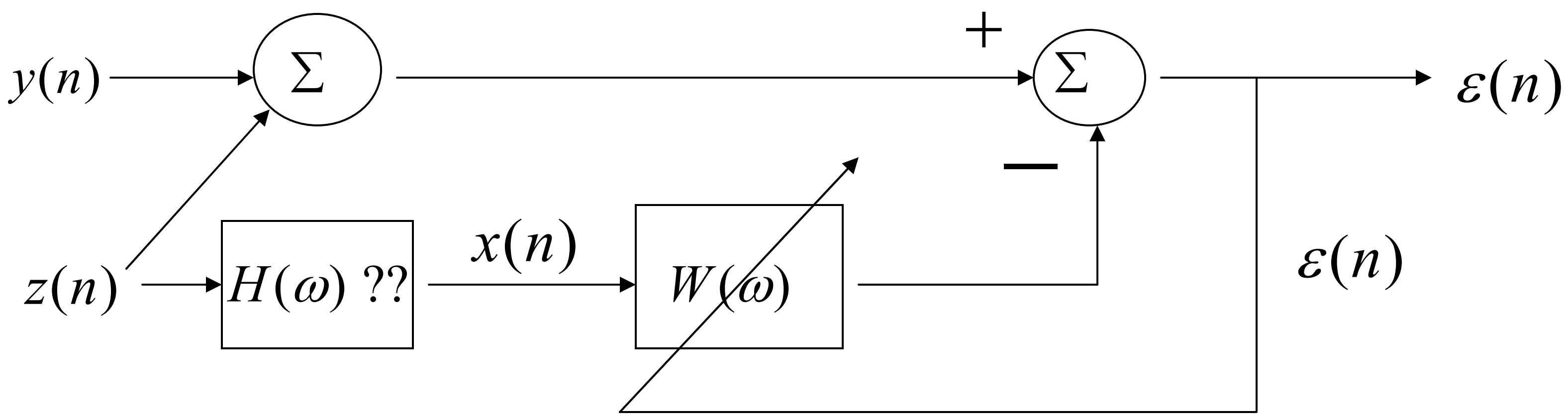
  track noise, wind, ???
- Tones 30-40 db stronger than BB

Ground array sensors

- Wind, other ambient noise

Noise Cancellation With Noise Reference

- Primary signal: signal of interest with additive noise.
- Reference signal: some coherent version of the noise
- Subtract filtered reference from primary.



References

- Wiener, 1940's linear prediction, estimation, etc.
- Widrow, Hoff 1959. LMS algorithm.

Adaptive Filters:

- Widrow and Stearns.
- S. Haykin.
- Honig and Messerschmidt
- Zeidler, 1990- adaptive prediction filters

Spectral Estimation

- S. Kay
- L. Marple

MSE/Stationary Signals

Error

$$\varepsilon(n) = y(n) + z(n) - \sum_{k=0}^{L} w_k x(n-k)$$

$$= y(n) + z(n) - \mathbf{w}\mathbf{x}'$$

vector of filter weights, $\mathbf{w}$,

vector of past $L+1$ $x-$samples, $\mathbf{x}$

MSE

$$E(\varepsilon(n)^2) = E(y(n)^2) + E(z(n) - \mathbf{w}\mathbf{x}')^2$$

So output is min $\Leftrightarrow$ noise is min

$$\mathbf{w}_{opt} = \mathbf{R}_{xx}^{-1}\mathbf{P}_{xz}$$

Stationary process; known statistics:

- Wiener-Hopf equations $\mathbf{w}_{opt} = \mathbf{R}_{xx}^{-1}\mathbf{P}_{xz}$
- Noise covariance matrix, $\mathbf{R}_{\mathbf{xx}}(i,j) = E(x(i)x(i+j))$
- Cross-covariance vector, $\mathbf{P}_{zx}(j+1) = E(z(n)x(n-j))$
- Quadratic error surface

Least-mean square (LMS) algorithm :

$$\varepsilon(n)^2 = y(n)^2 + (z(n) - \mathbf{w}_n\mathbf{x}_n')^2$$

Weight updates $\mathbf{w}_{n+1} = \mathbf{w}_n - \mu\nabla(n)$

$$= \mathbf{w}_n + \mu\varepsilon(n)\mathbf{x}_n$$

$$\nabla(n) = Grad(\varepsilon(n)^2), \quad \mu = step\ size.$$

LMS limitations

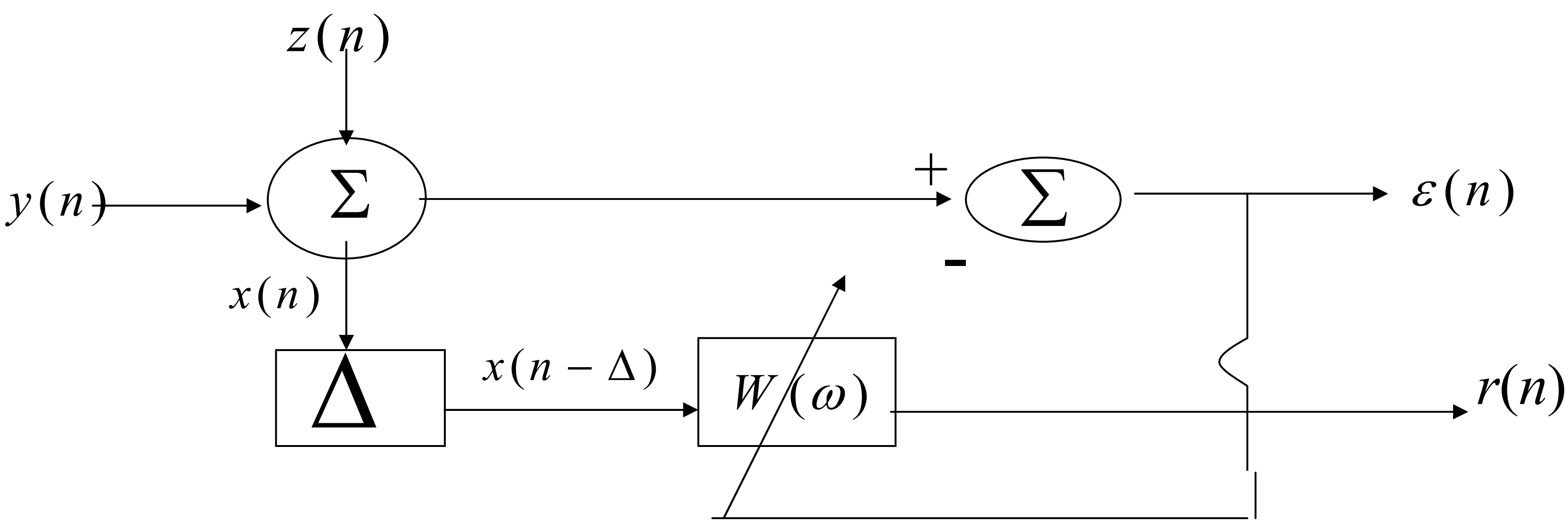
- Noisy
- Performance dependent upon coherence of reference and primary noise
- Modes of convergence -> high-power tones, narrowband components attenuated.

LMS attributes

- Robust- H^{∞} optimality criterion
- Simple; computationally efficient.

Adaptive Prediction Filters

- Effective when noise de-corellates more rapidly than the signal of interest, hence effective on broadband noise.
- Choose "bulk delay" so that cor. time $z(n) < \Delta <$ cor. time $y(n)$
- Use $x(n-\Delta)$ to predict $y(n)+z(n)$ (as in standard LMS).



Performance/gain

- Sinusoids in WGN (Zeidler, 1990)
- Filter gain

$$G(\omega) = \frac{P_O(\omega_1)P_I(\omega)}{P_O(\omega)P_I(\omega_1)}; \qquad |\omega - \omega_1| >> \frac{2\pi}{L}$$

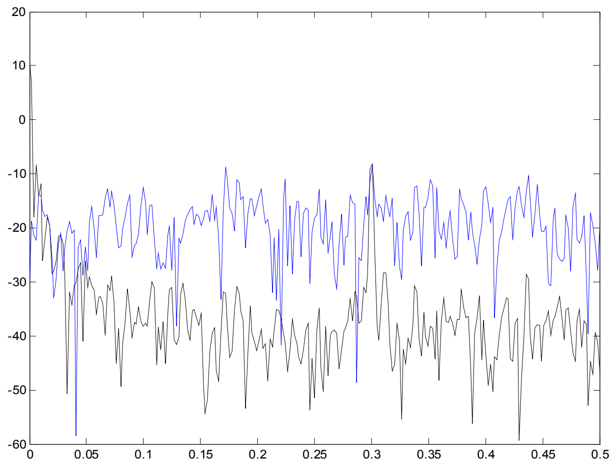
$$= 1 + \frac{a^2}{\mu \upsilon^2 L}$$

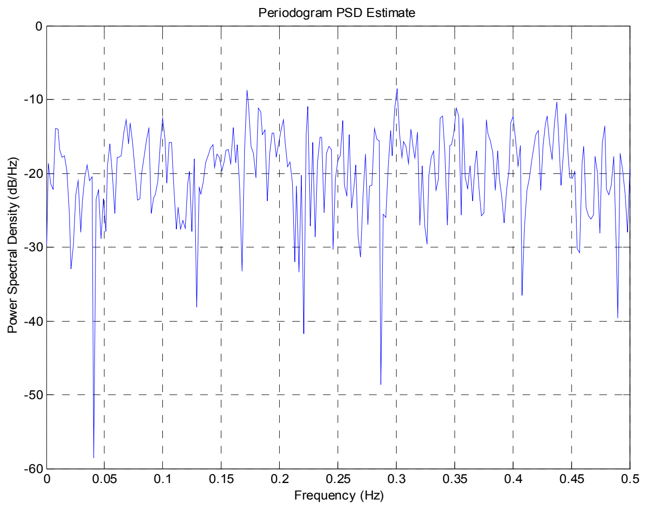
where $a \approx 1$ for large L= filter length; υ^2 = noise variance.

$$L_{opt} \approx \frac{\text{sampling freq.}}{\text{signal bandwidth}}$$

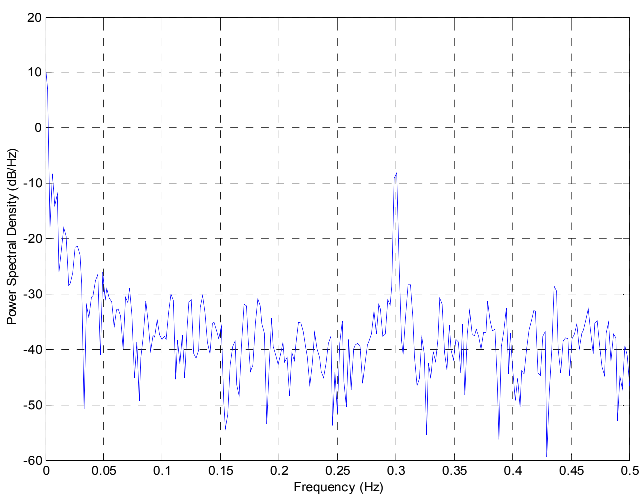
Prediction filter simulations: Sinusoids in WGN

- One sinusoid, freq.= 0.3
- Step size = .0015
- Filter order = 1024
- SNR=1, noise variance=.01
- Filter gain ~ 20 dB
- Bulk delay = 1





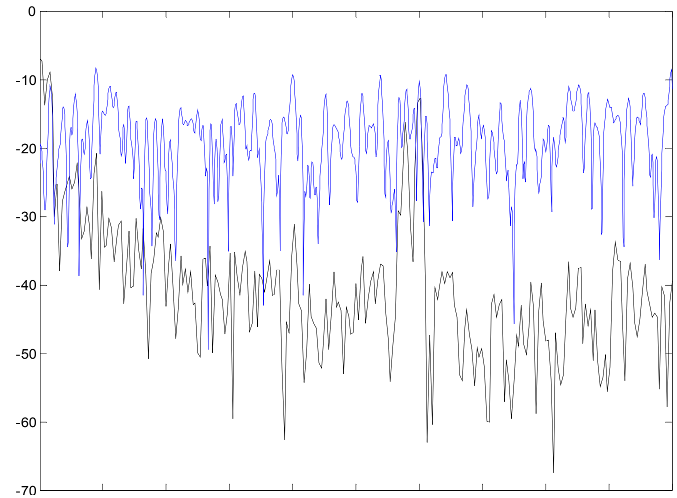
Filter input

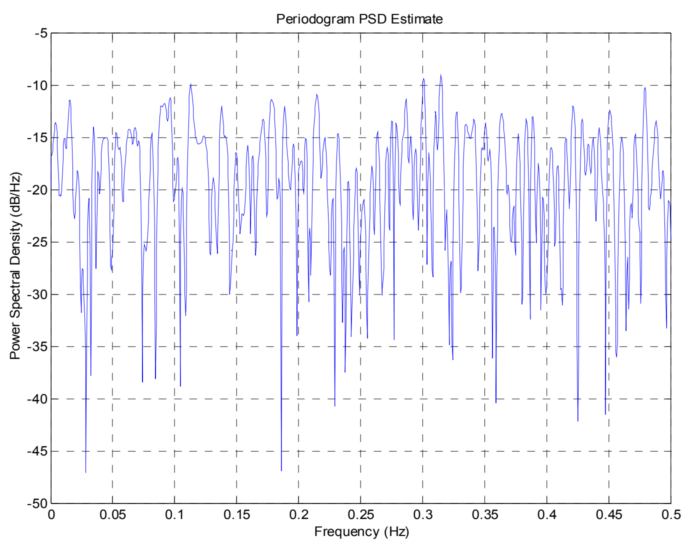


Filter output

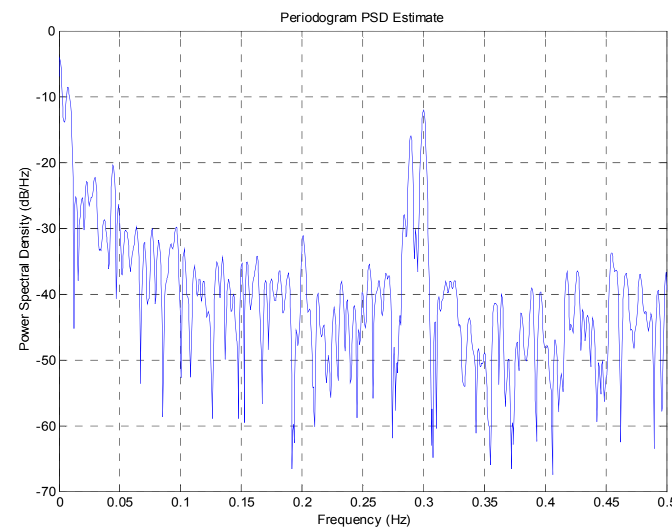
Prediction filter simulations: Sinusoids in WGN

- Two sinusoids: 0.29 ,0.3
- Step size = .0015
- Filter order = 1024
- SNR=1, noise variance =.01
- Filter gain ~ 20 dB
- Bulk delay = 1





Filter input



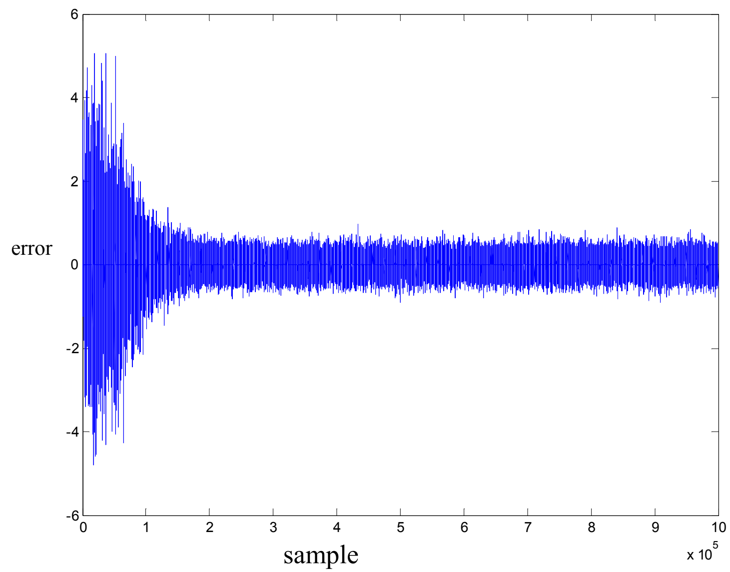
Filter output

Error/Convergence

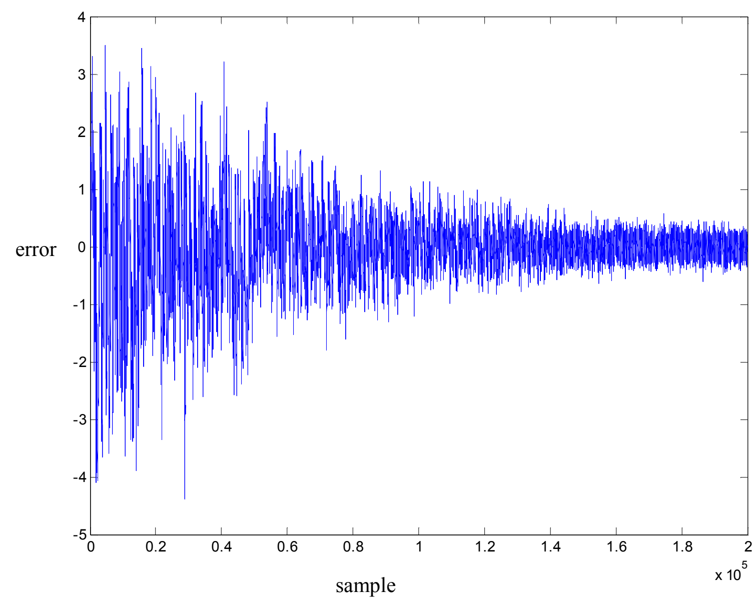
200,000 samples

Error $\varepsilon(n) = x(n) - \mathbf{w}(n)\mathbf{x}(n-\Delta)'$

Filter order $= 1024$, $\mathbf{w}_o = [1,1,1,\ldots,1]$



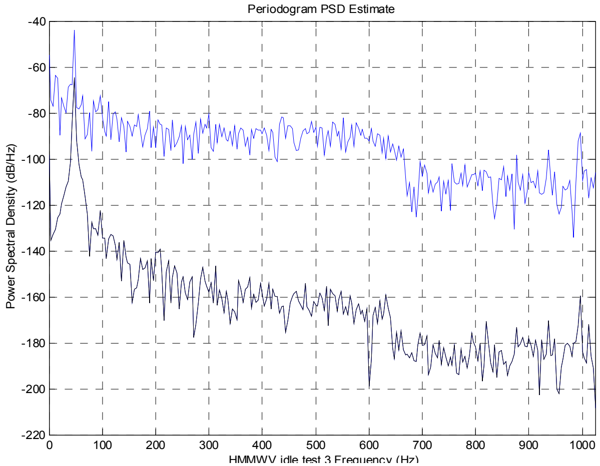
Step size = .015

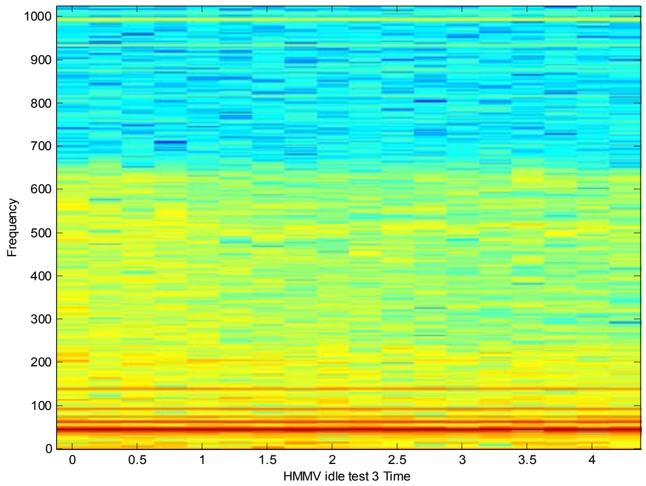


Step size = .0015

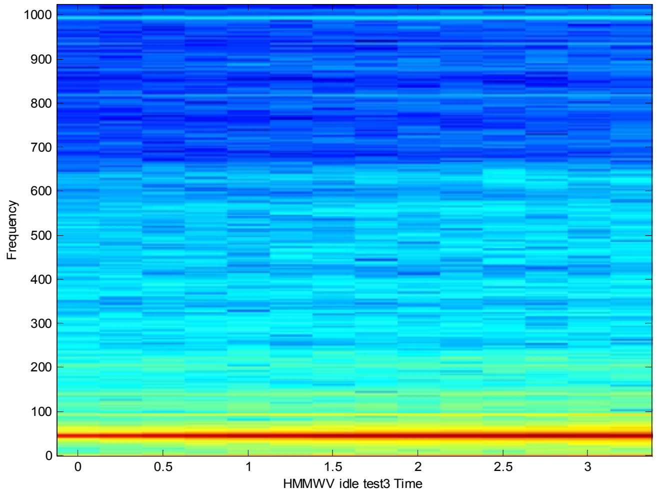
ARL Test Data 9/02

- HMMWV at idle
- 2048 samples/sec
- Roof mic
- Step size = .0015, bulk del.=25
- Filter order = 2048
- Filter gain ~ 35 dB near 50 Hz.



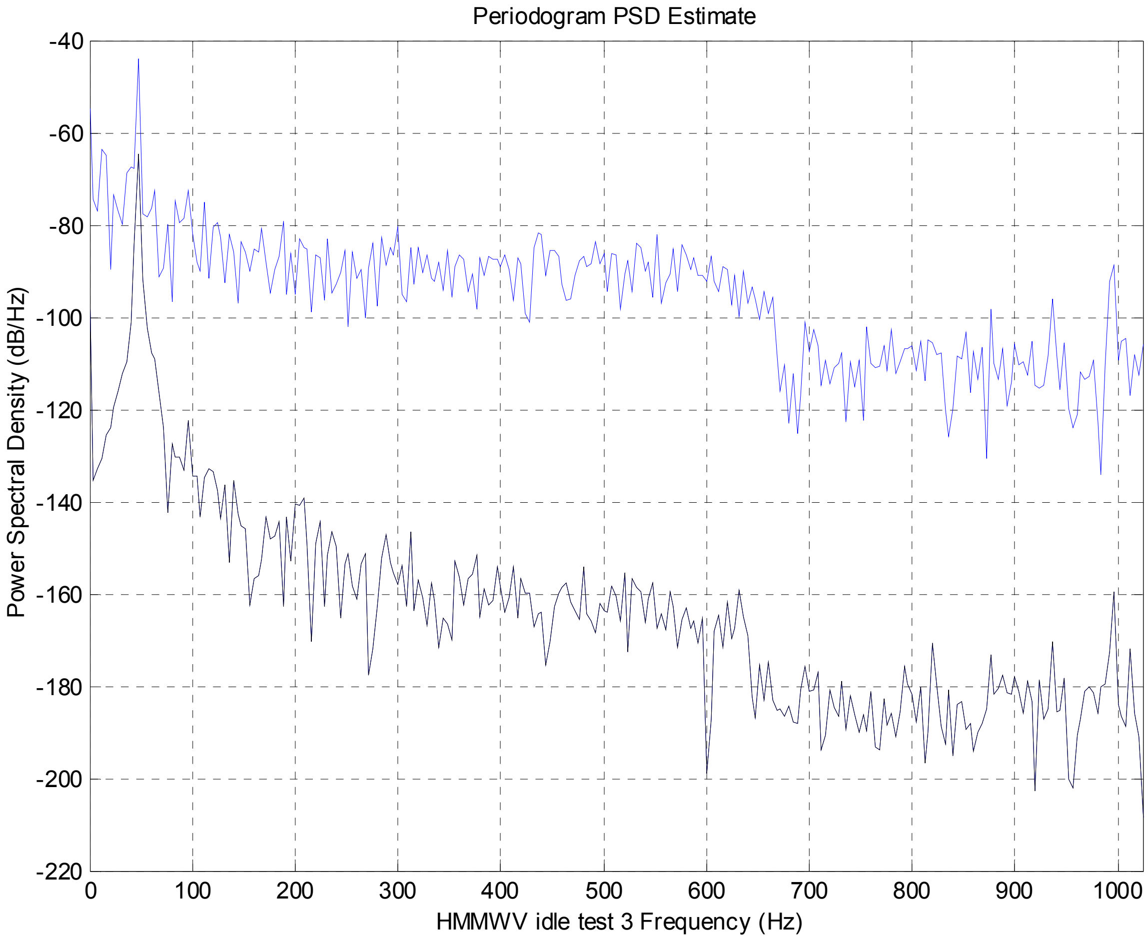


5 sec. filter input



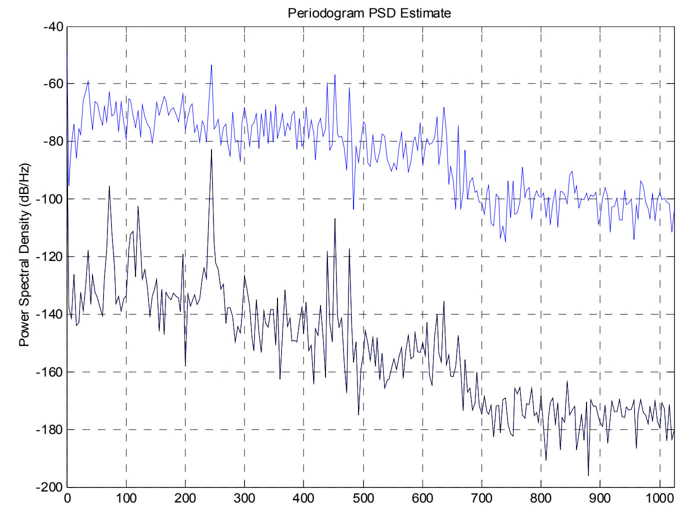
5 sec. filter output

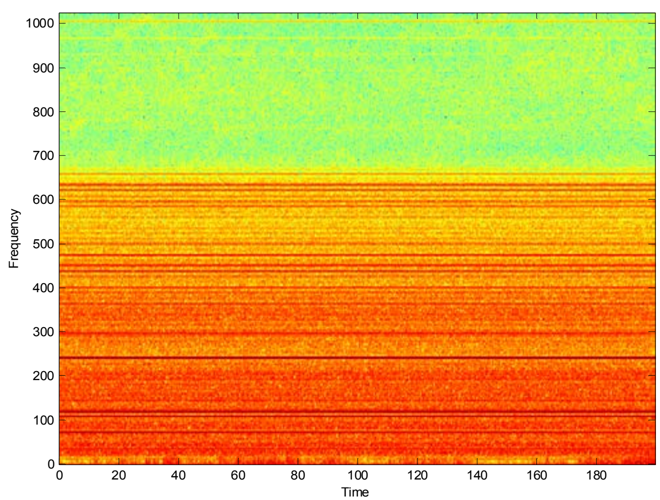
HMMWV at idle.
Filter gain ~ 35 dB



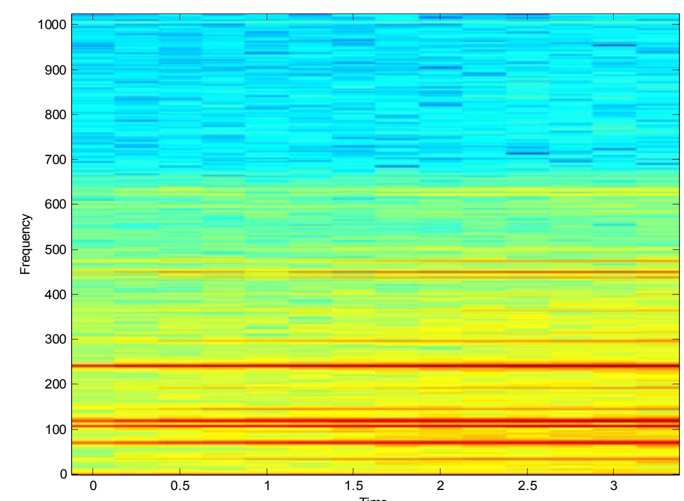
ARL Test Data 9/02

- M113 at idle
- 2048 samples/sec
- Roof mic
- Step size = .0015, bulk del.=25
- Filter order = 2048
- Filter gain ~ 25 dB near 250 Hz.



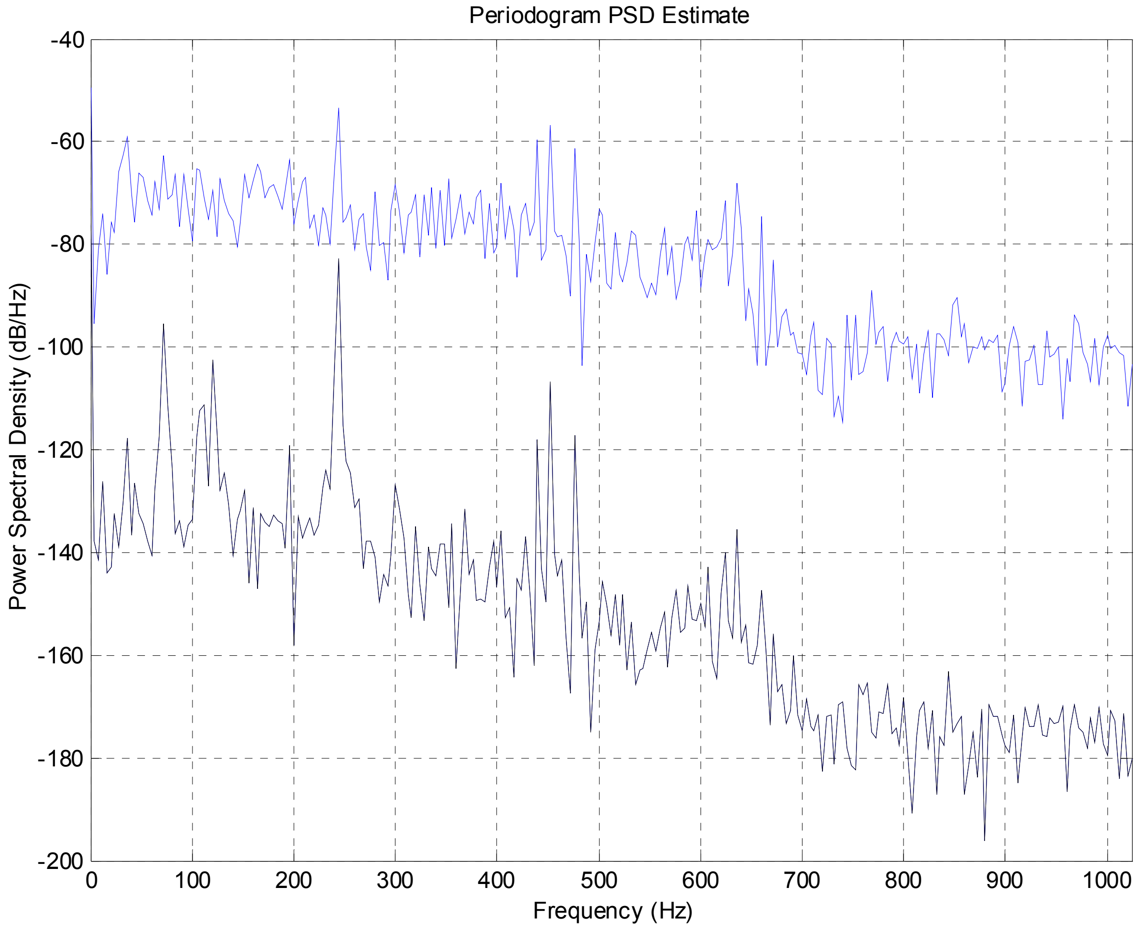


180 sec. filter input



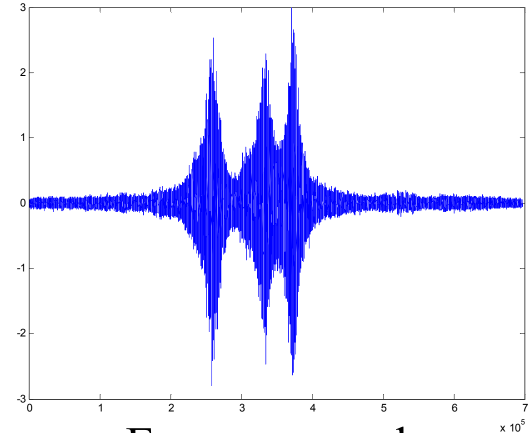
Filter output

HMMWV at idle
Filter gain ~ 25 dB

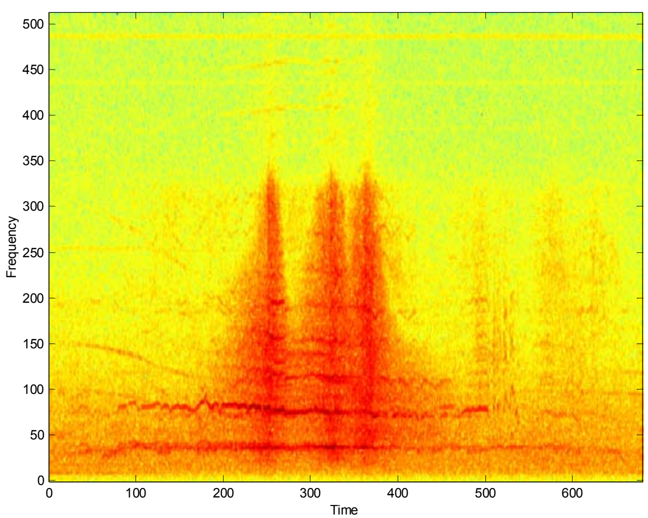


ARL Test Data 9/02

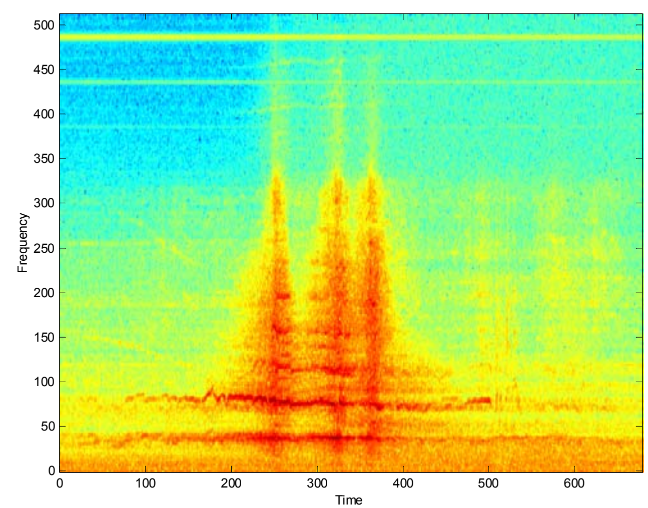
- Ground array sensor
- Three track vehicles approach; pass within a few meters.
- Range ~ 2 km at 80 sec.



Error vs sample

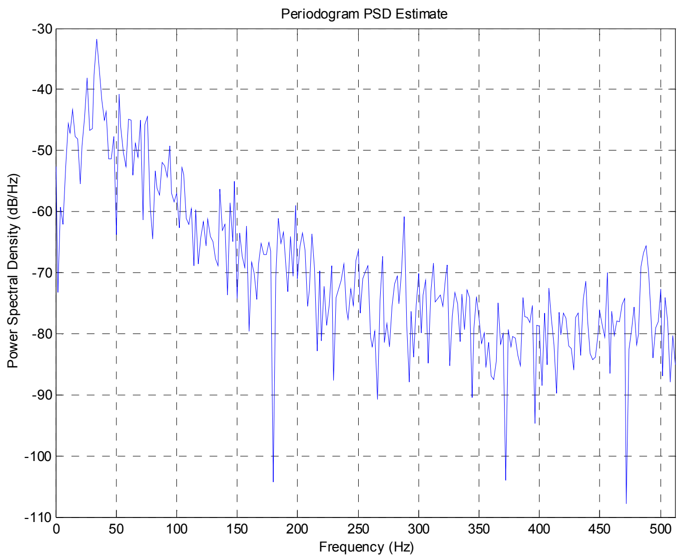


670 sec. filter input

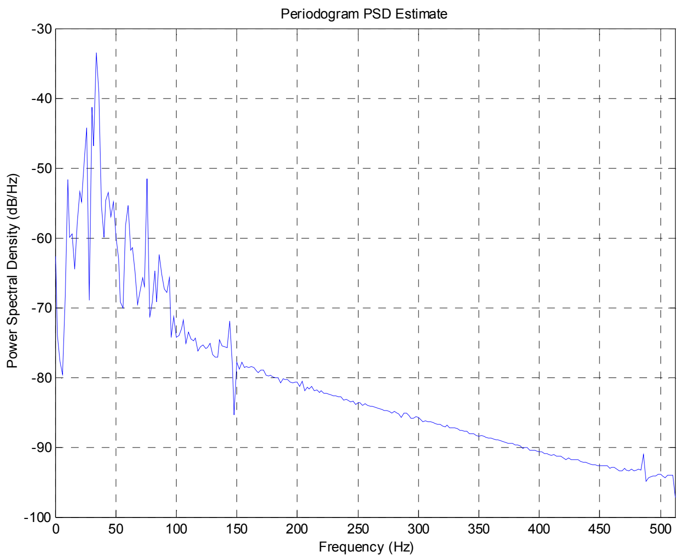


670 sec. filter output

Ground array sensor at 80 seconds



Filter input



Filter output

To address; current work

- Convergence/tracking
- Filter parameters
- Performance - statistical analysis of test data
- Normalized LMS
  - Adjusts step size with each sample
  - Minimizes a-posteriori error
  - Equivalent to "converging" filter weights with each sample.
- Block LMS
  - Adjust filter weights in "blocks" of data samples.
  - Less variance, due to averaging of errors
- Lattice filter
  - Non-LMS approach.