Building a Robust Explanatory Model from a Large Data Set

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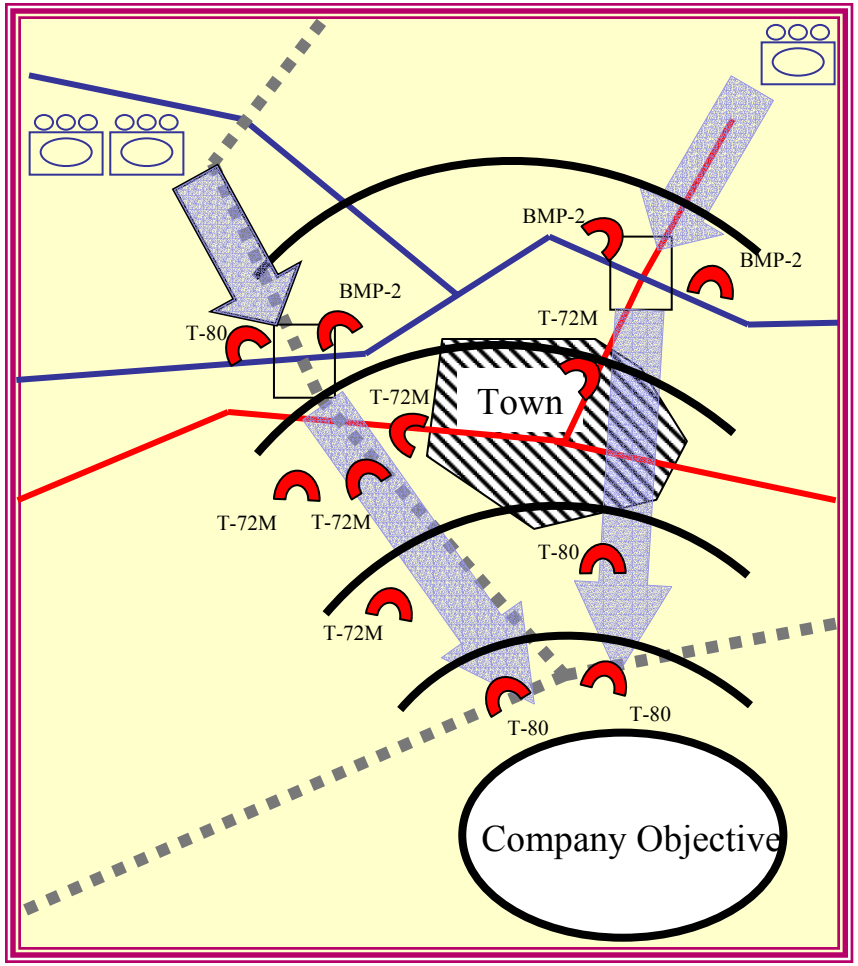
Outline

- An example of application.
- A method of robust variable selection using L_2E applied to the example.
- “BLUF”

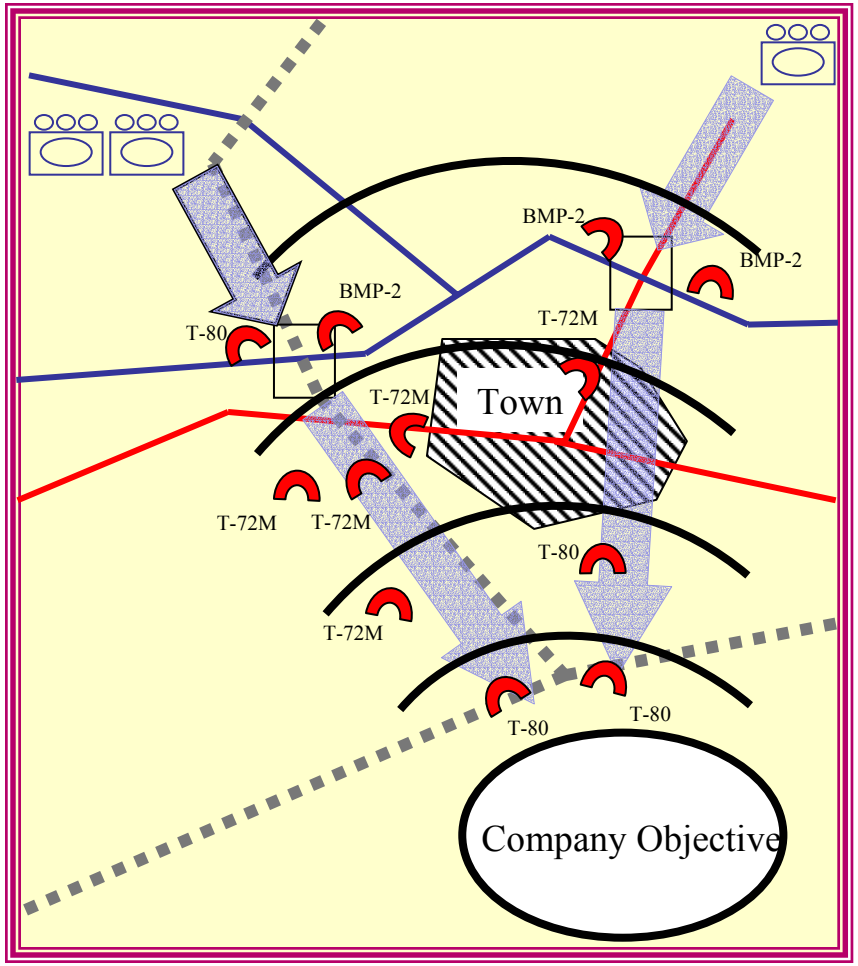
An Analysis of OneSAF Runs

- **228 OneSAF ("Semi Automated Force") runs**
- **3 situational time slices per run**
- **429 data points per run (143 per time slice)**
  - **Number of K, M/F, F, and M kills**
  - **Ammunition levels**
  - **Number of hits delivered**
  - **Range of hits**
  - **Number of side hits delivered**
  - **Distance to objective**
  - **Number of Blue on objective**
- **Response Variable: MBT Score**
- **228x430 data matrix.**



An Analysis of OneSAF Runs

- **Battle results are infinite in scope, yet the outcome of any one battle is defined by a finite set of battlefield interactions.**
- **The goal is to identify battle plan evaluation metrics based on statistical analyses of parameterized combat influences.**
- **Not about "predicting" the battle outcome; rather about explaining potential scenario developments.**



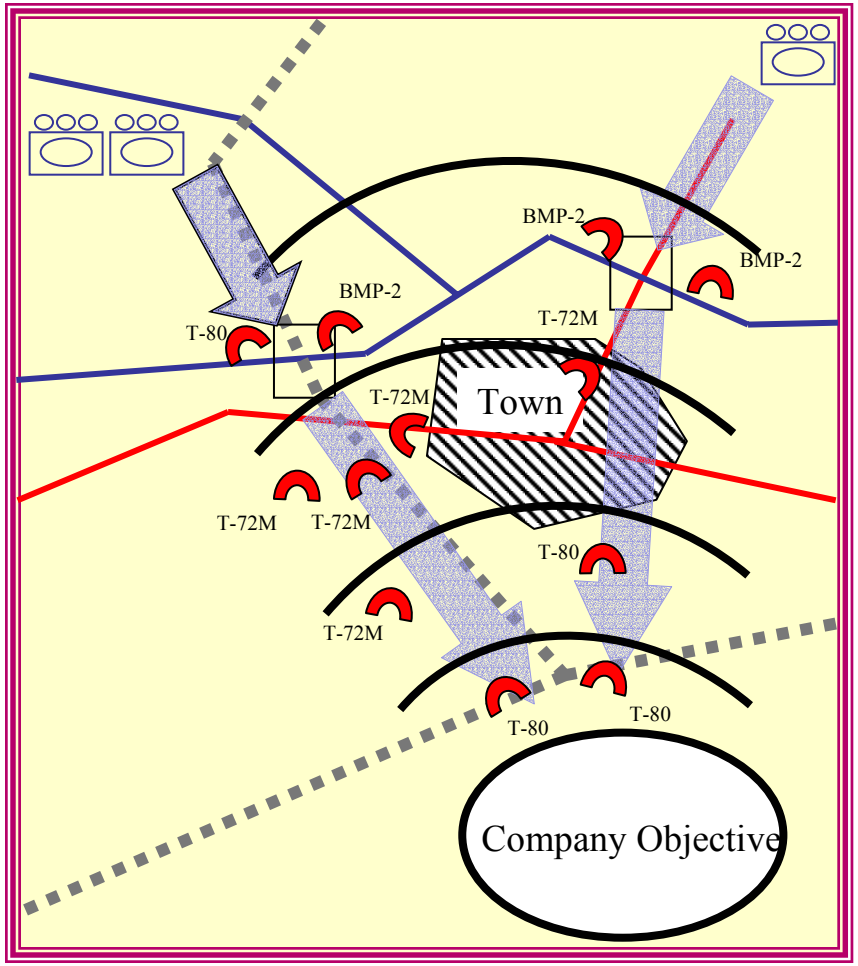
“L2E” Variable Selection
Applied to143 Variables at the Time Slice 1

- MF3S1: # tanks in P3 that are MF’d.
- F3S1:# tanks in P3 that are F’d.
- NKB1S1: # K kills on BMP by P1.
- R05HB2S1: Avg range 105 HEAT on BMP by P2
- MF1S1: # tanks in P1 that are MF’d.
- N05HB3S1: # hits 105 HEAT on BMP by P3.
- N05ST2S1: # hits 105 SABOT on T80 by P2.

Var	L_2E Coeff
Int	584.01
MF3S1	-127.12
F3S1	-108.10
NKB1S1	111.64
R05HB2S1	0.047
MF1S1	-152.55
N05HB3S1	26.36
N05ST2S1	-51.03

An Analysis of OneSAF Runs

- **What does it mean for N05ST2S1 (# hits 105 SABOT on T80 by P2) to have a negative coefficient?**
- **P2 follows behind P1 in the layout.**
- **P2 hitting T80 at the time slice 1 means P1 has been hit hard.**



Variable Selection

- Is an exploratory technique.
- Recall John Tukey's distinction between exploratory data analysis (EDA) and confirmatory data analysis.
- Tests and confidence/prediction intervals may not be valid if the same set of data is used to select variables in the model.

L_2E Regression

- Robust regression method based on L_2E criterion of Scott (2001).
- For a linear model

$$y = b_0 + b_1 x_1 + \cdots + b_k x_k + \varepsilon$$

where the error term is a normal random variable with mean 0 and variance σ^2, estimate the parameters b_i's and σ^2 by minimizing

$$L_2E = \frac{1}{2\sqrt{\pi}\sigma} - \frac{2}{n}\sum_{i=1}^{n}\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{\varepsilon_i^{\,2}}{2\sigma^2}\right)$$

Why L_2E?

- L_2E regression has been shown to have robustness properties similar to least median of squares (LMS) regression and other robust estimators.
- The functionals to be minimized in other robust methods need sorting of *n* residuals, whereas L_2E does not.

Comparison of multiple regression parameter estimates for L_2E vs LS

- No contamination (n=100)

	b_0	b_1	b_2	b_3	b_4	σ
True Values	0	-1	2	-1	2	0.2
Least Squares Estimates	0.0006	-1.0050	1.9982	-1.0015	2.0020	0.2055
L_2E Estimates	0.0104	-1.0068	2.0013	-1.0100	2.0022	0.2182

- 20% contamination (n=100)

	b_0	b_1	b_2	b_3	b_4	σ
True Values	0	-1	2	-1	2	0.2
Least Squares Estimates	0.0139	0.1592	1.6500	0.1679	2.0401	8.65
L_2E Estimates	-0.0357	-1.0286	2.0137	-1.0005	2.0110	0.2118

Toward Robust Variable Selection

- Most of classical variable selection methods available today are based on LS regression.
- Can we come up with a variable selection method that is not easily influenced by outliers and other data contamination?
- One of the criticisms of many variable selection schemes: the selected models often depend on the influential observations.

Example

- Simulated data set of 100 points using the model $y=3-x_2+4x_3-7x_4$ (with $\sigma=.2$).
- We want to look at how σ_{est} and L_2E_{est} vary across different subsets of explanatory variables.

Exp var	L_2E_{est}	σ_{est}
x_1	-.0159	18.19
x_2	-.0168	16.86
x_3	-.0164	18.52
x_4	-.0297	9.982

Example (cont'd)

- Simulated data set of 100 points using the model $y=3-x_2+4x_3-7x_4$ (with $\sigma=.2$).
- We want to look at how σ_{est} and L_2E_{est} vary across different subsets of explanatory variables.

Exp var	L_2E_{est}	σ_{est}
x_1 ,x_4	-.0299	9.86
x_2 ,x_4	-.0304	9.87
x_3 ,x_4	-.1114	2.69

Exp var	L_2E_{est}	σ_{est}
x_1 , x_3 ,x_4	-.0305	9.80
x_2 , x_3 ,x_4	-1.419	0.19

Heuristic Variable Selection (suggested by the prev. example)

- Choose the variable that gives the smallest L_2E and/or σ at each forward selection step, and stop when adding any of the remaining variable gives no improvement.
- The correct models were arrived at in many different models and at reasonable contamination levels.
- Interesting result: When the contamination level goes up, adding a redundant variable usually results in increase of L_2E and σ!
- The following table shows that what happens to the previous example when we have 20% contamination.

Exp var	L_2E	σ
x_2 , x_3 ,x_4	-1.000	0.19
x_1 , x_2, x_3 ,x_4	-.0557	6.73

Ongoing and Future Work

- Developing the criteria analogous to AIC and others.
- Robust F ratio?
- Continuing applications to combat simulation data.