Application of Time Scales to Real Time Communications Networks

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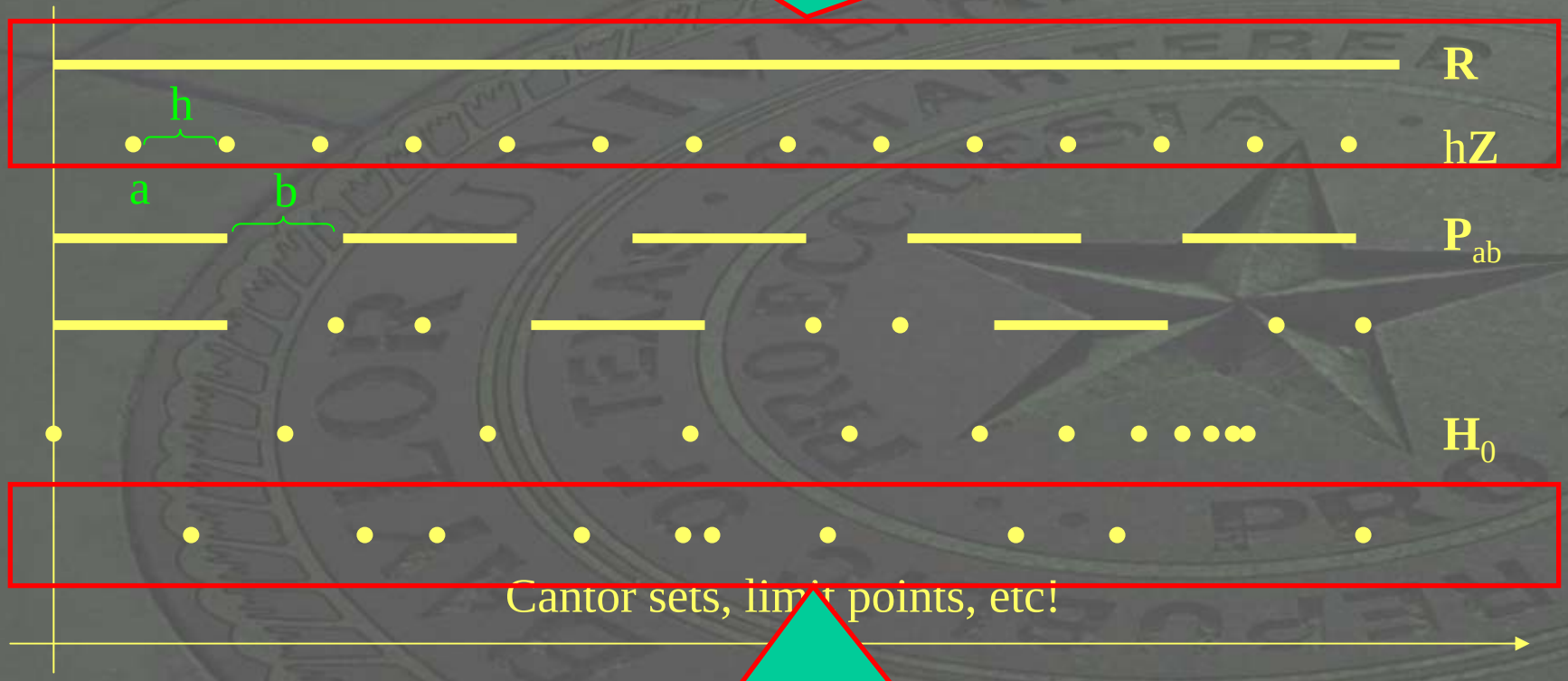
Outline

- Background and motivation
- The controller area network (CAN)
  - System architecture
  - Timing considerations
- A Time Scales Primer
- TS model of a linear plant on a CAN
- Stability issues
- Effects of dynamically adjusting the sample period
- Conclusions

Time Scales and Engineering

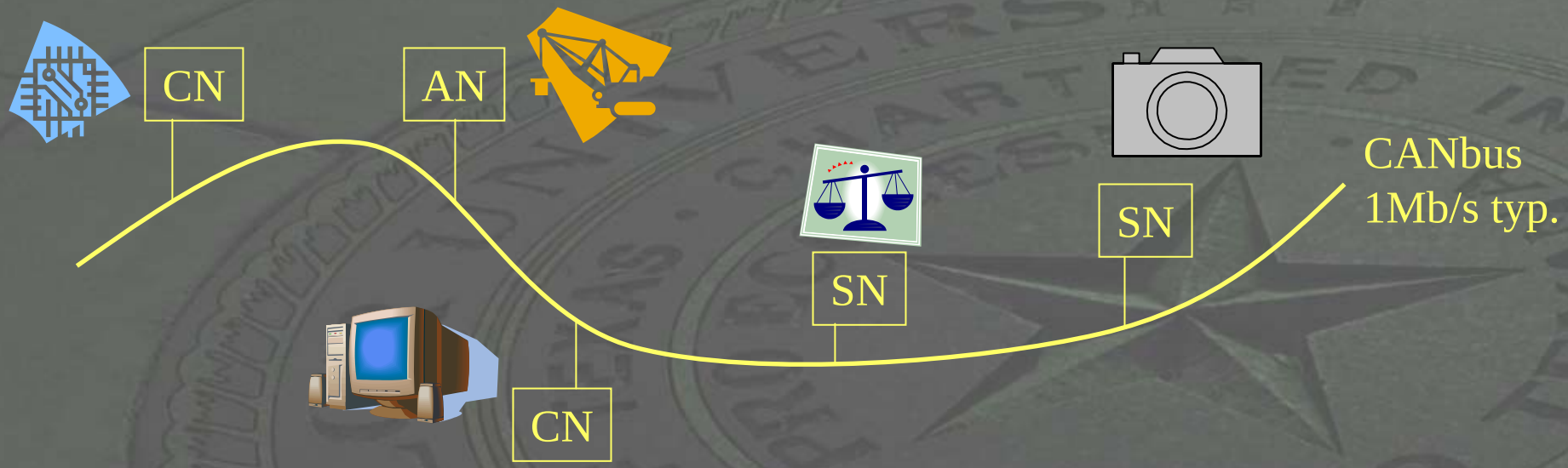
Where 99.99 % of engineering has taken place up to now…



The real world that engineers have tried to avoid.

Application: CAN

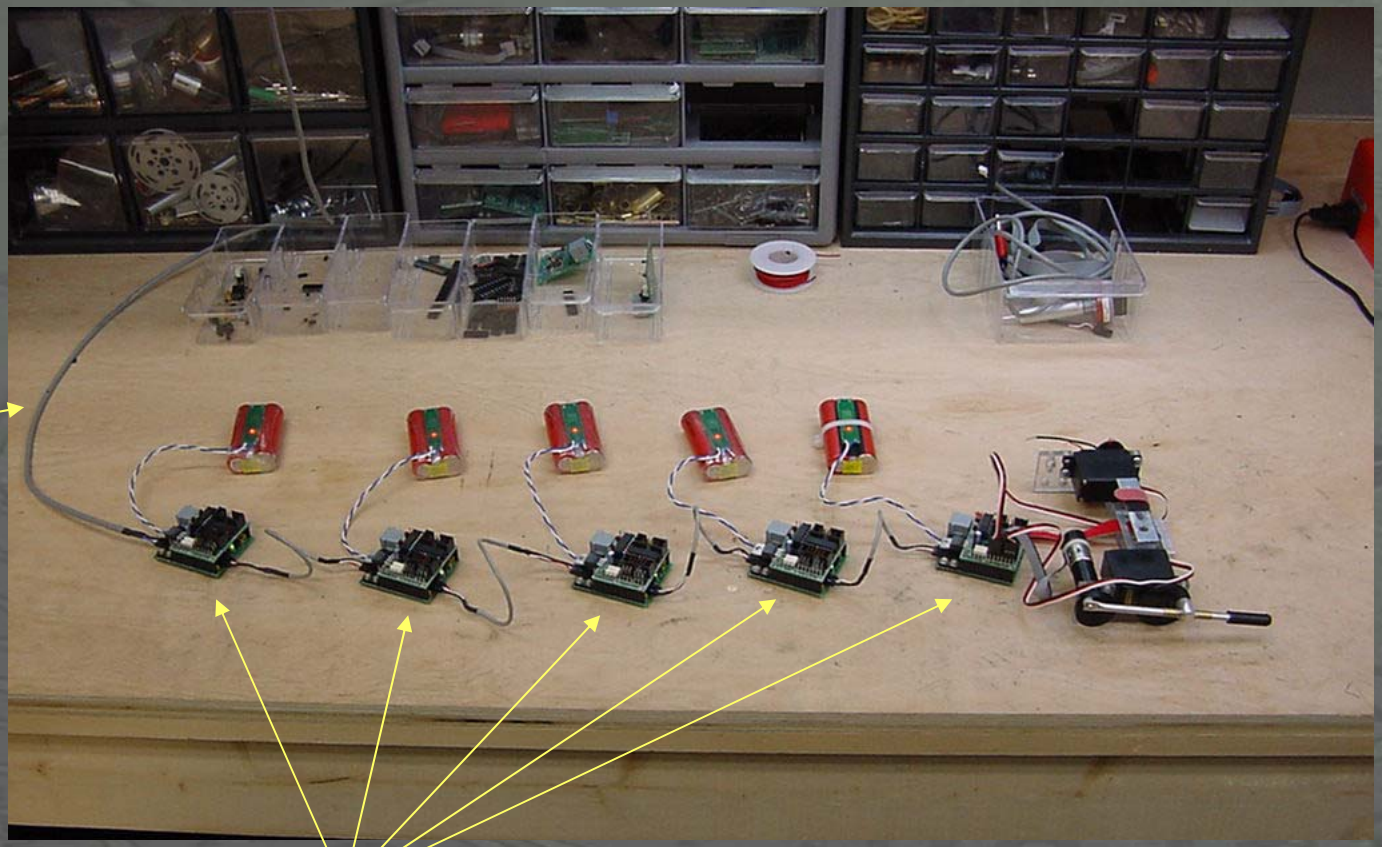
The “Controller Area Network” (CAN) protocol defines a communication standard for real-time systems using a centralized comm bus.



Features of CAN make it very good for control problems: real-time collision arbitration, works over variety of modes, redundancies, message priorities, all nodes “hear” every message.

Widely adopted by automotive manufacturers (e.g. GMLAN)…

A CAN system



CAN bus

SN/AN based on PIC 18F248

Automotive Use

A typical modern vehicle has 6 to 12 computers (CNs) and upwards of 50 SNs/ANs:

- Battery Voltage
- Batter Current
- Battery Temp
- Accelerator Position
- Brake Pressure, Master
- Brake Pressure, Line
- Transaxle Pressure
- Clutch Line Pressure
- Speed
- Battery Ground Fault
- Hi/Lo Contactor State
- Key Switch Run
- Key Switch Start
- Brake Switch
- Emergency Brake
- Shift Lever
- Transaxle Temp
- Transmission Temp
- Engine Temp
- 12V Power Ack Control
- 12V Power Ack Inverter
- SOC Reset
- Reverse and 2nd Gear Clutches
- Clutch Pressure Control
- DC/DC Converter
- DC/DC Converter Current
- 12V Power relay
- Brake Solenoid
- Warning Lights
- Main Contactor close
- Torque measured
- Idle
- Shift in Progress
- Inverter Temp

Key Observation:
Some of these signal sources are *periodic* and some are *sporadic*.

Periodic signals are required to complete control loops.

Sporadic signals usually of very high priority

A Time Scales Primer

- A time scale T is any closed subset of the real line.
- We denote the successor of $t \in T$ as $\sigma(t)$. The graininess of an element $t \in T$ is the distance from that element to the next, i.e.

$$\mu(t) = \sigma(t) - t$$

In the reals, the graininess of every element is 0 and in the integers, the graininess of every element is 1.

- We have a generalized delta derivative for a function $f : T \to R$ which is defined by $f^{\Delta}(t) := \lim_{s \to \mu(t)} \frac{f(t+s) - f(t)}{s}$
- The integral is based off of the Riemann integral and is defined by

$$\int_a^b f(s)\Delta s = \begin{cases} \int_a^b f(s)ds \\ \sum_{s=a}^{\rho(b)} f(s)\mu(s) \end{cases}$$

A Time Scales Primer

- We say the function p is regressive ($p \in \Re$) if $1+\mu(t)p(t) \neq 0$ and positively regressive ($p \in \Re^+$) if $1+\mu(t)p(t) > 0$, for all $t \in T$

- We now define the regressive first order linear dynamic initial value problem:

$$x^{\Delta}(t) = p(t)x(t), \quad x^{\Delta}(t_0) = x_0$$

- We define the unique solution to this IVP, given by

$$x(t) = e_p(t, t_0)x_0$$

- The ***generalized exponential*** function is defined by

$$e_p(t, t_0) = \exp\left(\int_{t_0}^{t} \lim_{s \downarrow \mu(\tau)} \frac{\mathrm{Log}(1+sp(\tau))}{s} \Delta\tau\right)$$

A Time Scales Primer

- A first order linear dynamic IVP is exponentially stable if we have

$$p \in \mathbf{S}_{\mathbf{C}} := \left\{ \lambda(t) : \limsup_{t \to \infty} \frac{1}{t - t_0} \int_{t_0}^{t} \lim_{s \downarrow \mu(\tau)} \frac{\mathrm{Log}\left|1 + s\lambda(\tau)\right|}{s} \Delta\tau < 0 \right\}$$

Where $\mathbf{S}_{\mathbf{C}}$ is called the ***<u>regressive stability region</u>***.

- If this "time average" is negative, then we will have an exponentially stable solution.
- We note some interesting features of this region:
  - It is always contained in the left half plane
  - It is a possibly disconnected region

Time Varying Systems

- We now ready to consider the stability characteristics of the regressive first order time varying linear dynamic system

$$x^{\Delta}(t) = A(t)x(t), \qquad x(t_0) = x_0$$

- We use a Unified Lyapunov Theory. Let $P(t)$ be a symmetric positive definite matrix. Let $x(t)$ be a solution to the above system. By taking the delta derivative of the weighted norm of our solution, we obtain the following (***<u>hopefully negative definite</u>***) quadratic form:

$$\begin{aligned}[x^T P x]^{\Delta} &= x^T[A^T P + PA + \mu A^T PA + (I + \mu A^T)P^{\Delta}(I + \mu A)]x \\ &= x^T[A^T P + PA + \mu A^T PA + \Sigma]x \\ &\le -\eta\, x^T x \quad \text{where} \quad \eta > 0 \quad \text{and} \quad -\eta \in \Re^+\end{aligned}$$

- If this quadratic form is negative definite at each element t in the time scale, then the solution to the system is exponentially stable.

Time Varying Systems

- We do this by employing the generalized version of Gronwall's Inequality:

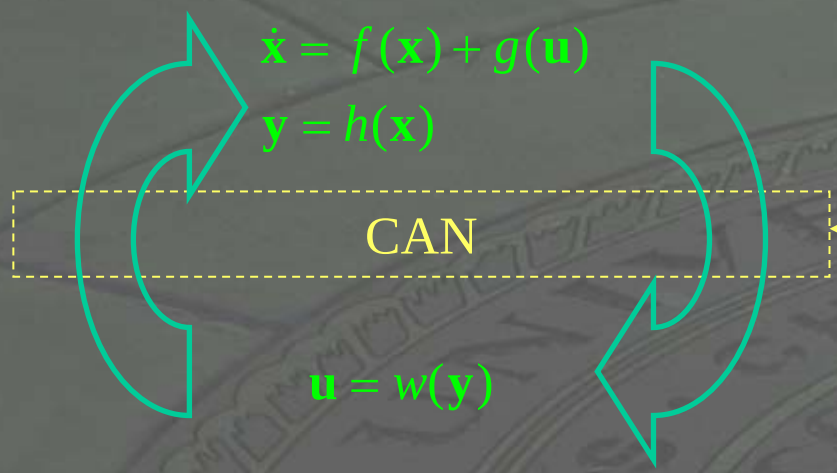
Theorem: If $y, f \in C^1_{rd}$ and $p \in \Re^+$, then

$$y^{\Delta}(t) \le p(t)y(t) \text{ for all } t \in T$$

implies

$$y(t) \le y(t_0)e_p(t,t_0) \quad \text{for all } t \in T.$$

CAN control

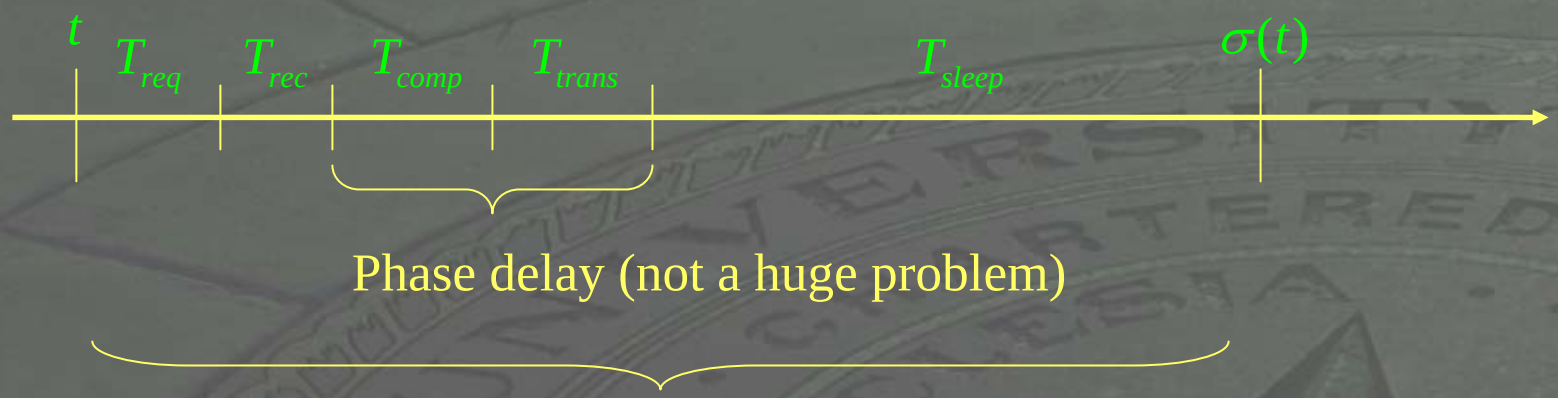


Timing is very important!
Delays can cause instability.
Sources of delay:
- Traffic
- Message length
- Transmission delay

CAN is real-time ➔ trans delay is deterministic (prop to message length).
Messages limited to 8 bytes. Main source of delay is TRAFFIC on bus.

Traditional method keeps $\mu(t)$ constant.

Problems…



μ would be constant and predictable if all loops on the bus operated in synchrony.

Can't do this: high-priority sporadic messages compete for bandwidth and cause ***jitter***.

Previous thinking was to design message schedulers to minimize jitter; attempt to run control loops using "rule of 10"; keep bus utilization below 50%.

Solutions?

SOLUTION? Is it possible for servo loops to adaptively sample? Temporarily relinquish strict timing but still maintain stability?

- Need a loop stability metric that accounts for traffic conditions.
- Use time scale models since system timing is now not known *a priori*.
- Use metric to increase sampling rate or raise priority for plants that are nearing instability.

Leads to an interesting concept: " μ-dynamics": <u>the timescale on which a system is defined is itself determined by the dynamics of the system and any relevant external inputs.</u>

$$\mu(t) = g(\mathbf{x}(t), \mathbf{u}(t))$$

$$\mathbf{x}^{\Delta}(t) = f(\mathbf{x}(t))$$

A Linear Plant on CAN

To start, work with minimum-phase linear systems of form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x}$$

Under a "sampled-data" feedback compensator, this transforms to

Exists for all A

$$\mathbf{x}^{\Delta} = \underbrace{\operatorname{expc}(\mu\mathbf{A})\hat{\mathbf{A}}}_{\mathcal{A}(\mu(t))}\mathbf{x}$$

Desired closed-loop state matrix

Definition: $\operatorname{expc}(X) := I + \frac{1}{2}X + \frac{1}{6}X^2 + \ldots + \frac{1}{n!}X^{n-1} + \ldots$

Notes:

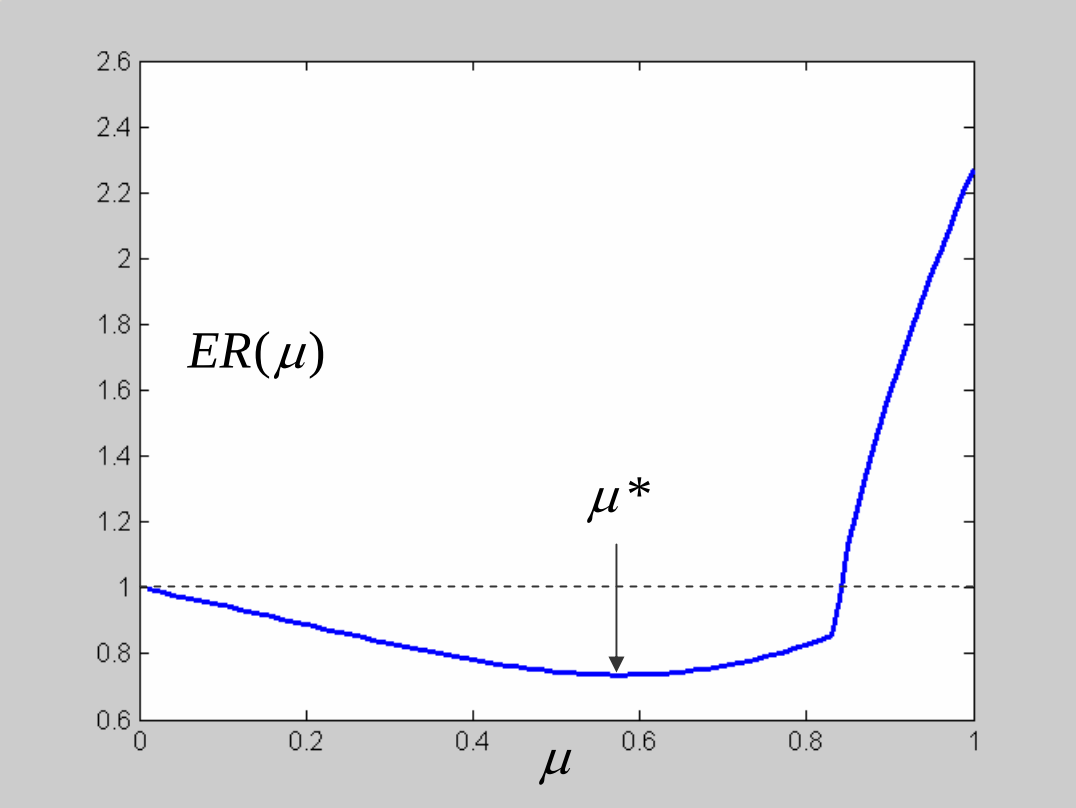
$$\operatorname{expc}(X) = (e^X - I)X^{-1} \text{ when } X^{-1} \text{ exists}$$

$$\operatorname{expc}(\mu X) = \xi_\mu^{-1}(X)X^{-1} \text{ when } X^{-1} \text{ exists}$$

$$\operatorname{expc}(\mu X) \to I \text{ as } \mu \downarrow 0.$$

Stability

Choose a nominal sampling rate $\mu^* = const$



Look at maximum eigenvalue radius to determine a good sample period

$$ER = \max_i \left| 1 + \mu\lambda\{\mathcal{A}(\mu)\} \right|$$

ER reflects how much control "effort" required to stabilize a system.

Require ER<1 or system is "instantaneously unstable".

Stability

Design a symmetric matrix $P>0$ s.t.

$$\mathcal{A}(\mu^*)^T P + P\mathcal{A}(\mu^*) + \mu^* \mathcal{A}(\mu^*)^T P\mathcal{A}(\mu^*) = -I$$

$$P = \int_0^\infty e_{A^T(\mu^*)}(s,0) e_{A(\mu^*)}(s,0)\Delta s, \qquad s \in \mathbf{S} := \mu^* \mathbf{Z}$$

Elements of $\mathcal{A}$ are rd-continuous functions of graininess, so

$$\mathcal{A}(\mu)^T P + P\mathcal{A}(\mu) + \mu\mathcal{A}(\mu)^T P\mathcal{A}(\mu) = -I + \Sigma(\mu)$$

$\Sigma(\mu)$ is continuous, symmetric, PSD and $\Sigma(\mu^*) = 0$

Choose Lyapunov candidate

$$\left.\begin{aligned} V &= \mathbf{x}^{\mathbf{T}} P\mathbf{x} \\ V^\Delta &= \mathbf{x}^{\mathbf{T}}[\mathcal{A}(\mu)^T P + P\mathcal{A}(\mu) + \mu\mathcal{A}(\mu)^T P\mathcal{A}(\mu)]\mathbf{x} \\ &= \mathbf{x}^{\mathbf{T}}[-I + \Sigma(\mu)]\mathbf{x} \\ &\le \frac{-1 + \lambda_{\max}(\Sigma(\mu))}{\lambda_{\min}(P)} V := \eta(\mu) V \end{aligned}\right\}$$

After a lengthy calculation, we can show that

$$\eta(\mu) \in \mathcal{R}^+$$

The Cool Part

Thus, by Gronwall's Inequality, we have that

$$V(t) \le e_{\eta(\mu(t))}(t,t_0)V_0$$

and furthermore by Potzche, Siegmund, Wirth (2003), if

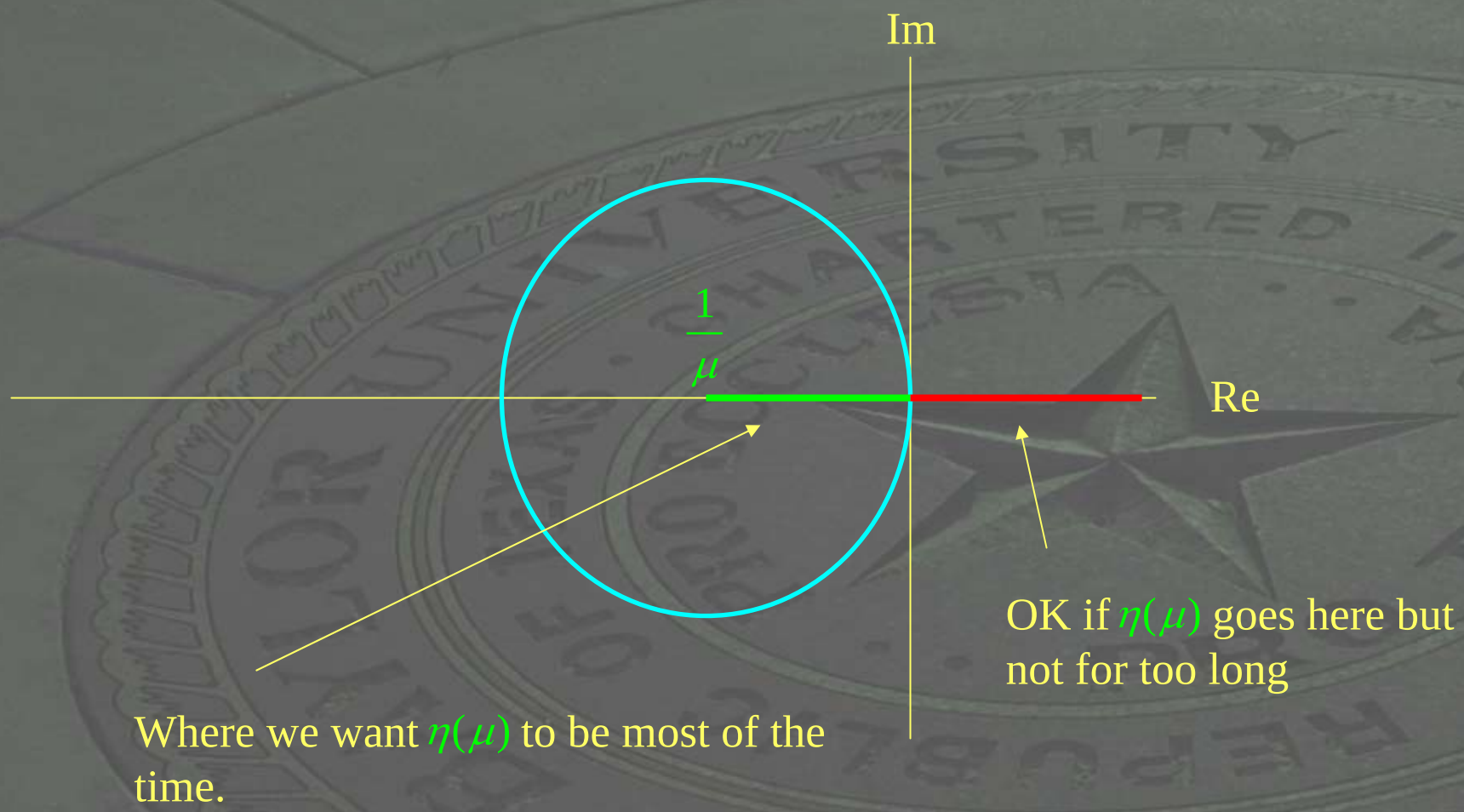
$$\eta \in \mathbf{S_C} := \left\{ \lambda(t) : \limsup_{t\to\infty} \frac{1}{t-t_0} \int_{t_0}^{t} \frac{\log\left|1+\mu(t)\lambda(t)\right|}{\mu(t)} \Delta t < 0 \right\}$$

V is bounded above by a standard real exponential (for ease of interpretation):

$$V(t) \le Ke^{-\alpha t}, \quad \alpha > 0$$

<u>The cool part</u>: the system may be instantaneously unstable, but remains stable on average.

Basic Idea

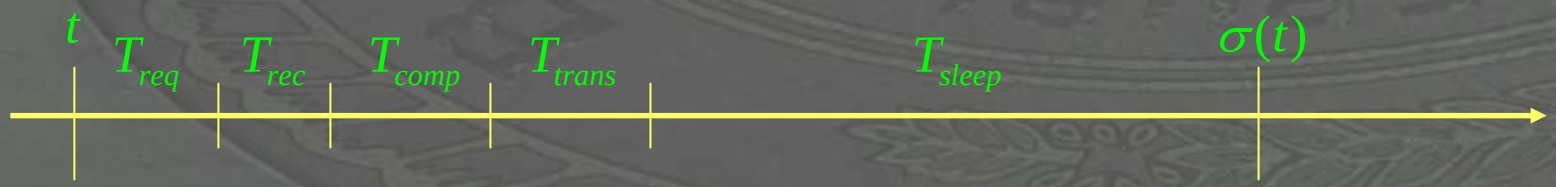

Im
Re
$\frac{1}{\mu}$
OK if $\eta(\mu)$ goes here but not for too long
Where we want $\eta(\mu)$ to be most of the time.

Timing

- We know we have a purely isolated (discrete) time scale.
- If the average over a moving finite window is negative, the infinite-window average must necessarily be negative too.
- Refine $\mathbf{S_C}$ to get a useable form:

$$\frac{1}{t-\rho^k(t)}\sum_{\rho^k(t)}^{t}\log\left|1+\mu(t)\eta(t)\right|<0 \qquad \Rightarrow \prod_{\rho^k(t)}^{t}\left|1+\mu(t)\eta(t)\right| := P(t)<1$$

- Set T_{sleep} so that $\mu(t)=\mu^*$ in absence of traffic.
- In traffic, monitor $\mu(t)$ and update $P(t)$.
- If $P(t)$ rises too near to 1, reduce T_{sleep}



Example

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -4 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ (Oscillatory, unstable)

Choose a feedback compensator that gives closed-loop system

$$\hat{\mathbf{A}} = \begin{bmatrix} 0 & 1 \\ -4 & -1 \end{bmatrix}$$ (Oscillatory, stable; poles at $-0.5 \pm 1.94\,j$)

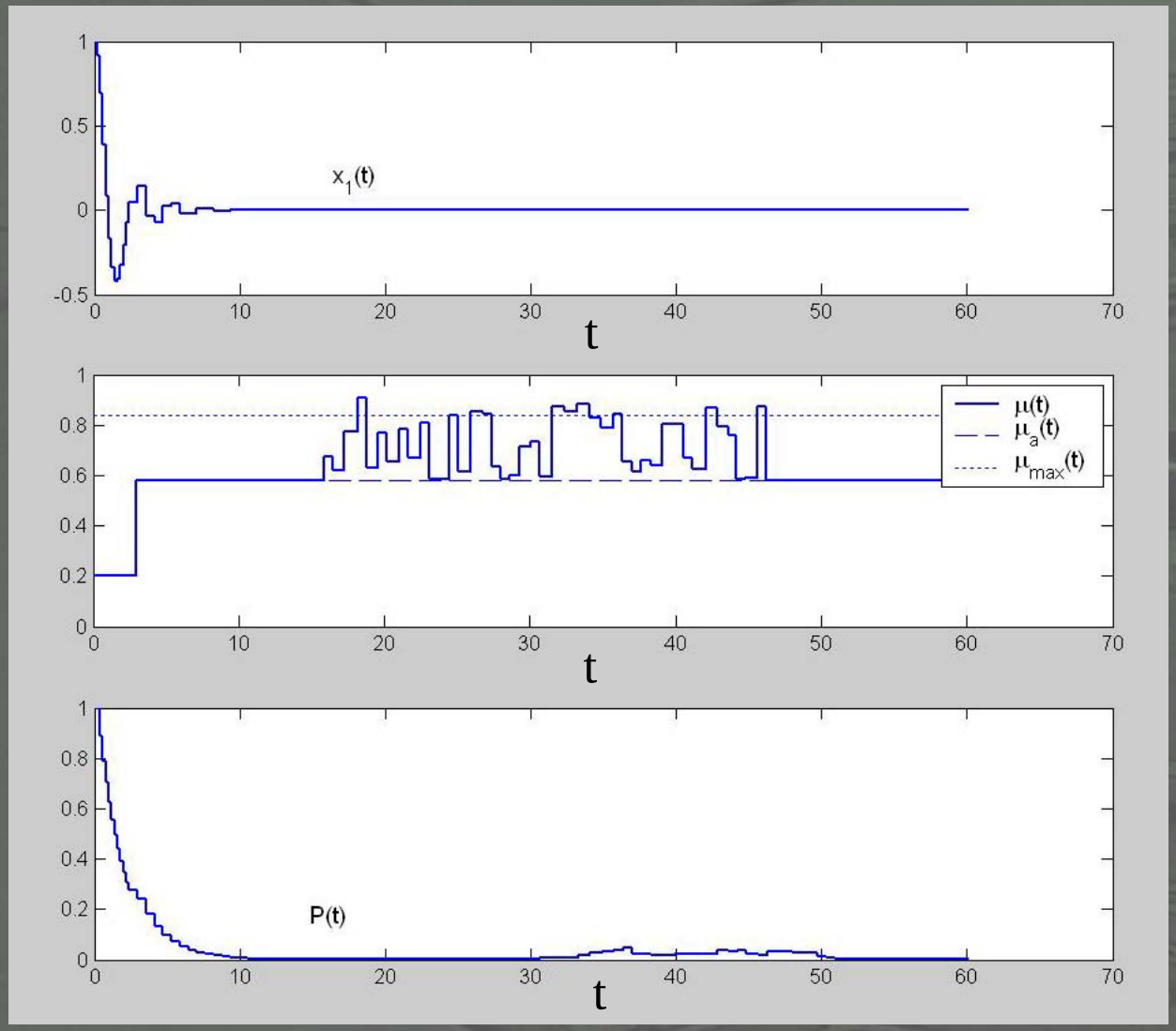
$$\mu^* = \min_{\mu} \max_{i} \left| 1 + \mu\lambda_i(\mathcal{A}(\mu)) \right| = .580$$

Control period that will stabilize system in minimum # of steps (optimal bandwidth)

$$\mu_{\max} = \min_{\mu} \{ \max_{i} \left| 1 + \mu\lambda_i(\mathcal{A}(\mu)) \right| = 1 \} = .837$$

Control period that will make system “instantaneously unstable”

Simulation 1

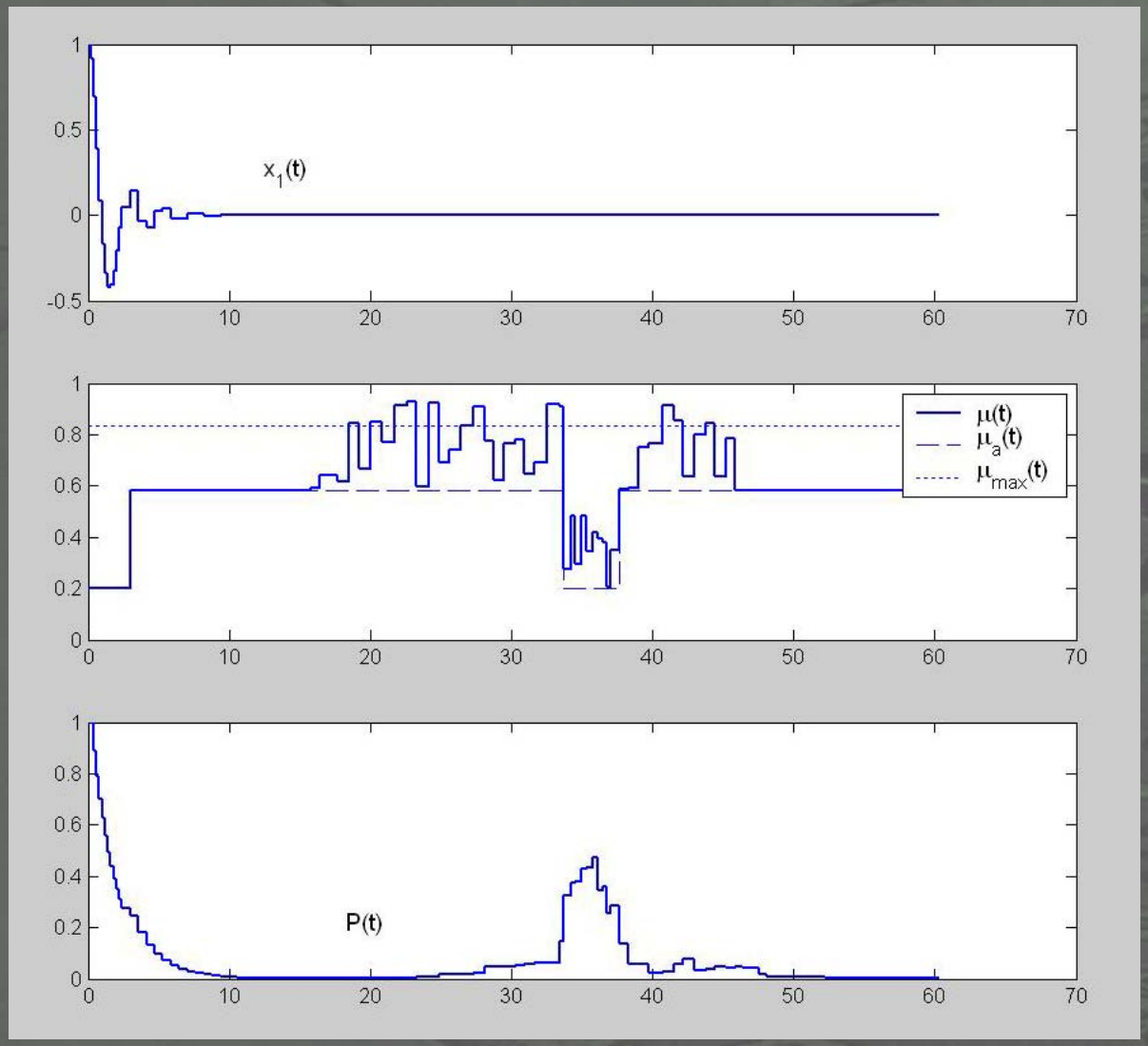


System experiences burst of network traffic between 15 and 45 seconds.

Averaging window is 25 samples.

During high-traffic period, system uses 19.2% less bandwidth than a constant-period system would and ***remains stable***!

Simulation 2



Same setup, but now burst causes *P(t)* to exceed preset threshold of 0.25. Controller reduces *μ(t)*→ *P(t)* falls to "safe" level.

System remains stable using 13.5% less band-width than constant period system would.

In Practice…

Stuff engineers care about:

• Can T_{trans} really be measured (in order to measure $\mu(t)$)? Yes – consists of two parts: waiting for a collision-free transmission (can be measured by an internal timer) and actual transmission transit time. Since we have a real-time network, transit time is deterministic.

• What about the computation of $|1+\mu(t)\eta(t)|$? No need to compute – this function is predictable once the system model is known, can be stored in a lookup table over a sufficiently representative span of μ values.

• What happens if many nodes try this all at the same time? GREAT QUESTION!! How to model/analyze multiple dynamical systems interacting on different (but coupled) time scales?.

Conclusions

• Time scale methods hold promise for problems that are purely discrete (or at least discretized) but of non-constant step size.

• Time scale methods can help in design/analysis problems where discrete and continuous solutions to the problem are both relevant (e.g. high-gain adaptive controllers, Gravagne/Davis/DaCunha '04).

• Questions remain in general about the nature of solutions to "μ-dynamics" problems:

$$\mathbf{x}^{\Delta}(t) = f(\mathbf{x}(t))$$

$$\mu(t) = g(\mathbf{x}(t))$$