11th ARL/USMA Technical Symposium

ANALYTICAL DETERMINATION OF SHOCK RESPONSE SPECTRA, FOR A PROPORTIONALLY DAMPED SYSTEM

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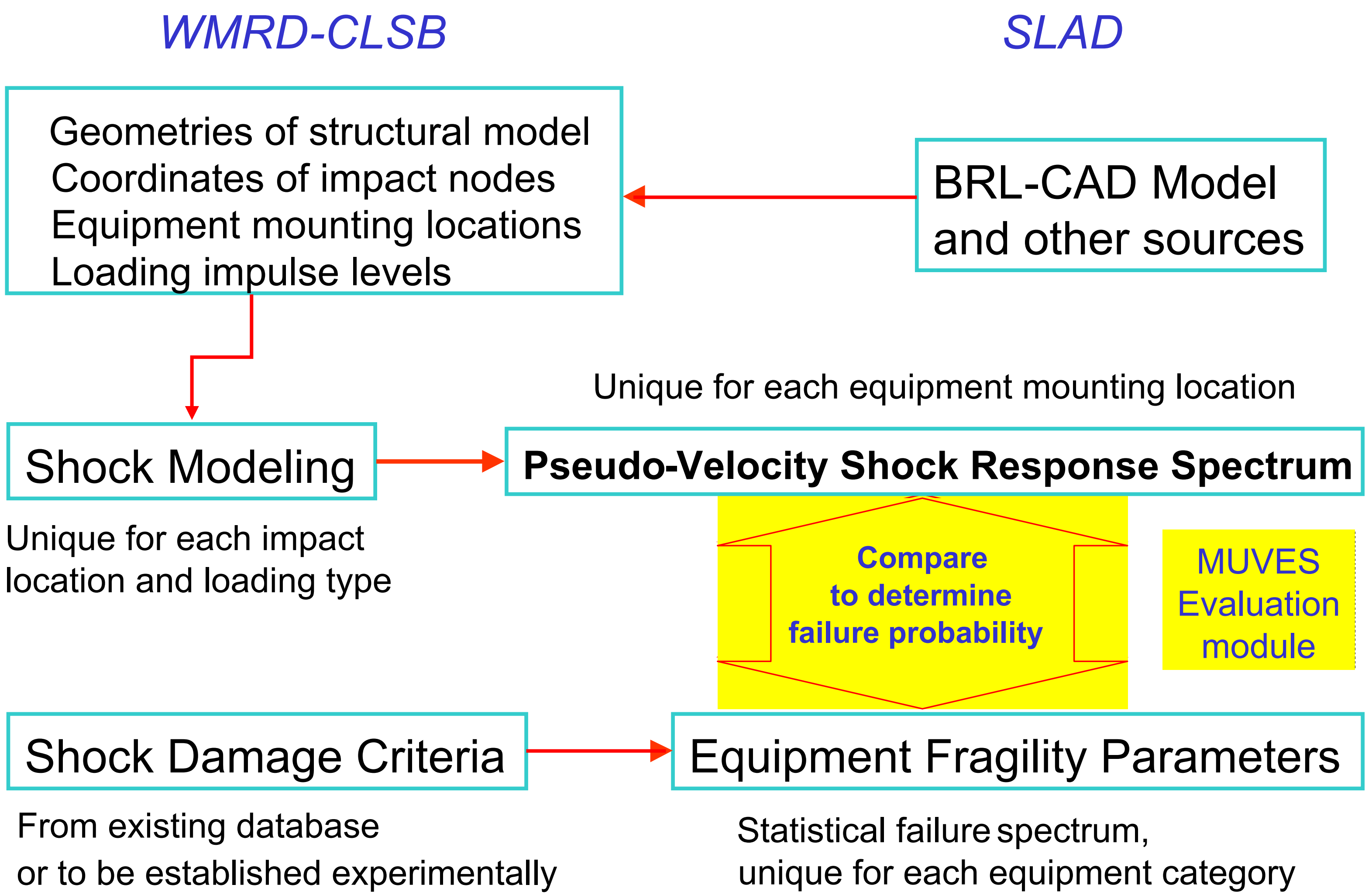
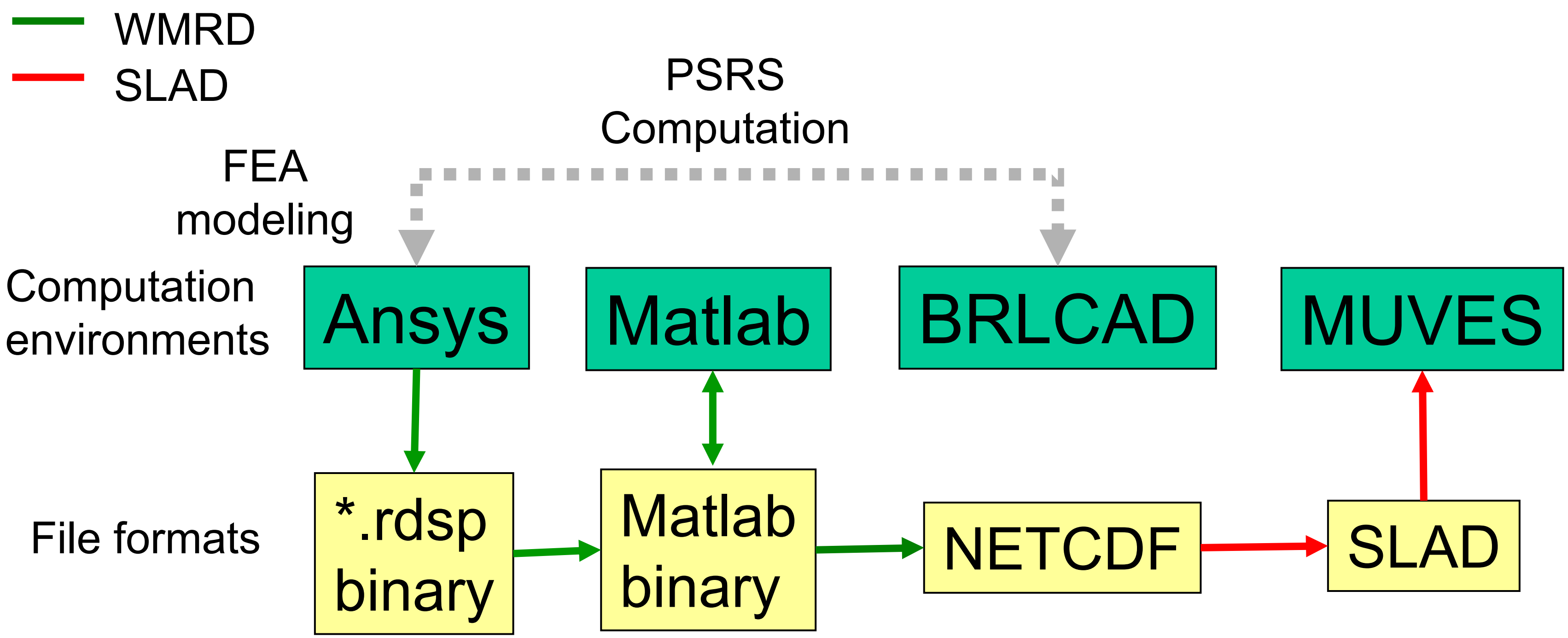
INTRO: Overview of Vibration Analysis Approach



Medium of Exchange & File Formats



- Different computational environments need to communicate with each other by correct file conversion.
- To handle large amounts of data better, conversion codes are being rewritten to improve efficiency.

FEA Shock Models for Armored Vehicles: Reducing the time for impact calculations

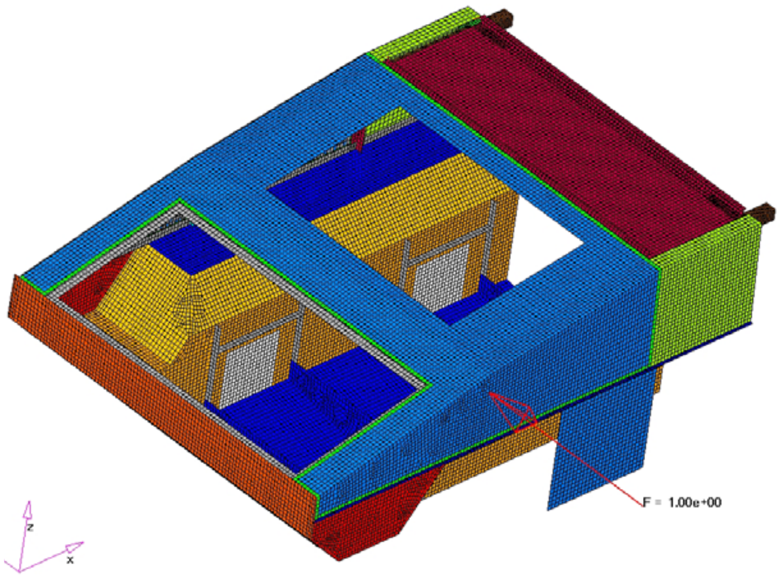
$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\}$$

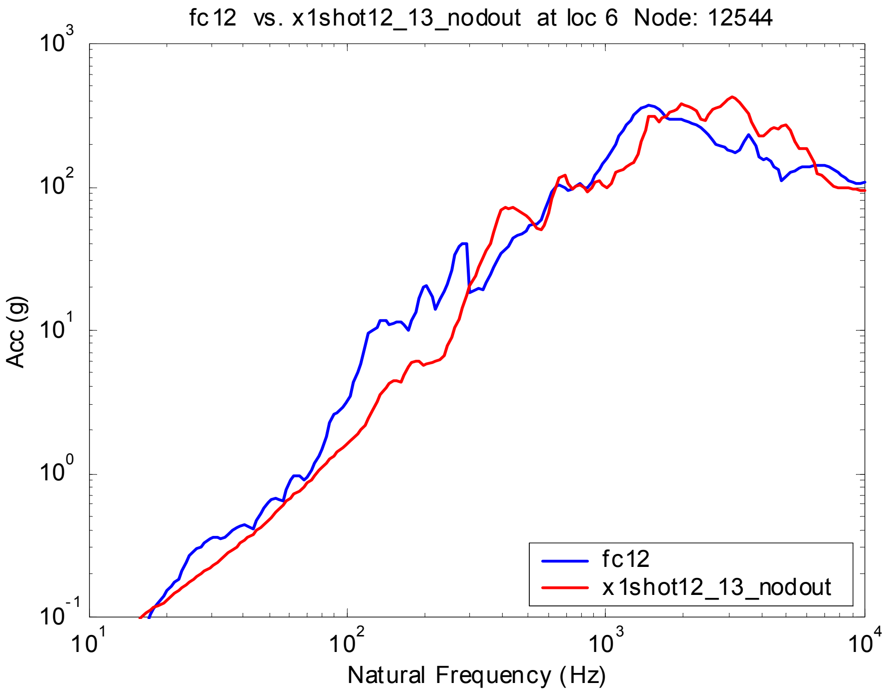
FEA Structural Transient Analysis

- **Direct Integration**
  - Used by most analysts
  - Current shock model: ~2 days/impact
  - Too slow for many impacts
  - But can be used for comparison in a few cases
  - **Explicit**: Dyna, Abacus
  - **Implicit**: Nastran, Ansys
- **Modal superposition**: Ansys
  - Very fast for large number of impacts
  - For linear system only (based on linear superposition of modes)
  - Clear-cut upper frequency, which is limited by number of modes; possible in eigen analysis

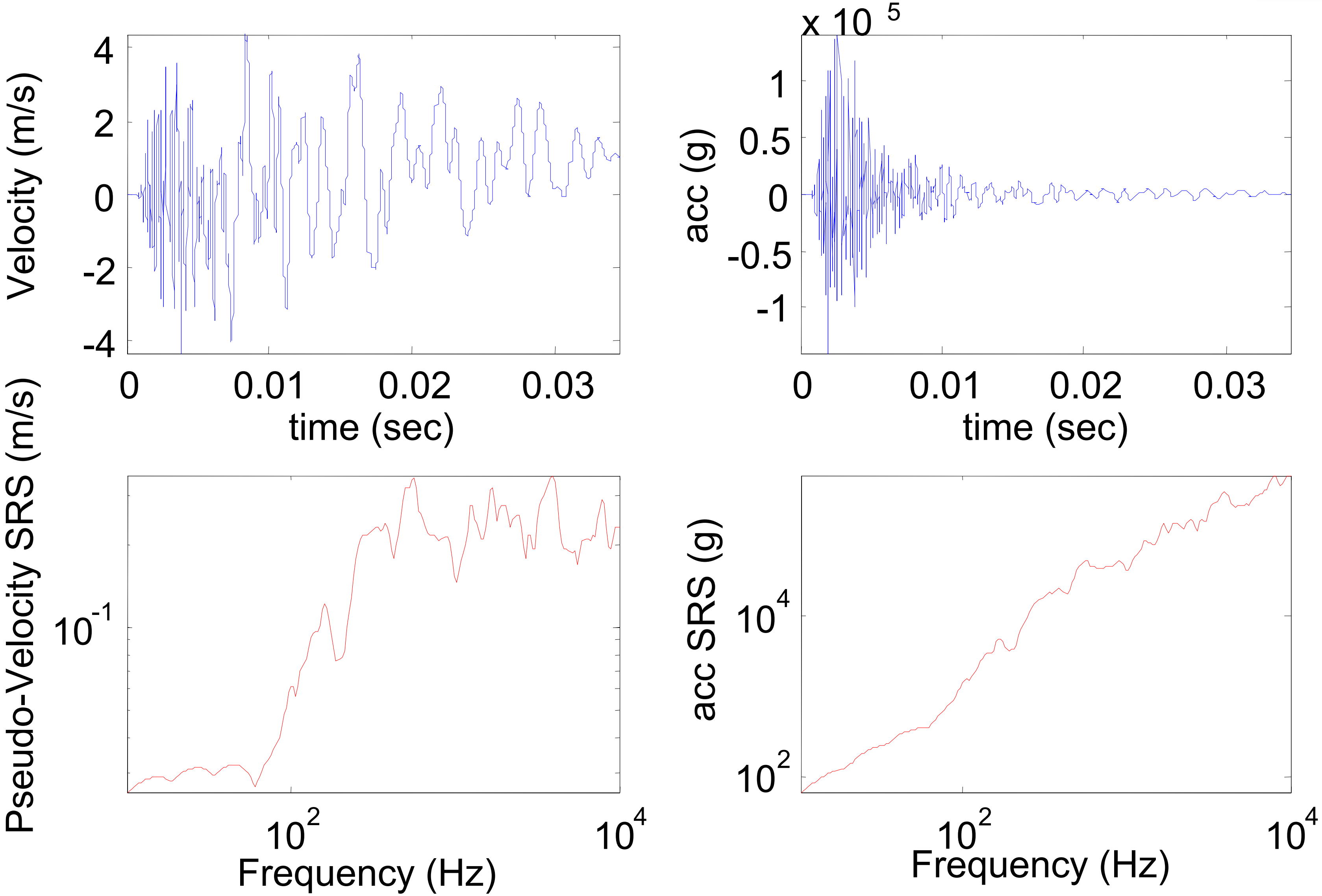
FCS-X1 shock model

Blue: Shock response measurement from ballistic test
Red: Corresponding response from a FEA shock model





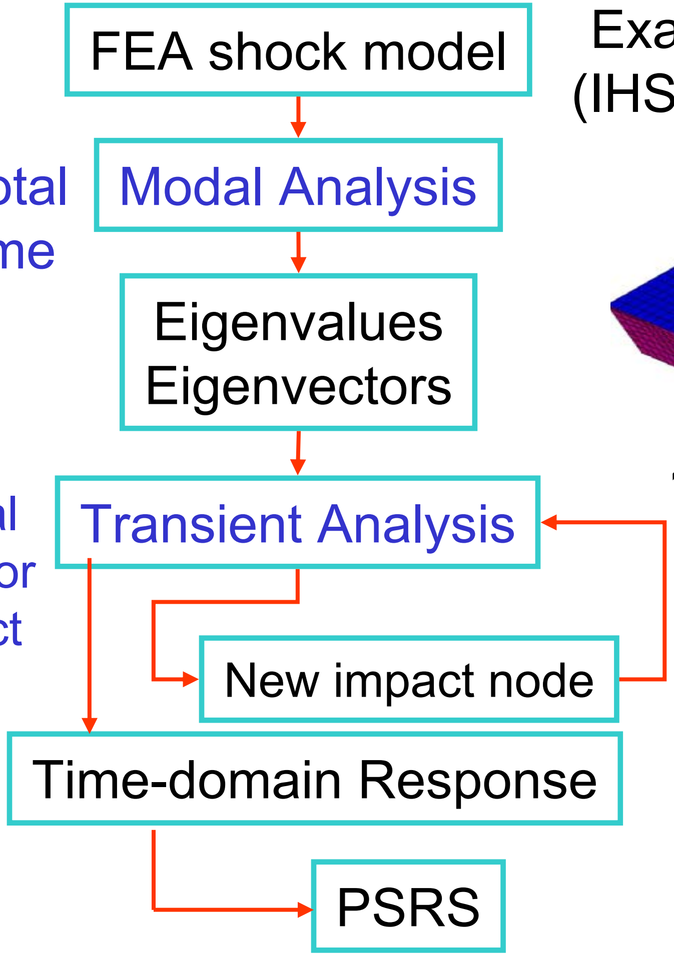
Typical SRS Data and Plots



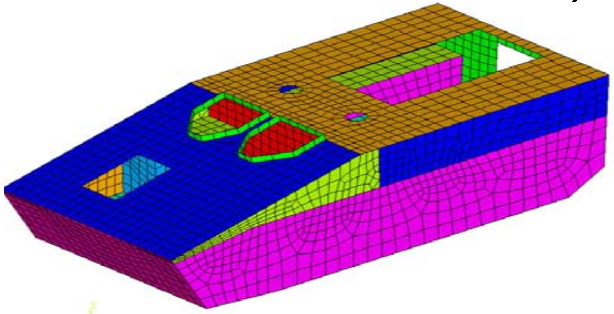
FEA Modal Superposition Method

~99% of total solution time

~1% of total sol'n time for each impact node

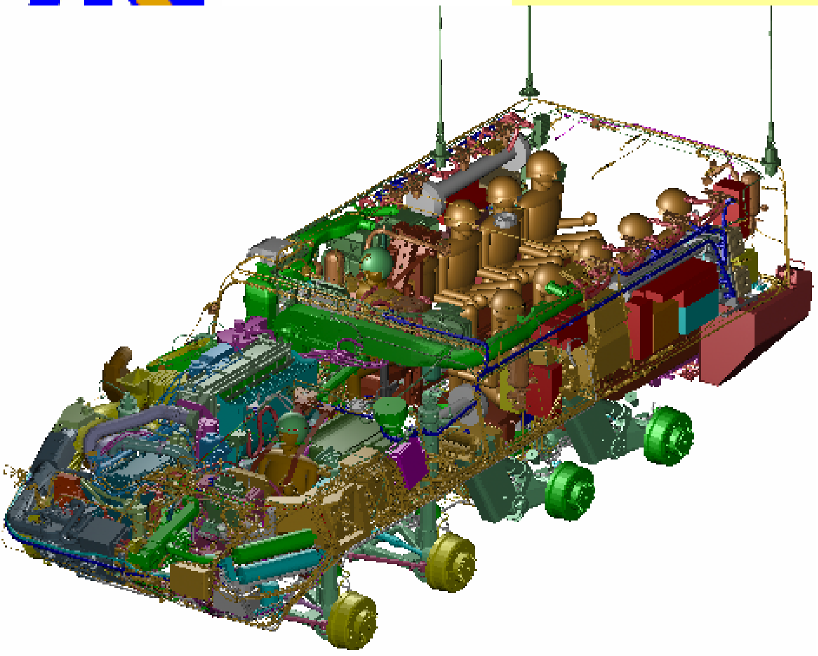


Example of solution time (IHS coarse mesh model)

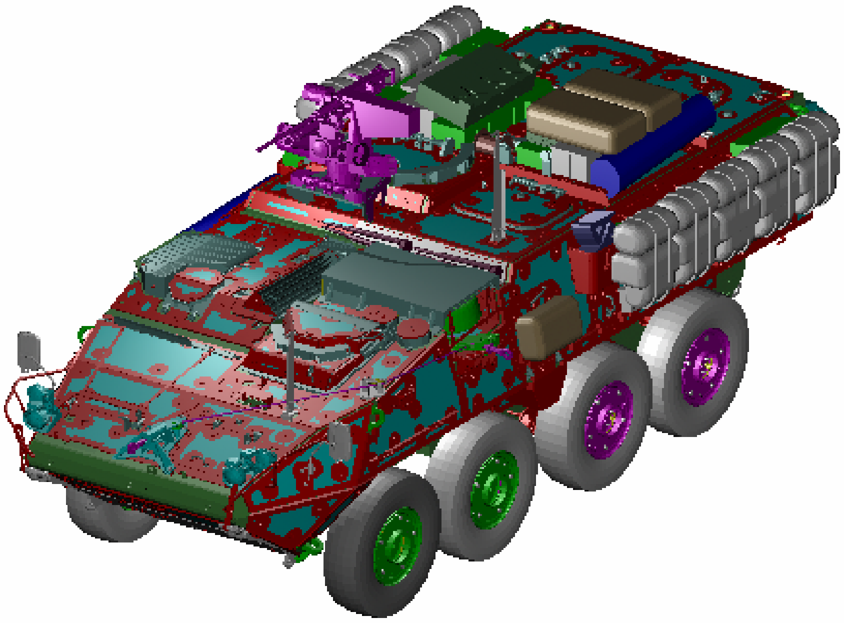


~4000 shell elements
450 modes
Modal analysis time: 67 hrs
Transient solution time: 0.17 hr (for one impact)
0.17 x 600 ~ 100 Hrs (4 days)

Stryker BRLCAD Model – Basis for the FEA Model

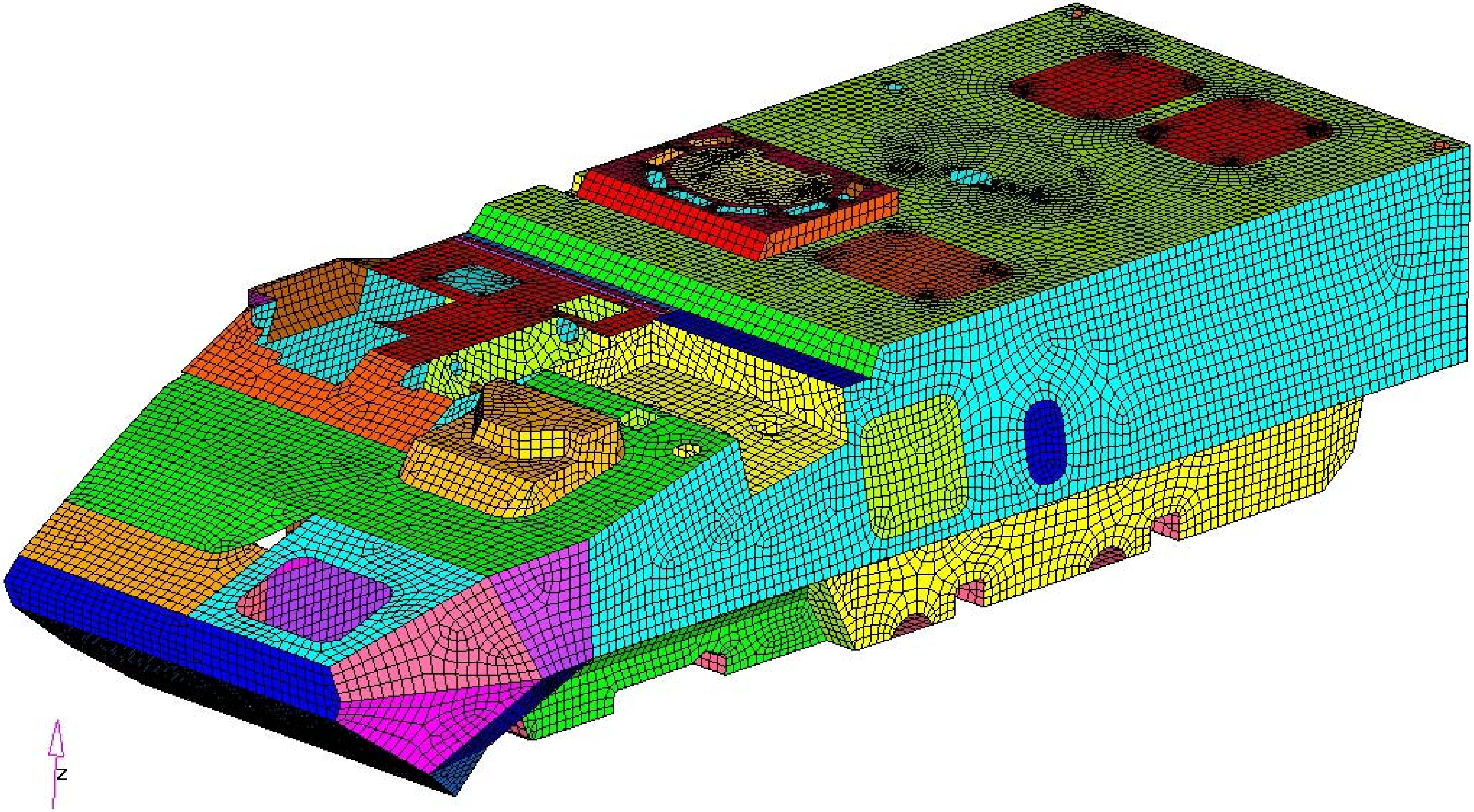


Interior Components

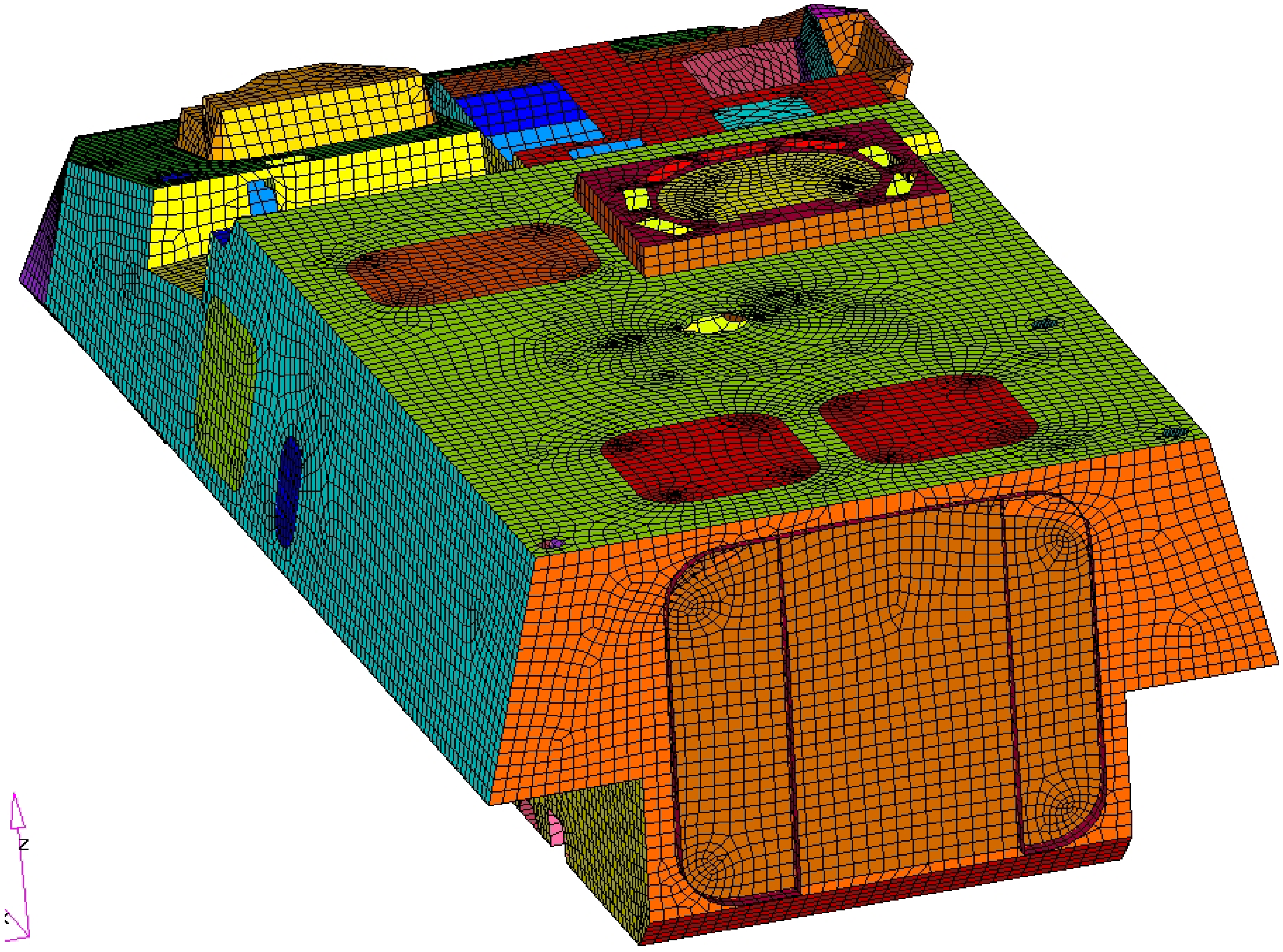


Exterior Components

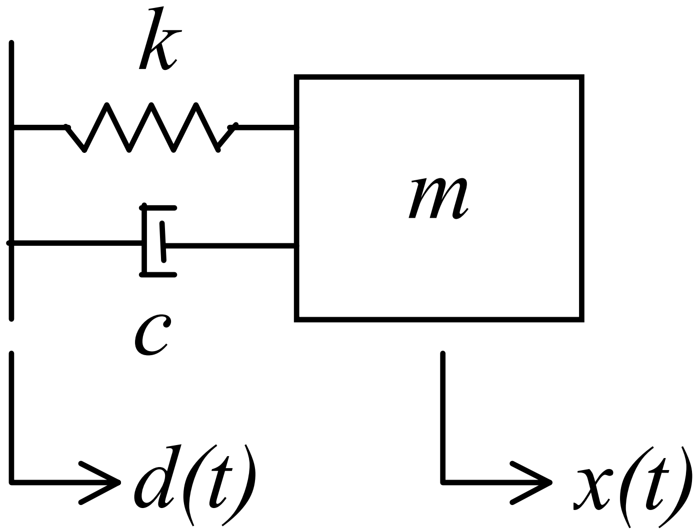
Stryker FEA Model



Stryker FEA Model



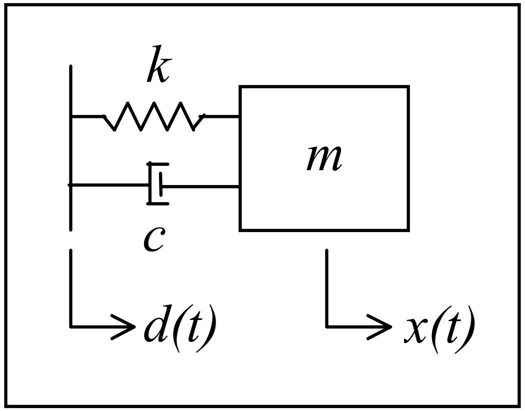
RESEARCH OBJECTIVE: Analytical SRS Determination



Hypothetical SDOF MSD system,
for SRS determination

Analytical SRS Determination

SRS Definitions:



Spectral displacement SRS:

$$S_D(\omega_n) := |x(t) - d(t)|_{\max}$$

Spectral velocity SRS:

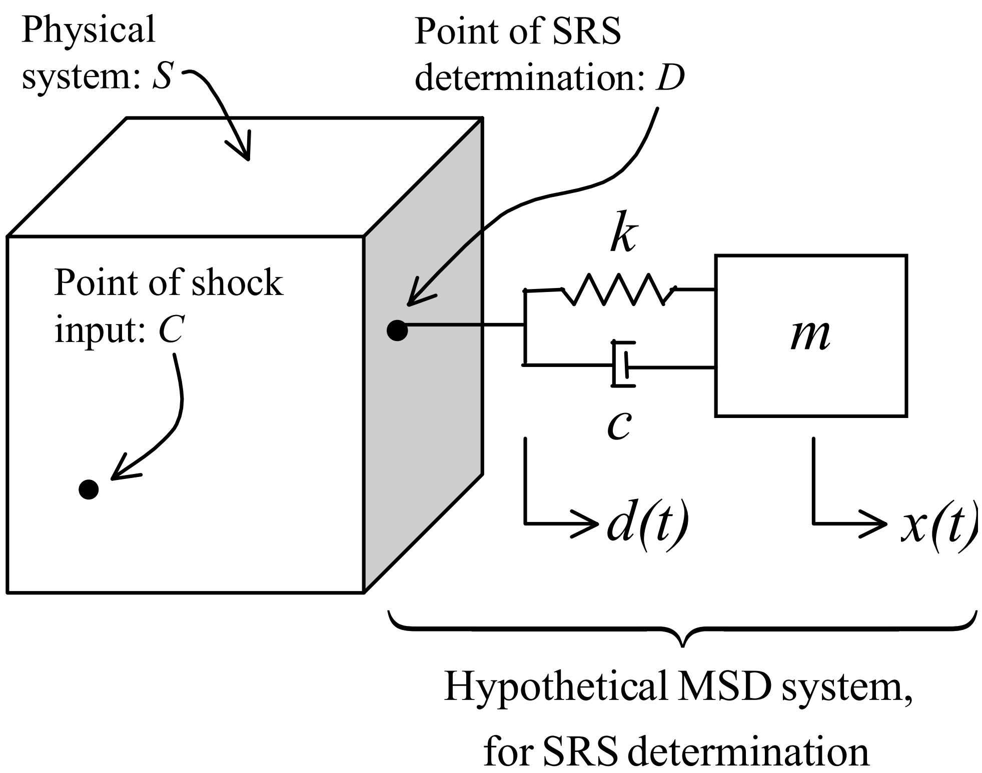
$$S_V(\omega_n) := \omega_n |x(t) - d(t)|_{\max} = \omega_n S_D(\omega_n)$$

Spectral acceleration SRS:

$$S_A(\omega_n) := |\ddot{x}(t)|_{\max}$$

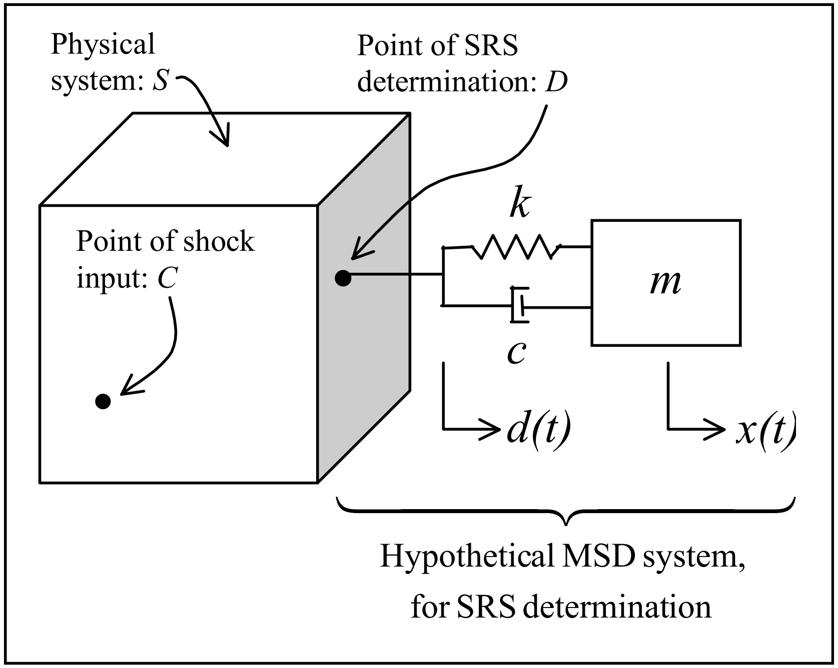
Analytical SRS Determination

Linear system model:



Analytical SRS Determination

USING LINEAR MODEL

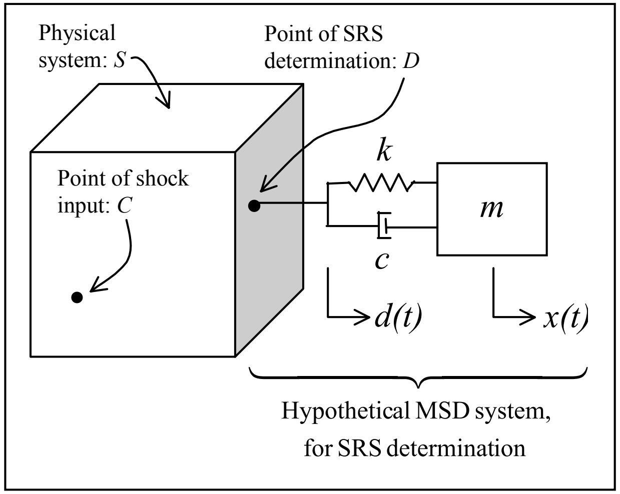


<u>Disturbance *d(t)*</u>:

$$d(t) = \sum_{i=1}^{\nu} d_i(t)$$

$$d_i(t) = D_i e^{-\alpha_i t} \sin(\omega_i t + \phi_i) u_{-1}(t)$$

Analytical SRS Determination



SOLUTION

Displacement *x(t)*:

$$x(t) = \beta e^{-\varsigma\omega_n t}\sin(\omega_d t + \psi)$$

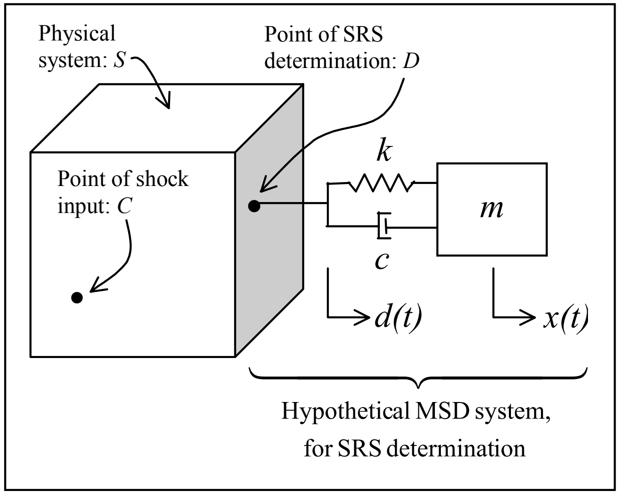
$$+\gamma\sum_{i=1}^{\nu} D_i\Big\{ e^{-\varsigma\omega_n t}\big[\delta_{1i}\sin(\omega_d t + \phi + \phi_i + \theta_{1i})$$

$$+\delta_{2i}\sin(\omega_d t + \phi - \phi_i + \theta_{2i})\big]$$

$$+e^{-\alpha_i t}\big[\delta_{3i}\sin(\omega_i t + \phi_i + \theta_{3i})\big]\Big\}$$

SOLUTION

Relative Displacement:



$$\delta(t) = x(t) - d(t)$$

$$= \beta e^{-\varsigma\omega_n t} \sin(\omega_d t + \psi)$$

$$+ \gamma \sum_{i=1}^{\nu} D_i \Big\{ e^{-\varsigma\omega_n t} \big[\delta_{1i} \sin(\omega_d t + \phi + \phi_i + \theta_{1i})$$

$$+ \delta_{2i} \sin(\omega_d t + \phi - \phi_i + \theta_{2i})\big]$$

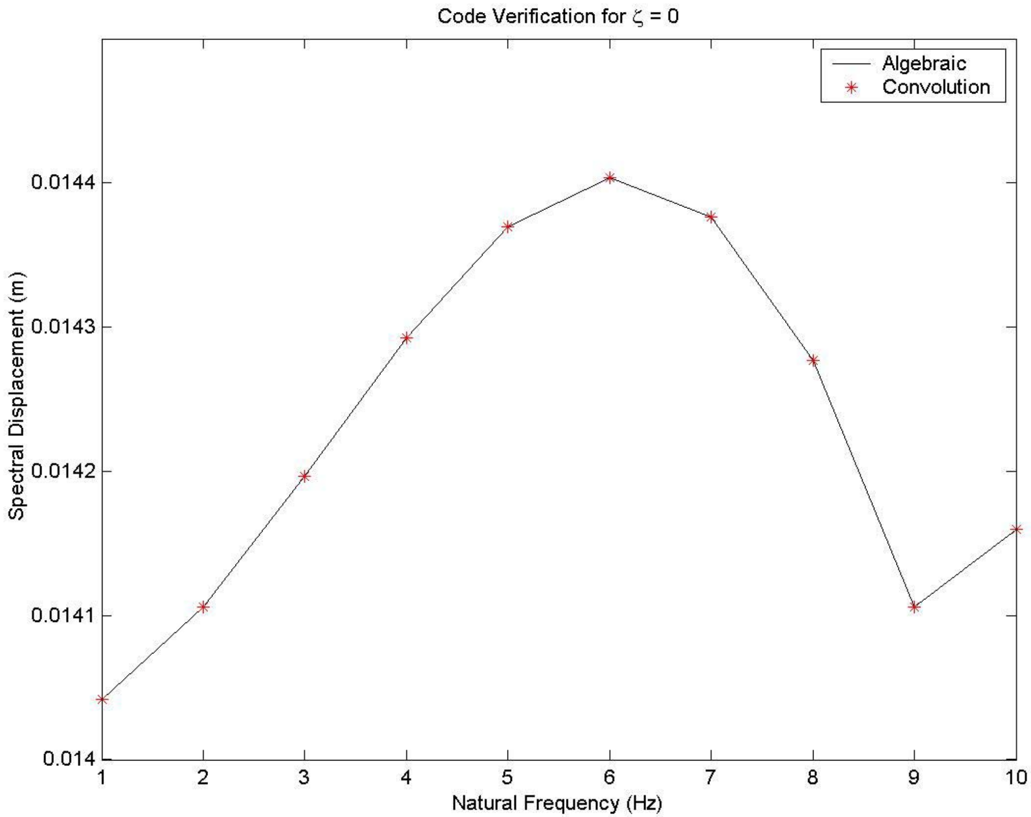
$$+ e^{-\alpha_i t} \big[\delta_{3i} \sin(\omega_i t + \phi_i + \theta_{3i})$$

$$- \sin(\omega_i t + \phi_i)/\gamma\big] \Big\}$$

MATLAB Implementation

- Algebraic results used to generate Shock Response Spectra, via MATLAB code
- Implementation verified for small range of frequencies, undamped case:
  - By hand for algebraic results
  - Using MATLAB to calculate the convolution integrals

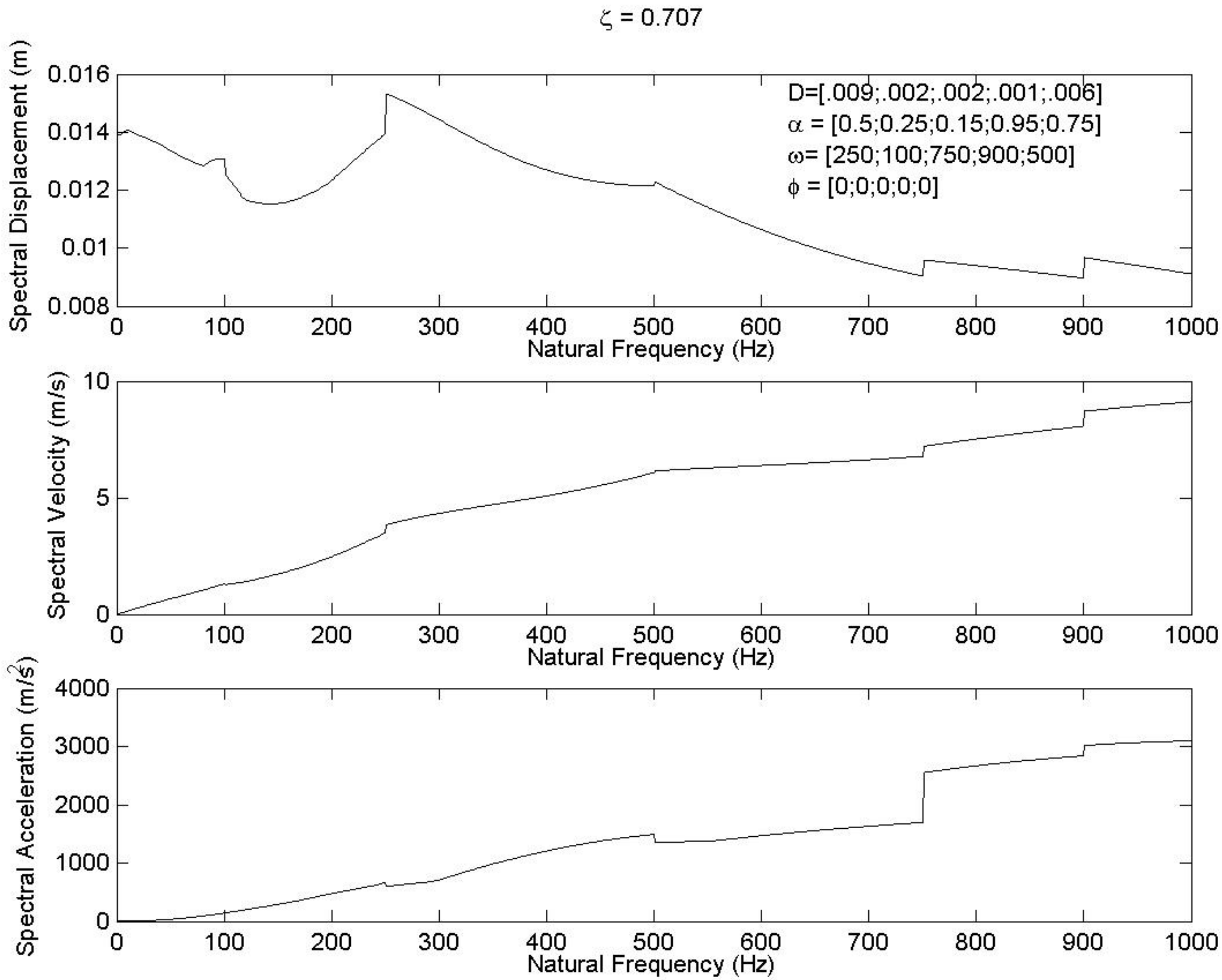
Code Verification



Natural Frequency	Algebraic solution data	Convolution solution data
1	0.01404178803909	0.01404178803909
2	0.01410566896492	0.01410566896492
3	0.01419624521110	0.01419624521110
4	0.01429238837489	0.01429238837489
5	0.01436953813222	0.01436953813222
6	0.01440384367614	0.01440384367614
7	0.01437636721478	0.01437636721478
8	0.01427672293821	0.01427672293821
9	0.01410558906370	0.01410558906370
10	0.01415993354654	0.01415993354654

-Numerical evaluation using convolution integrals agrees with numerical evaluation using algebraic solution.

MATLAB Output



CONCLUDING REMARKS

- **New method developed for SRS calculation**

  --Does not require numerical evaluation of
  convolution integral

  --Exact for impulsive shock disturbance

  --Valid for linearized system model

- **Implementation achieved in MATLAB**

- **Calculations useful**

  --*for evaluating other methods of SRS calculation*

  --*for determining minimum number of modes required for SRS of specified accuracy*