12th ARL/USMA Technical Symposium

QUASI-ANALYTICAL SHOCK RESPONSE SPECTRA, FOR PROPORTIONALLY DAMPED SYSTEMS, WITH REMOTE SHOCK LOADING

R. David Hampton & Kip P. Nygren
Department of Civil & Mechanical Engineering
U. S. Military Academy, West Point, NY

Ting H. Li
Weapons & Materials Research Directorate
U. S. Army Research Laboratory, Adelphi, MD

3 November 2004

PRESENTATION OUTLINE

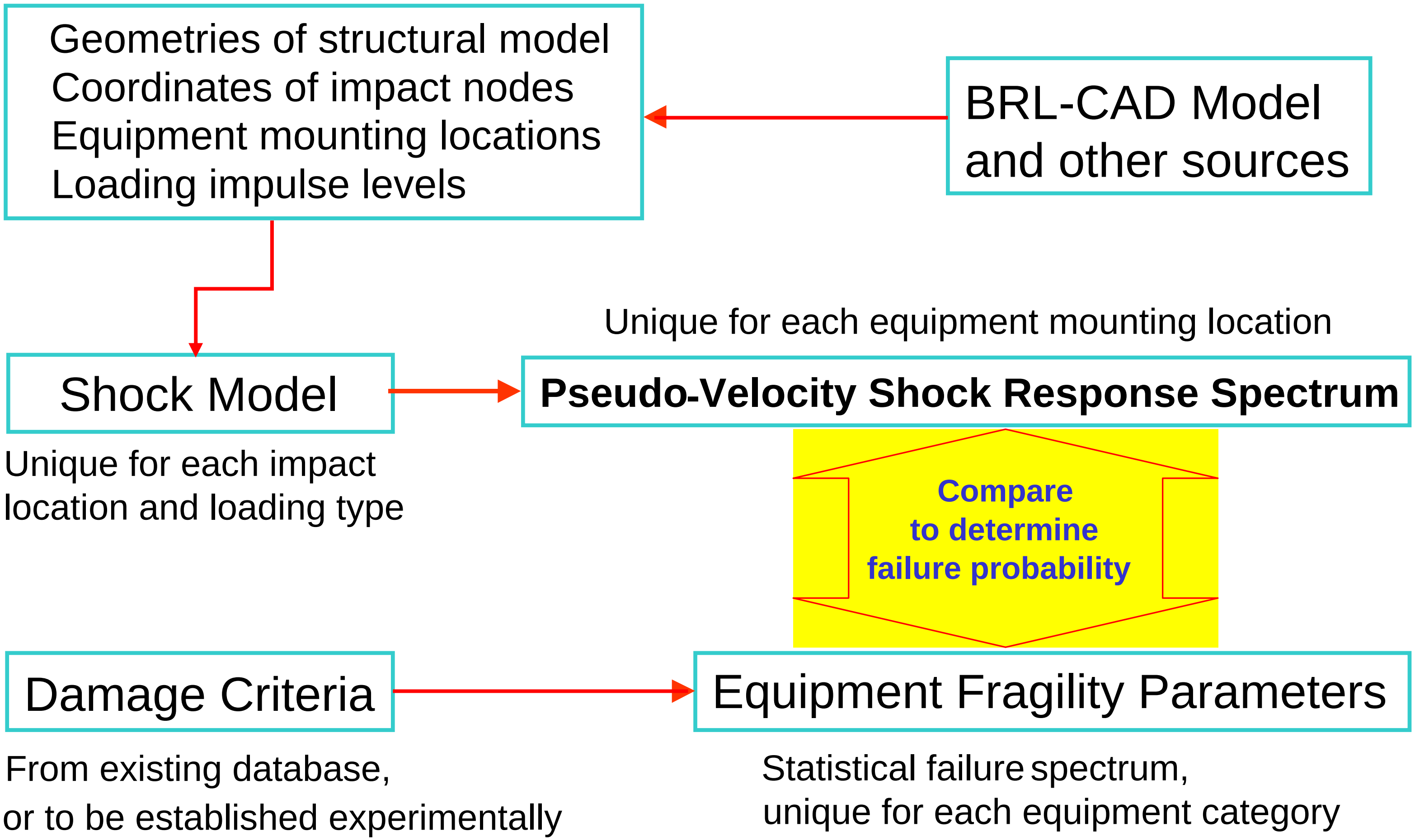
INTRODUCTION
- Research context
- SRS definition
- SRS for local damped harmonic input

OBJECTIVE
- SRS for remote impulsive input

SOLUTION
- Remote response of basic system
  - Modal response
  - Physical response
- Response of attached mass
  - For a remote arbitrary input
  - For a remote ideal impulse
  - For a remote rectangular pulse
  - For a remote saw-tooth pulse

CONCLUDING REMARKS

INTRO: Overview of Vibration Analysis Approach



INTRO: FEA Shock Models for Armored Vehicles: Reducing the time for impact calculations

FEA Structural Transient Analysis

Direct Integration

- Used by most analysts
- Current shock model: ~2 days/impact
- Too slow for many impacts
- But can be used for comparison in a few cases

Explicit — Dyna, Abacus

Implicit — Nastran, Ansys

Modal Superposition — Ansys

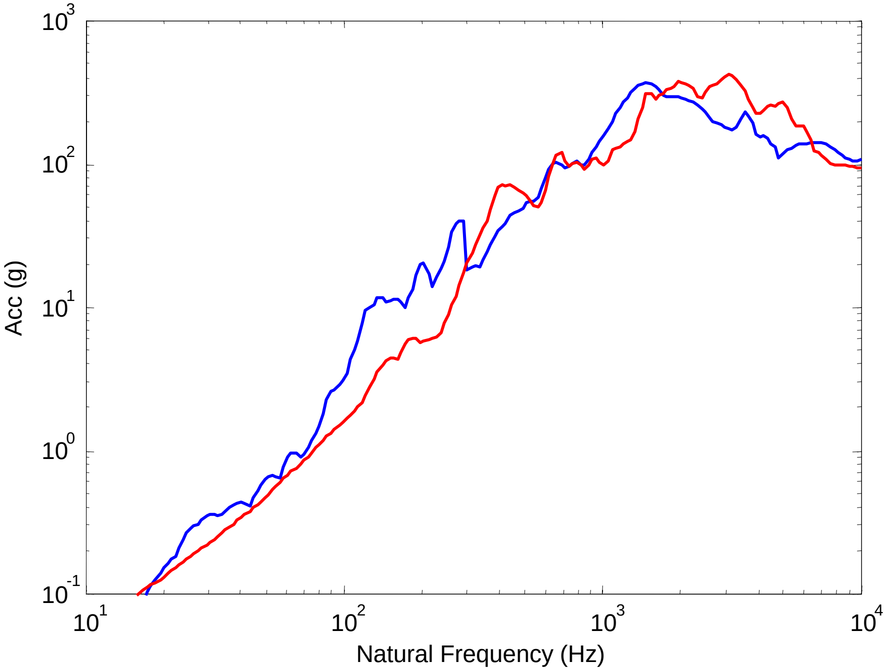
- Very fast for a large number of impacts
- Useful for a linear system only (based on linear superposition of modes)
- Permitting a clear-cut upper frequency, limited by the number of modes; possible in eigen-analysis

Linearized Equations: $$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\} = \{F\}$$

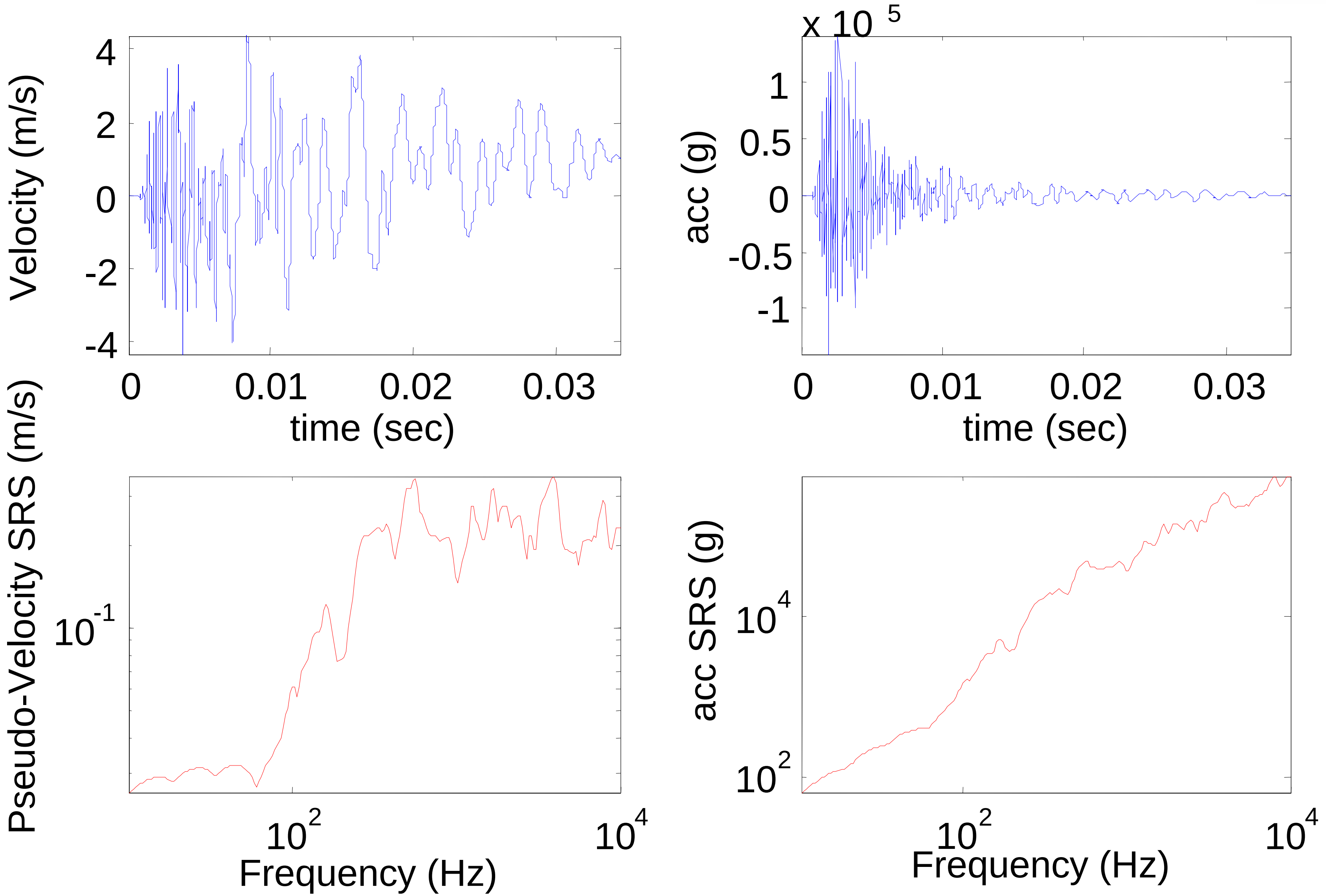
INTRO: Typical shock model

Blue: Shock response measurement from a ballistic test

Red: Corresponding response from an FEA shock model



INTRO: Typical SRS Data and Plots



INTRO: FEA Modal Superposition Method

FEA shock model

~99% of total solution time

Modal Analysis

Eigenvalues
Eigenvectors

~1% of total sol'n time for each impact node

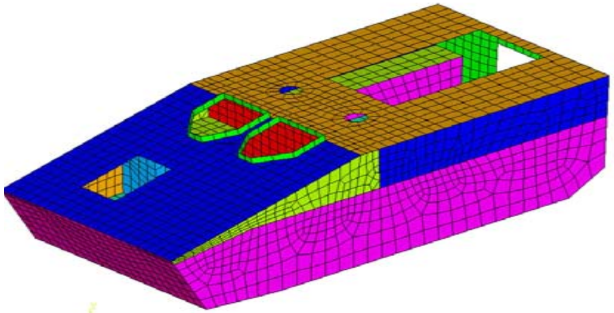
Transient Analysis

New impact node

Time-domain Response

PSRS

Example of solution time
(IHS coarse mesh model)



~4000 shell elements
450 modes

Modal analysis time:

67 hrs

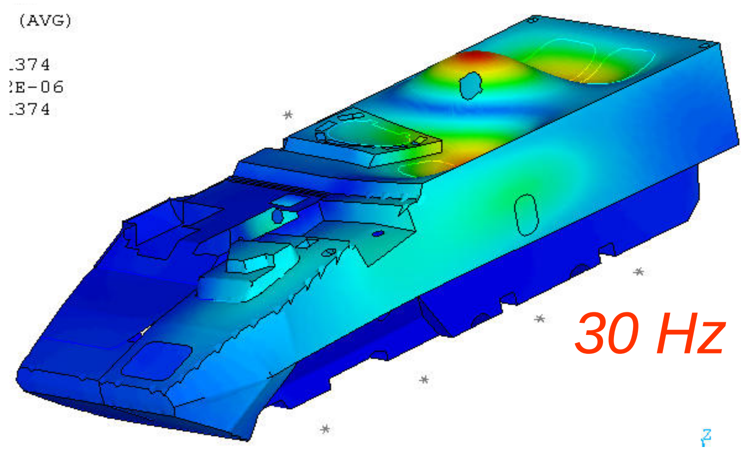
Transient solution time:

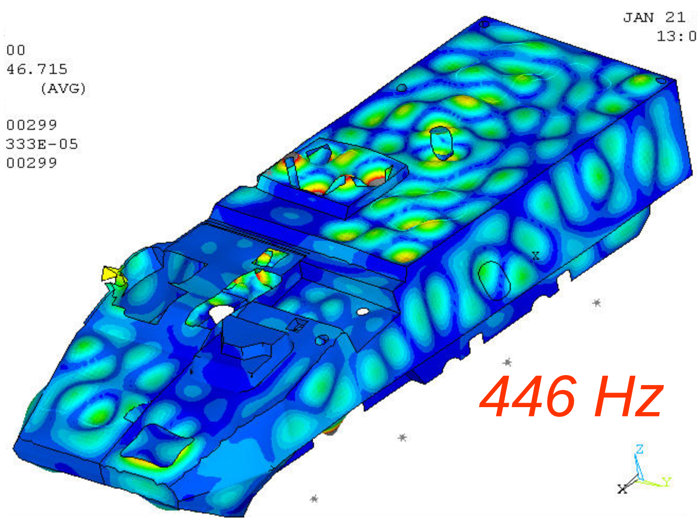
1 impact: 0.17 hr, or 10 min
600 impacts (typical): 100 hrs

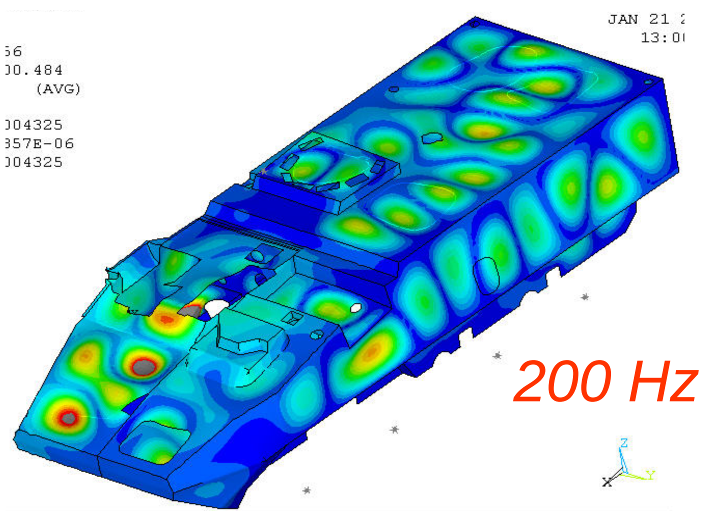
***INTRO*:** Modal Analysis

Needed for Modal Superposition Method (MSM)

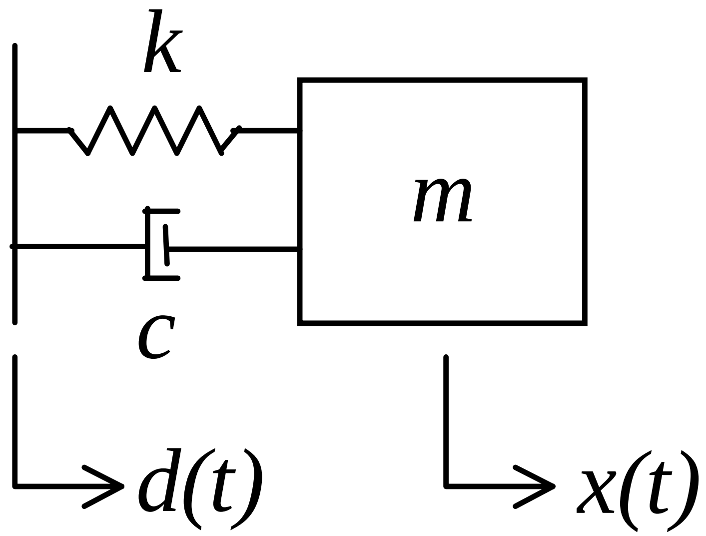
- *Eigen-analysis forms the basis of MSM transient analysis*
- Very fast for a large number of impacts because eigenvectors are used again and again for each impact
- For a linear system only
- Upper frequency is limited by the number of modes accurate and practical in the eigen-analysis





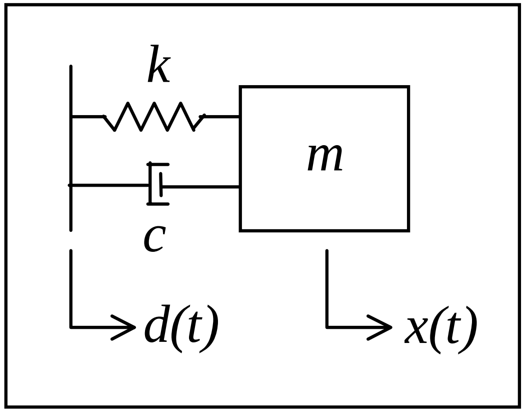


INTRODUCTION: SRS Definition



Hypothetical SDOF MSD system,
for SRS determination

SRS Definitions:



Spectral displacement SRS:

$$S_D(\omega_n) := \left| x(t) - d(t) \right|_{\max}$$

Spectral velocity SRS:

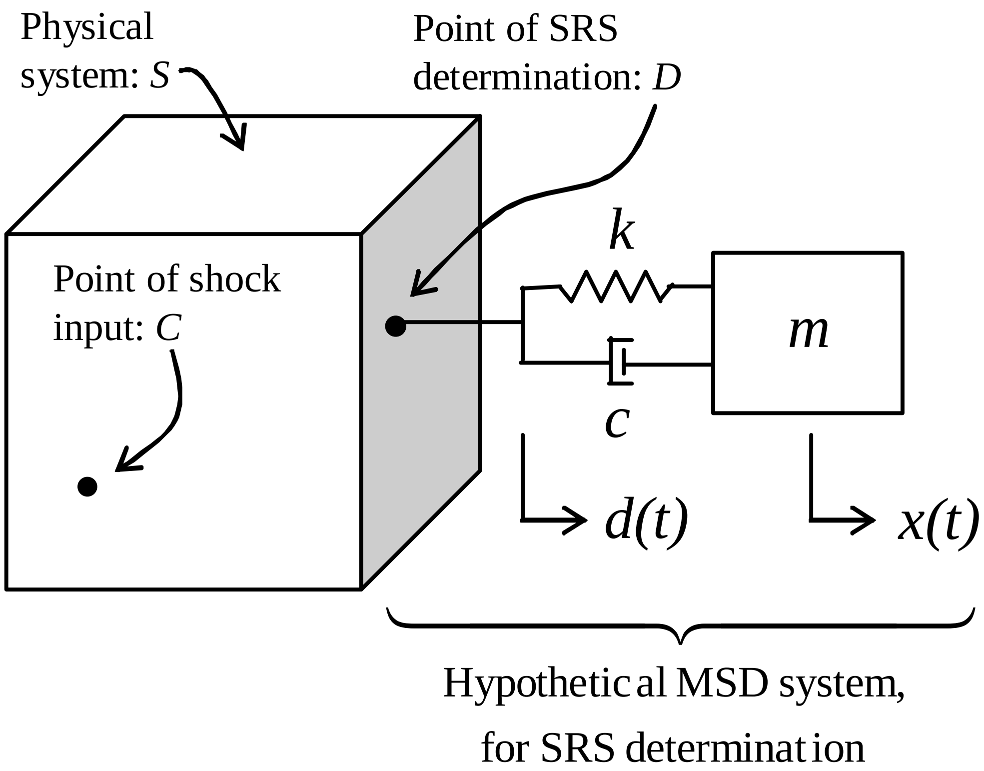
$$S_V(\omega_n) := \omega_n \left| x(t) - d(t) \right|_{\max} = \omega_n S_D(\omega_n)$$

Spectral acceleration SRS:

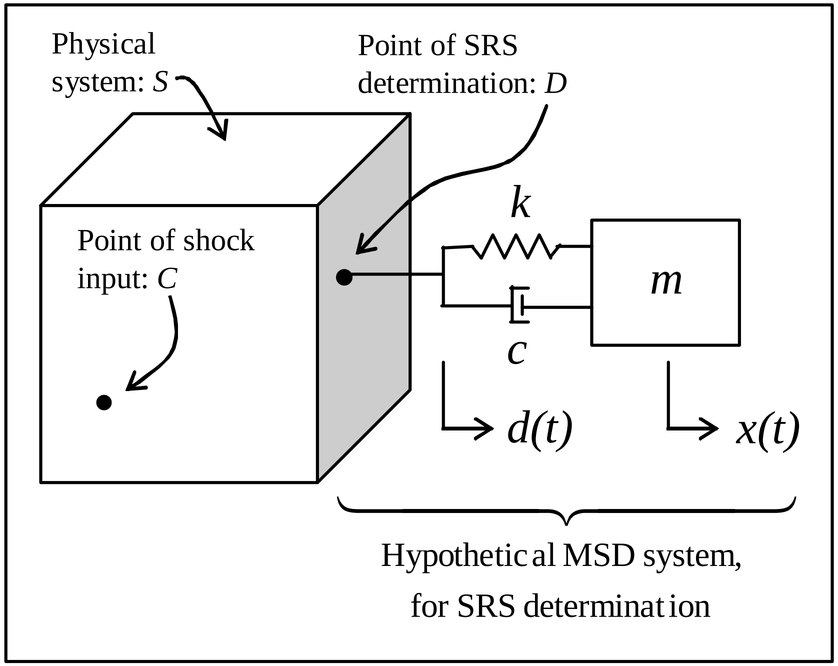
$$S_A(\omega_n) := \left| \ddot{x}(t) \right|_{\max}$$

INTRODUCTION: SRS Definition

Linear system model:



INTRODUCTION: SRS for Local Damped Harmonic Input



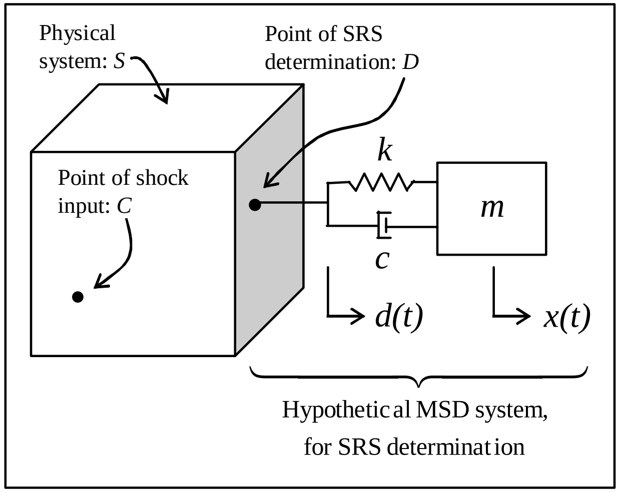
USING LINEAR MODEL

<u>**Local Disturbance *d(t)***</u>**:**

$$d(t) = \sum_{i=1}^{\nu} d_i(t)$$

$$d_i(t) = D_i e^{-\alpha_i t} \sin(\omega_i t + \phi_i) u_{-1}(t)$$

INTRODUCTION: SRS for Local Damped Harmonic Input



SOLUTION (U-086)

<u>Displacement *x(t)*</u>:

$$x(t) = \beta e^{-\varsigma\omega_n t}\sin(\omega_d t + \psi)$$

$$+\gamma\sum_{i=1}^{\nu} D_i\Big\{e^{-\varsigma\omega_n t}\big[\delta_{1i}\sin(\omega_d t+\phi+\phi_i+\theta_{1i})$$

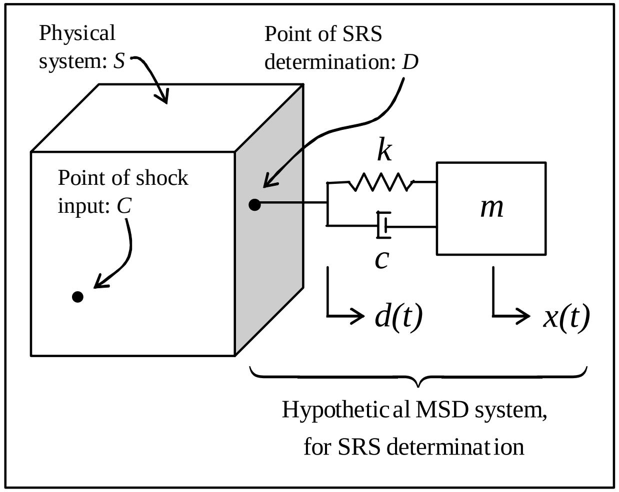
$$+\delta_{2i}\sin(\omega_d t+\phi-\phi_i+\theta_{2i})\big]$$

$$+e^{-\alpha_i t}\big[\delta_{3i}\sin(\omega_i t+\phi_i+\theta_{3i})\big]\Big\}$$

INTRODUCTION: SRS for Local Damped Harmonic Input

SOLUTION (U-086)

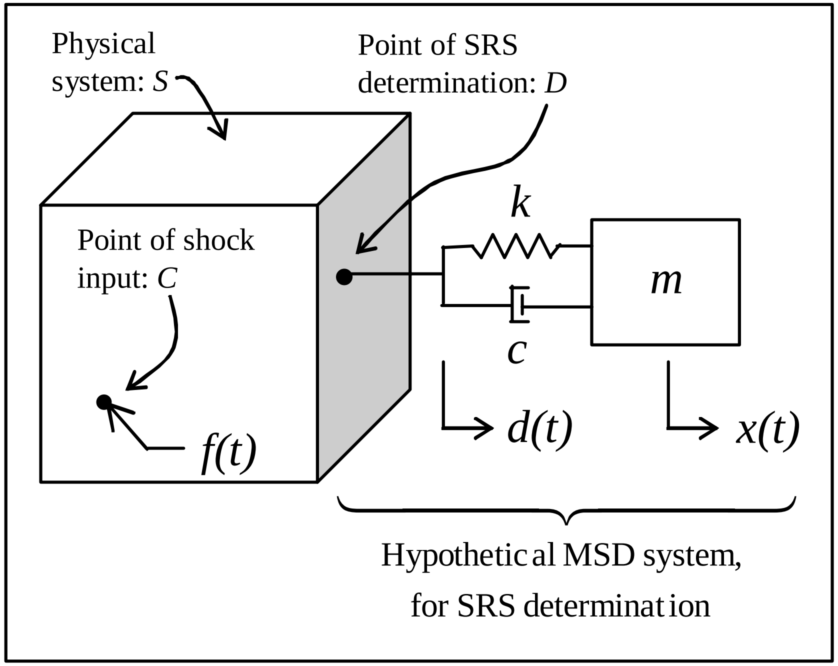
Relative Displacement:



$$
\begin{aligned}
\delta(t) &= x(t) - d(t) \\
&= \beta e^{-\varsigma\omega_n t}\sin(\omega_d t + \psi) \\
&\quad + \gamma \sum_{i=1}^{\nu} D_i \Big\{ e^{-\varsigma\omega_n t}\big[\delta_{1i}\sin(\omega_d t + \phi + \phi_i + \theta_{1i}) \\
&\qquad\qquad + \delta_{2i}\sin(\omega_d t + \phi - \phi_i + \theta_{2i})\big] \\
&\qquad + e^{-\alpha_i t}\big[\delta_{3i}\sin(\omega_i t + \phi_i + \theta_{3i}) \\
&\qquad\qquad - \sin(\omega_i t + \phi_i)/\gamma\big] \Big\}
\end{aligned}
$$

OBJECTIVE: SRS for Remote Shock Inputs

USING LINEAR MODEL



<u>Given remote disturbance</u>

$f(t)$: various impulses

1. Ideal impulse
2. Rectangular pulse
3. Sawtooth pulse

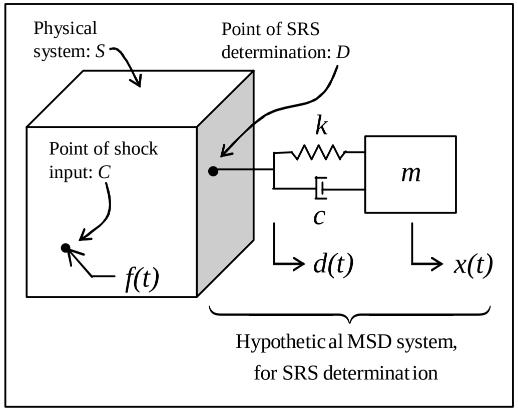
<u>Find kinematic responses</u>

$x(t)$: absolute displacement

$\delta(t)$: relative displacement

SOLUTION: Remote Response of Basic System

FOR BASIC SYSTEM



Modal coordinates

$$\underline{z} = \tilde{U}\,\underline{\eta}\text{, for } \tilde{U} = \left[\underline{\tilde{m}}_1, \cdots, \underline{\tilde{m}}_n\right]$$

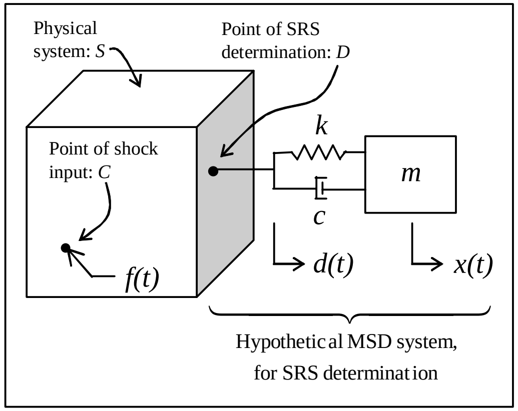
Modal equations of motion

$$\ddot{\eta}_i + 2\varsigma_i \omega_{ni} \dot{\eta}_i + \omega_{ni}^2 \eta_i = \tilde{f}_i$$

Modal solution

$$\eta_i(t) = e^{-\varsigma_i \omega_{ni} t}\left(A_i \cos \omega_{di} t + B_i \sin \omega_{di} t\right) u_{-1}(t) + \frac{1}{\omega_{di}}\left(e^{-\varsigma_i \omega_{ni} t} \sin \omega_{di} t\right) * \tilde{f}_i u_{-1}(t)$$

SOLUTION: Remote Response of Basic System



FOR BASIC SYSTEM

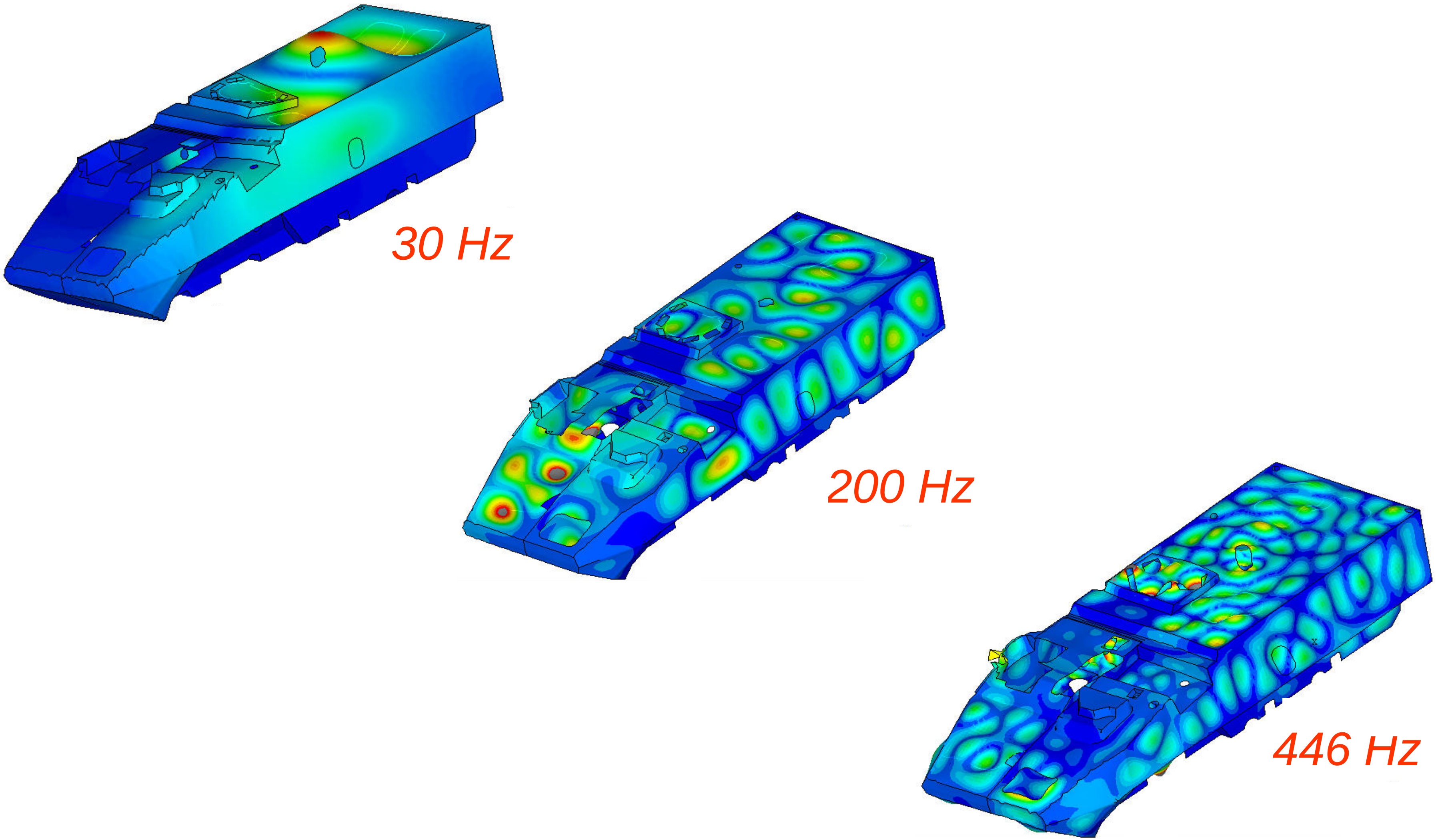
Physical response

$$\underline{z} = \tilde{U}\,\underline{\eta}$$

Induced displacement

$$d(t) = \left(\underline{z}\right)_j = \left(\sum_{i=1}^{n} \eta_i \underline{\tilde{m}}_i\right)_j$$

***EXAMPLE*:** Selected Modeshapes

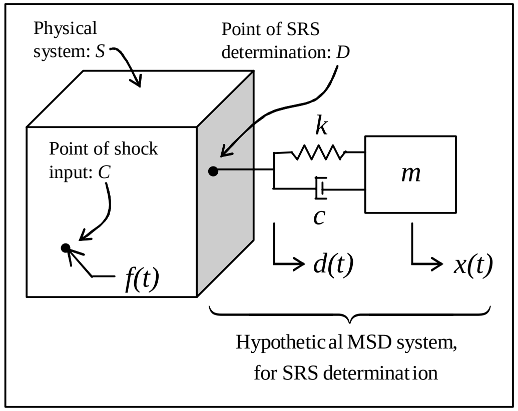


SOLUTION: Response of Attached Mass

FOR REMOTE ARBITRARY INPUT

Induced displacement

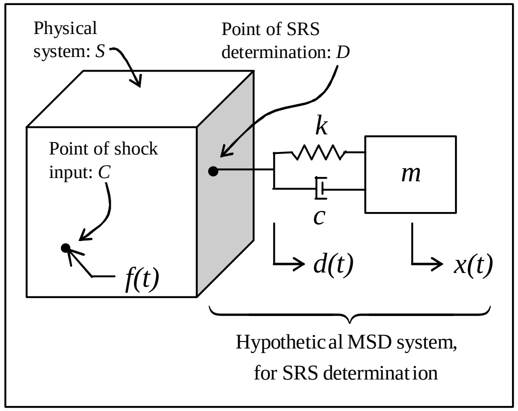
$$d(t) = (\underline{z})_j = \left(\sum_{i=1}^{n} \eta_i \underline{\tilde{m}}_i \right)_j$$



Displacement of attached mass

$$x(t) = e^{-\varsigma \omega_n t}\left(A\cos \omega_d t + B\sin \omega_d t\right)$$
$$+ C\left[e^{-\varsigma \omega_n t}\sin(\omega_d t + \phi)\right] * \sum_{i=1}^{n} \eta_i \tilde{m}_{ij}$$

SOLUTION: Response of Attached Mass



FOR REMOTE IDEAL IMPULSE

Shock input

$$f(t) = \gamma u_0(t)$$

Induced displacement

$$d(t) = \left[\sum_{i=1}^{n} D_{i1} e^{-\varsigma_i \omega_{ni} t} \cos \omega_{di} t + \sum_{i=1}^{n} D_{i2} e^{-\varsigma_i \omega_{ni} t} \sin \omega_{di} t \right] u_{-1}(t)$$

SOLUTION: Response of Attached Mass

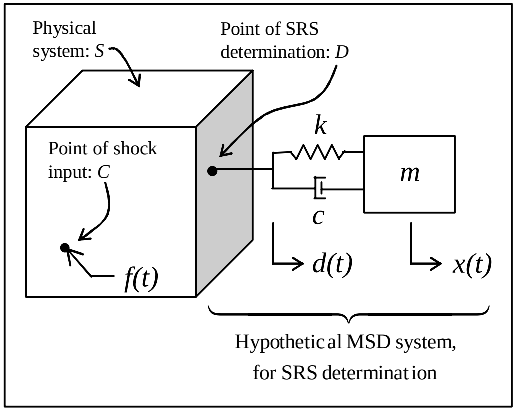
FOR REMOTE IDEAL IMPULSE

<u>Displacement of attached mass</u>

$$
\begin{aligned}
x(t) = \Big\langle & e^{-\varsigma\omega_n t}\left(A\cos\omega_d t + B\sin\omega_d t\right) \\
& + C\sum_{i=1}^{n} D_{i1}\Big\{ e^{-\varsigma\omega_n t}\left[-\delta_{1i}\cos\left(\omega_d t + \phi + \theta_{1i}\right) - \delta_{2i}\cos\left(\omega_d t + \phi + \theta_{2i}\right)\right] \\
& \qquad + e^{-\varsigma_i\omega_{ni} t}\left[\delta_{1i}\cos\left(\omega_{di} t + \phi + \theta_{1i}\right) - \delta_{2i}\cos\left(\omega_{di} t - \phi + \theta_{3i}\right)\right]\Big\} \\
& + C\sum_{i=1}^{n} D_{i2}\Big\{ e^{-\varsigma\omega_n t}\left[-\delta_{1i}\sin\left(\omega_d t + \phi + \theta_{1i}\right) + \delta_{2i}\sin\left(\omega_d t + \phi + \theta_{2i}\right)\right] \\
& \qquad + e^{-\varsigma_i\omega_{ni} t}\left[\delta_{1i}\sin\left(\omega_{di} t + \phi + \theta_{1i}\right) - \delta_{2i}\sin\left(\omega_{di} t - \phi + \theta_{3i}\right)\right]\Big\}\Big\rangle u_{-1}(t)
\end{aligned}
$$

SOLUTION: Response of Attached Mass

FOR REMOTE RECTANGULAR PULSE



<u>Shock input</u>

$$f_k(t) = \gamma\left[u_{-1}(t) - u_{-1}(t - t_1)\right]$$

<u>Induced displacement</u>

$$d(t) = \left[\sum_{i=1}^{n} D_{i1} e^{-\varsigma_i \omega_{ni} t} \cos\omega_{di} t + \sum_{i=1}^{\nu} D_{i2} e^{-\varsigma_i \omega_{ni} t} \sin\omega_{di} t\right] u_{-1}(t) + \left[\sum_{i=1}^{n} D_{i3} e^{-\varsigma_i \omega_{ni} t} \sin\left(\omega_{di} t + \phi_i\right)\right] u_{-1}(t)$$

$$-\left[\sum_{i=1}^{n} D_{i3} e^{-\varsigma_i \omega_{ni}(t - t_1)} \sin\left(\omega_{di}(t - t_1) + \phi_i\right)\right] u_{-1}(t - t_1) + \left[\sum_{i=1}^{n} D_{i4}\right]\left[u_{-1}(t) - u_{-1}(t - t_1)\right]$$

SOLUTION: Response of Attached Mass

FOR REMOTE RECTANGULAR PULSE

Displacement of attached mass

$$x(t) = e^{-\varsigma\omega_n t}\left(A\cos\omega_d t + B\sin\omega_d t\right)u_{-1}(t)$$

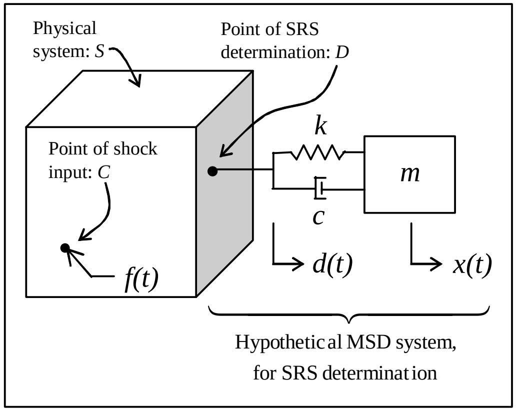
$$+\text{ a rectangular pulse from } t = 0 \text{ to } t = t_1$$

$$+12n \text{ damped sinusoidal terms starting at } t = 0$$

$$+6n \text{ damped sinusoidal terms starting at } t = t_1$$

where n = # of modes retained in original system

SOLUTION: Response of Attached Mass



FOR REMOTE SAWTOOTH PULSE

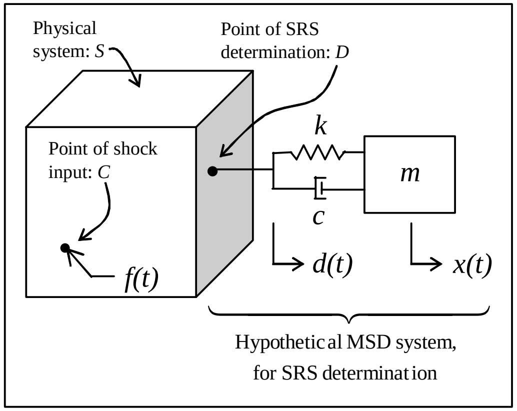
<u>Shock input</u>

$$f_k(t) = \sigma_1 t u_{-1}(t) - \sigma_1(t - t_1)u_{-1}(t - t_1)$$

$$+\sigma_2(t - t_1)u_{-1}(t - t_1) - \sigma_2(t - t_2)u_{-1}(t - t_2)$$

***SOLUTION*: Response of Attached Mass**

FOR REMOTE SAWTOOTH PULSE



Induced displacement

$$d(t)=\left[\sum_{i=1}^{n} D_{i1}e^{-\varsigma_i\omega_{ni}t}\cos\omega_{di}t+\sum_{i=1}^{n} D_{i2}e^{-\varsigma_i\omega_{ni}t}\sin\omega_{di}t\right]u_{-1}(t)$$

$+1$ constant, 1 ramp, and n more damped harmonic terms at $t = 0$

$+1$ constant, 1 ramp, and n damped harmonic terms at $t = t_1$

$+1$ constant, 1 ramp, and n damped harmonic terms at $t = t_2$

where n = # of modes retained in original system

***SOLUTION*: Response of Attached Mass**

FOR REMOTE SAWTOOTH PULSE

<u>Displacement of attached mass</u>

$$x(t) = \langle e^{-\varsigma\omega_n t}\left(A\cos\omega_d t + B\sin\omega_d t\right)$$

$$+1\ \text{constant},\ 1\ \text{ramp, and}\ 14n\ \text{more damped harmonic terms at}\ t = 0$$

$$+1\ \text{constant},\ 1\ \text{ramp, and}\ 6n\ \text{damped harmonic terms at}\ t = t_1$$

$$+1\ \text{constant},\ 1\ \text{ramp, and}\ 6n\ \text{damped harmonic terms at}\ t = t_2$$

where n = # of modes retained in original system

CONCLUDING REMARKS

- **New method developed for part of SRS calculation**

  --Does not require numerical convolution

  --Provides exact SRS-mass response for remote shocks

  Ideal impulses

  Rectangular pulses

  Sawtooth pulses

  --Valid for linearized system model

- **Calculations useful**

  --for evaluating other methods of SRS calculation

  --for determining minimum number of modes required for SRS of specified accuracy