# Stress Response Optimization of Elastic/Viscoelastic Strips

12th Annual ARL/USMA Technical Symposium

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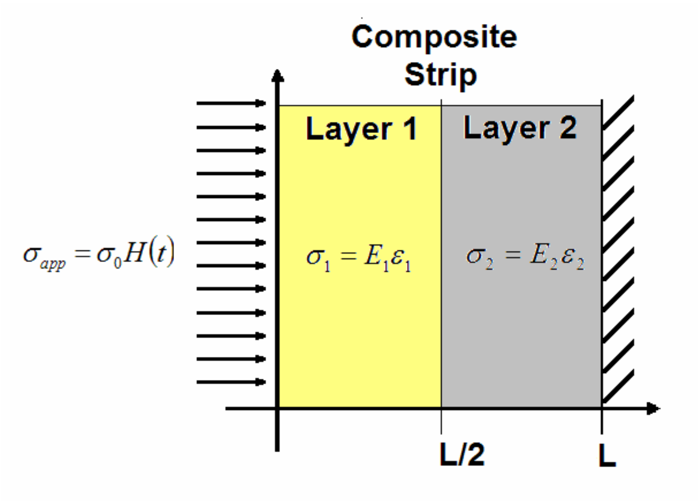
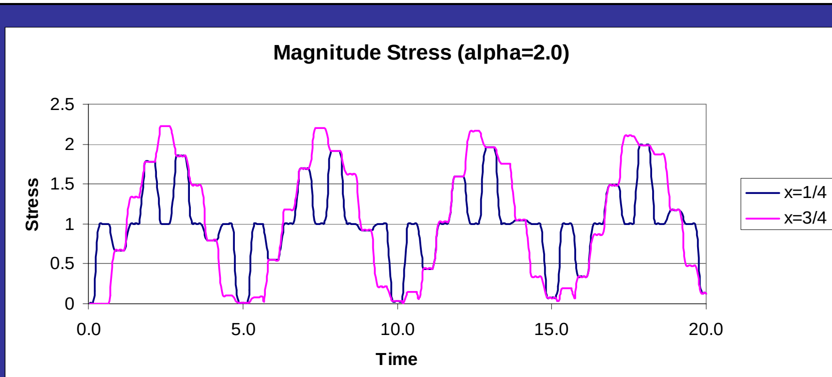
# Background

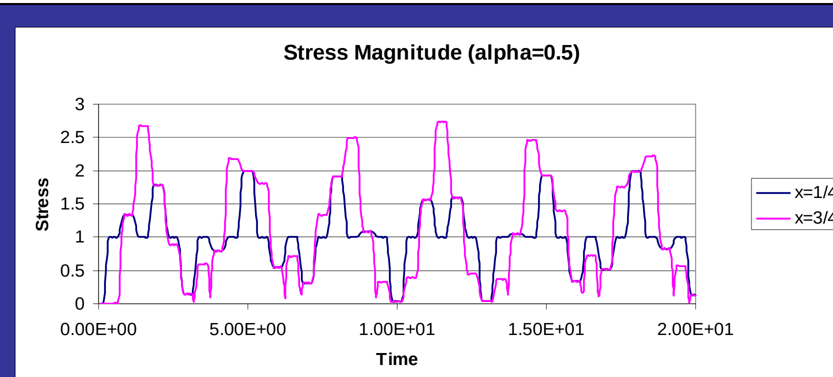



<u>To Limit Parameter Space</u>

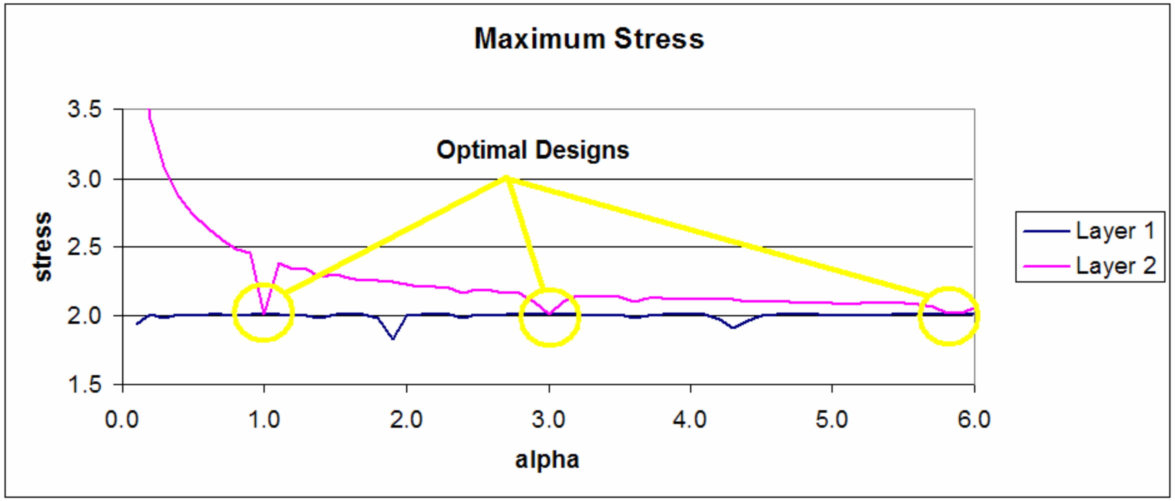
$$E_2 = \frac{E_1}{\alpha} \quad \text{and} \quad \rho_2 = \frac{\rho_1}{\alpha}$$

$$\boxed{\Rightarrow \quad c_1 = c_2}$$



# From Velo and Gazonas, IJSS, Vol. 40, 2003



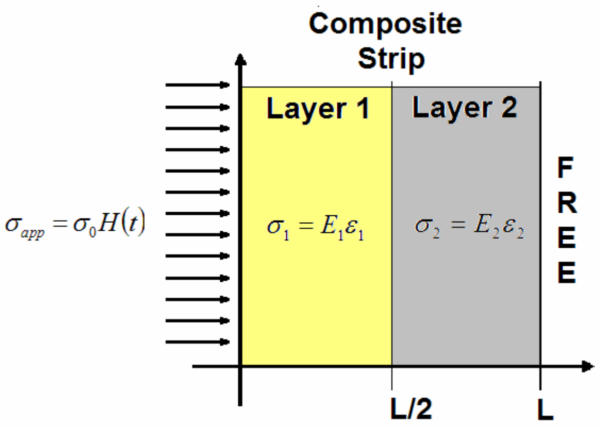

For Homogeneous Strip $\sigma_{1,\max} = \sigma_{2,\max} = 2$

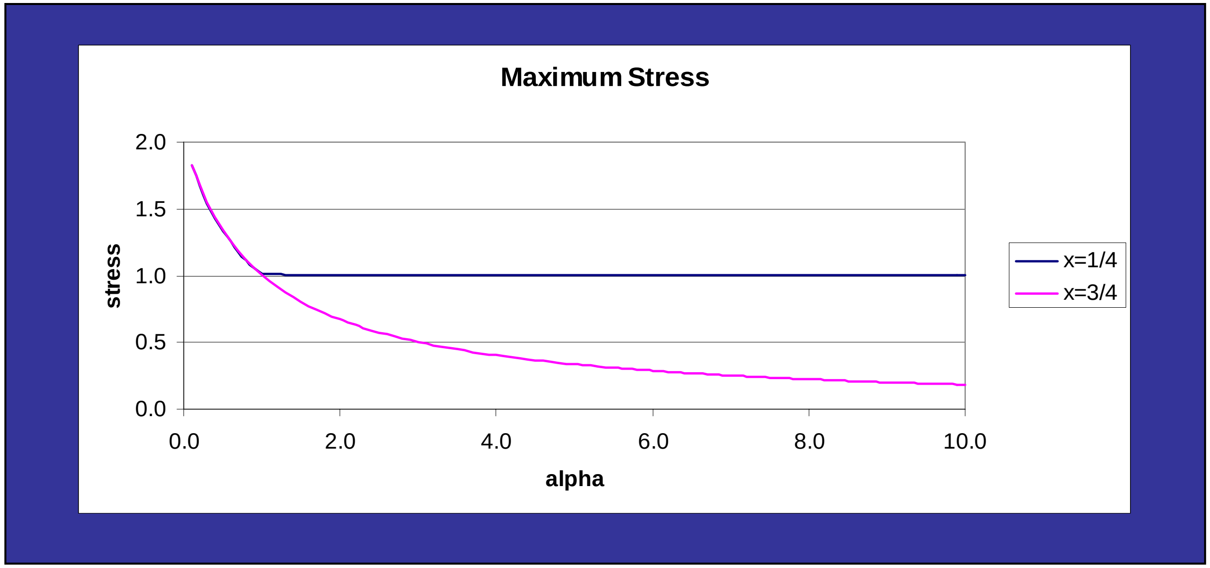
For Composite Strip $\sigma_{1,\max} \leq 2 \leq \sigma_{2,\max}$

Infinite optimal designs exist $\alpha = 1.0, 3.0, 5.8, \ldots$

# Not all models exhibit optimal designs

The same strip with a free back



# Putting the simple models in context

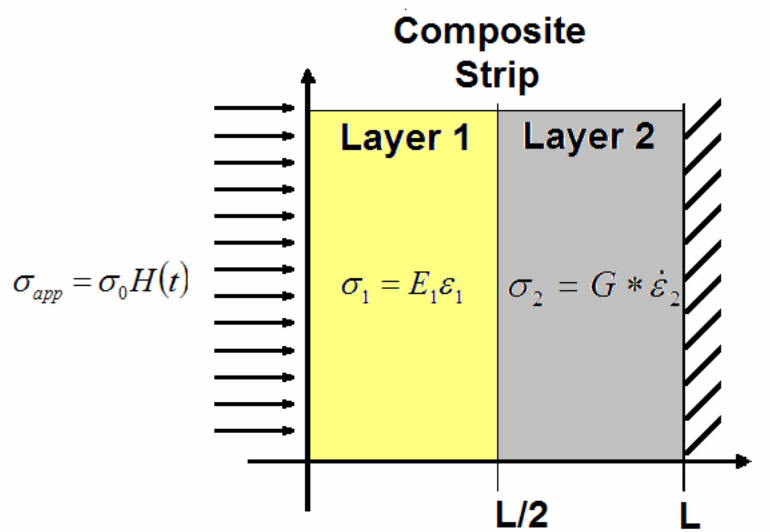
## Purpose of research

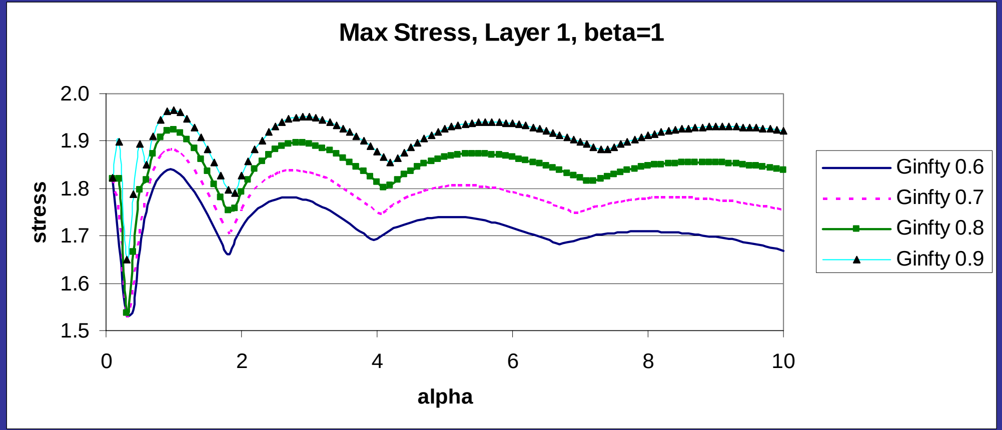
1. Find simple models that have optimal designs to benchmark larger codes (DYNA3D with GLO and others).
2. Solve basic problems in mechanics.

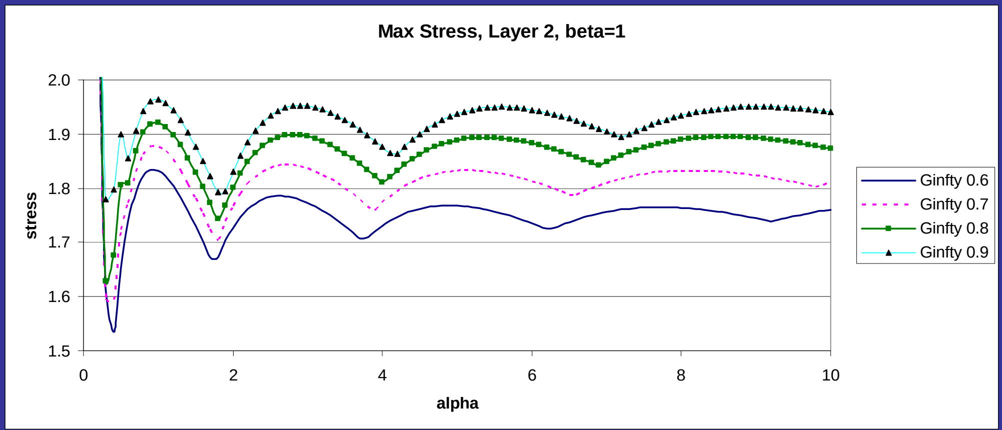
## Proposed 4 BVPs (extend to visco strips)

1. Elastic/Visco, stress step, fixed back
2. Elastic/Visco, stress step, free back
3. Elastic/Visco, rigid impact, fixed back
4. Elastic/Visco, rigid impact, free back

# Elastic/Visco Strip, subject to stress step with a fixed back



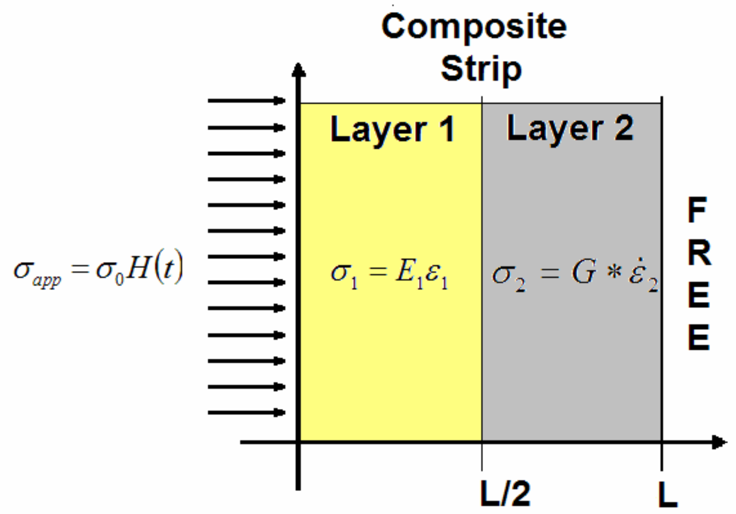



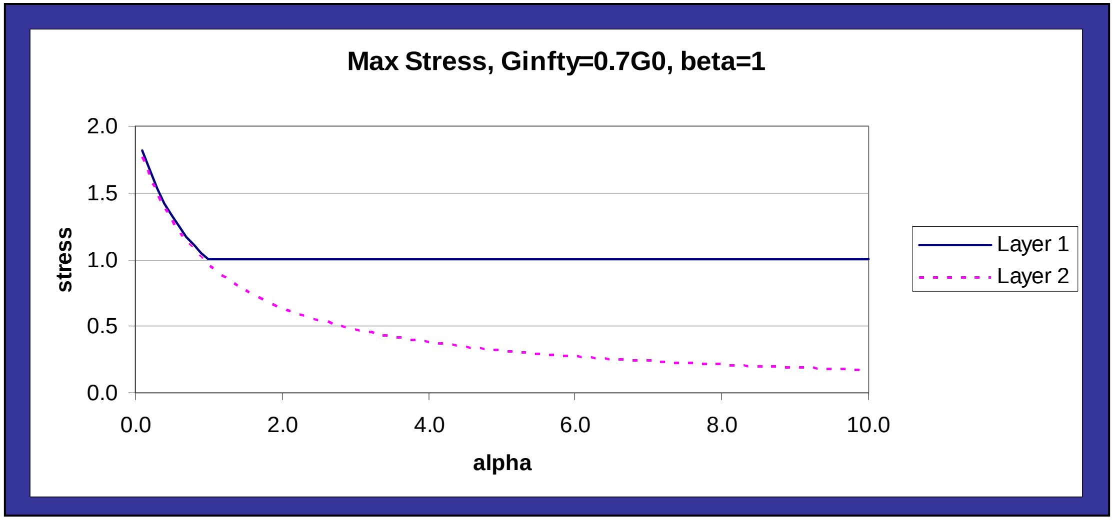




Relaxation Modulus for Standard Linear Solid

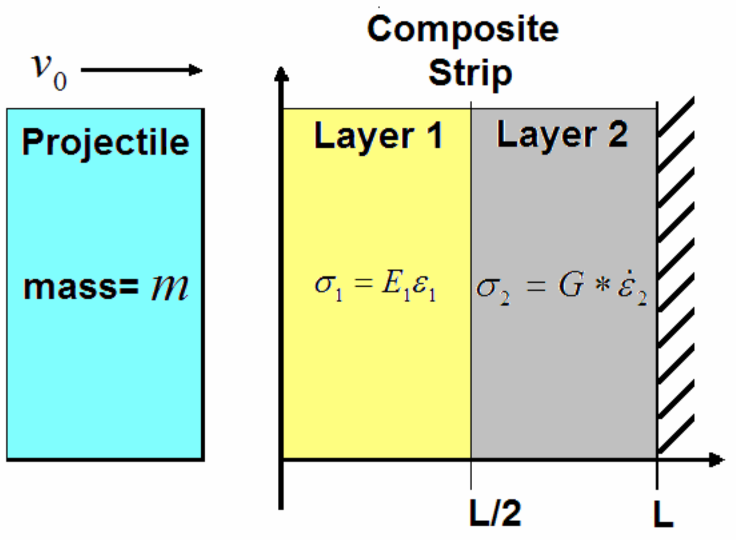
$$G(t) = G_\infty + \left(\frac{E}{\alpha} - G_\infty\right) e^{-\beta t}$$

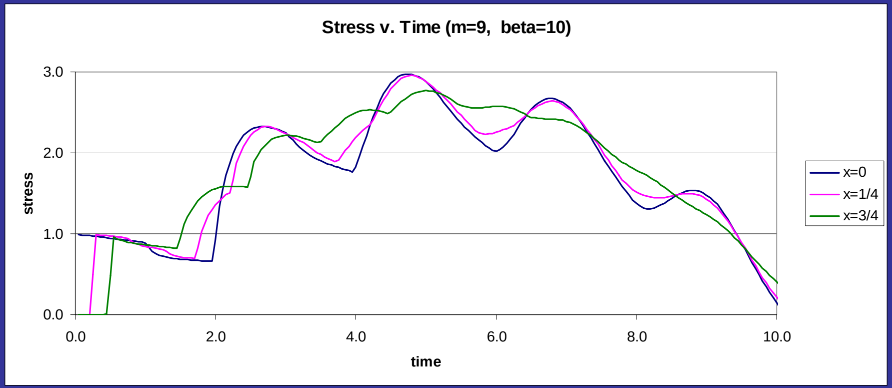
# Elastic/Visco strip, subject to stress step with free back

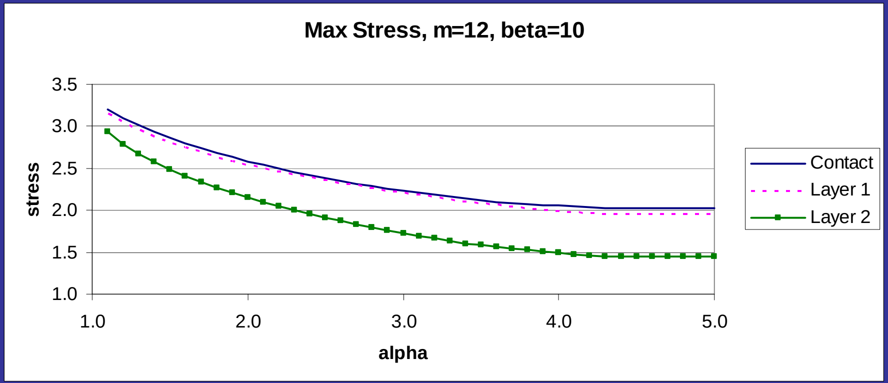


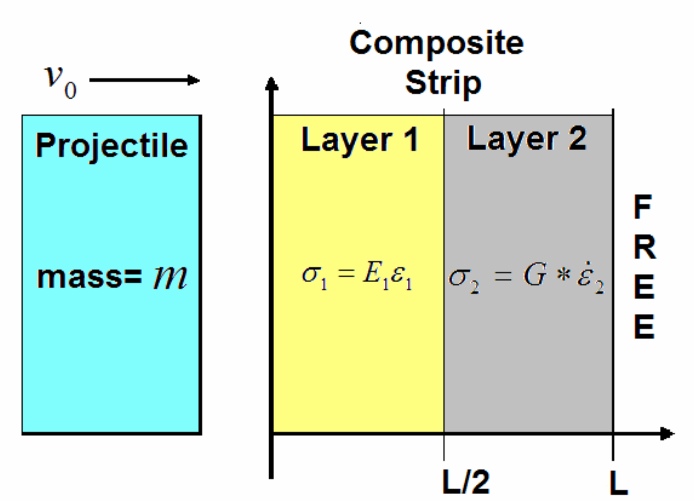
# Elastic/Visco strip subject to rigid impact with fixed back

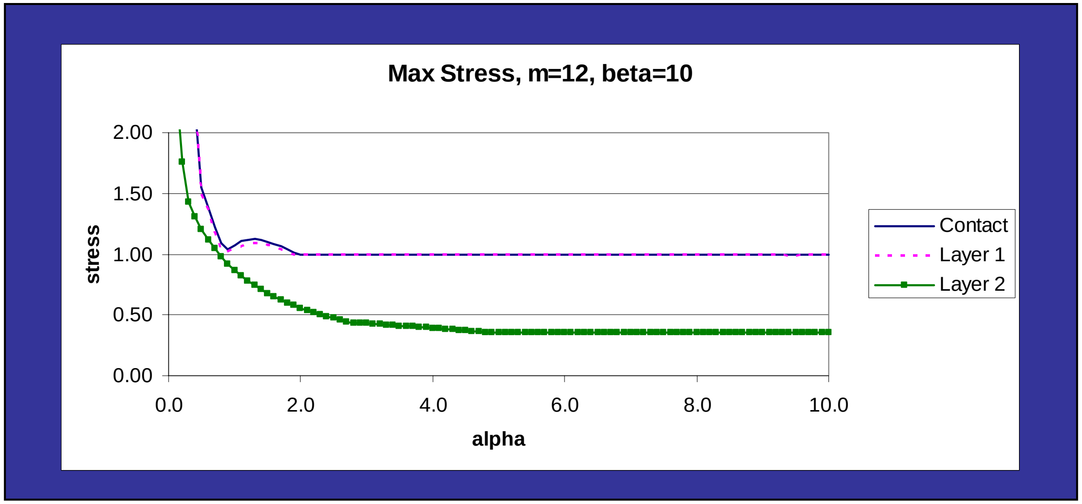


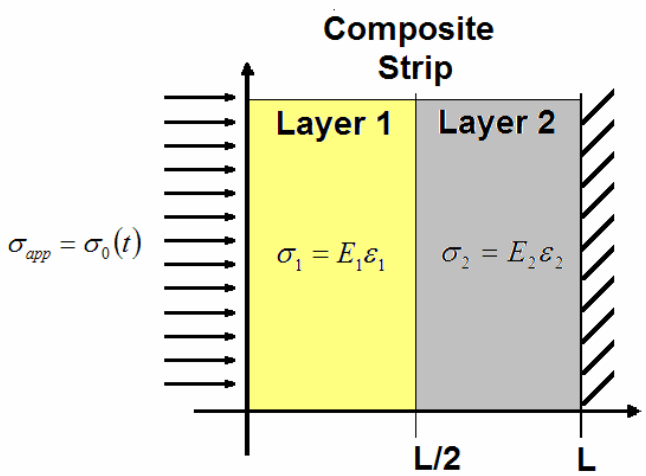
# Elastic/Visco strip subject to rigid impact with free back



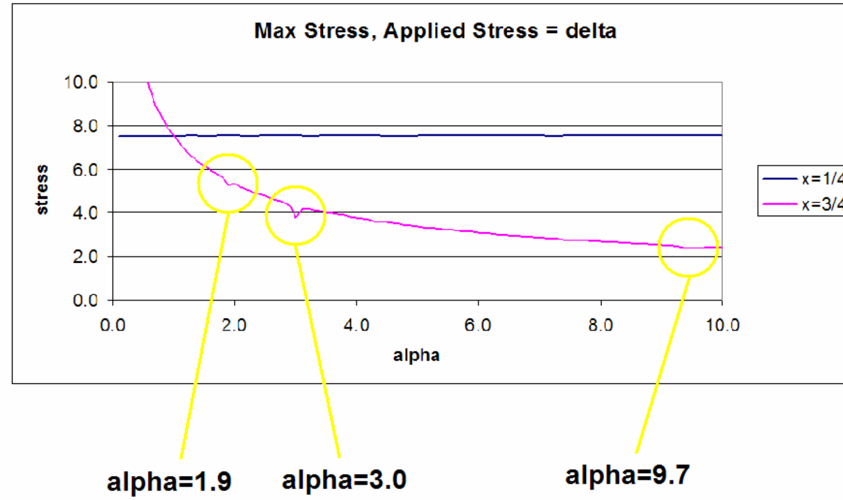



# 1st Summary

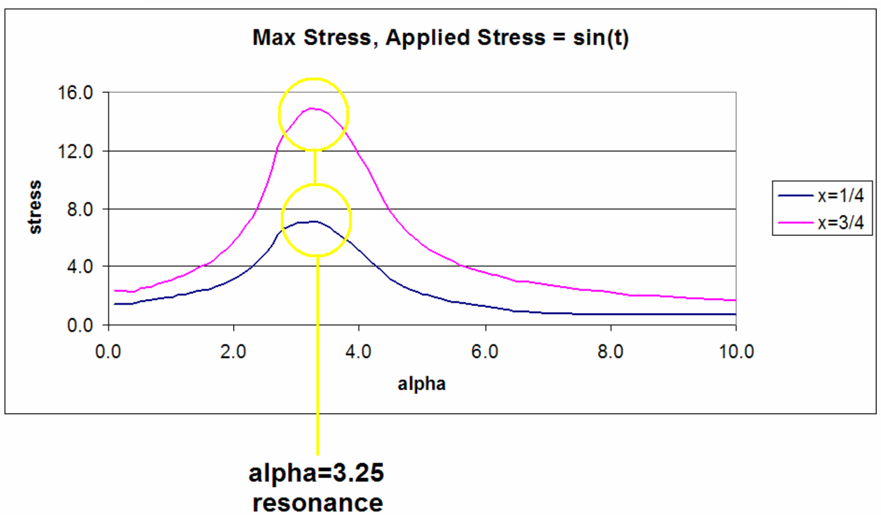
- Elastic/Visco subject to stress step and fixed back is only model with optimal designs.
- “Opimizability” appears to be material independent.
- Future problems:
  - Is applied stress related to optimizability? – Consider other stresses.
  - Is fixed/free end related optimizabilty? – Consider other end conditions.
  - Can we make stress greater in an elastic striker than the target?
  - Will extension to vibrations change results on rigid striker?



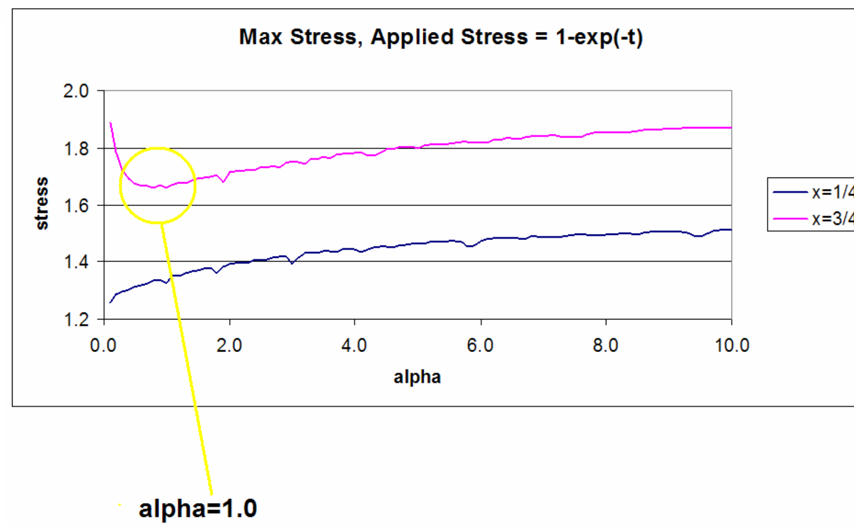
Composite Strip
Layer 1
Layer 2
$\sigma_{app} = \sigma_0(t)$
$\sigma_1 = E_1\varepsilon_1$
$\sigma_2 = E_2\varepsilon_2$
L/2
L

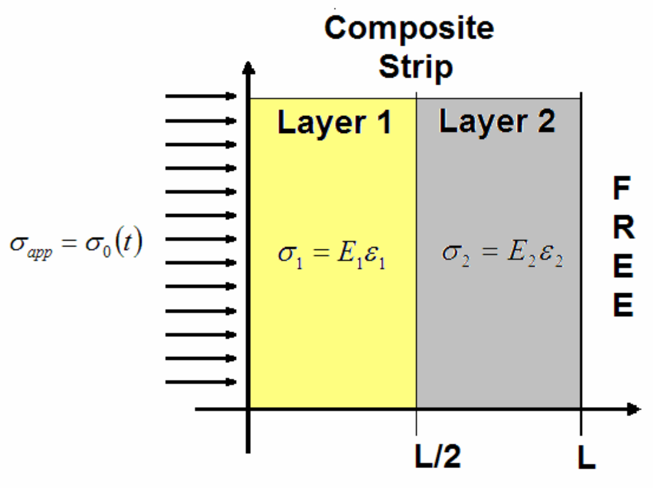
Max Stress, Applied Stress = delta
stress
alpha
x=1/4
x=3/4
alpha=1.9
alpha=3.0
alpha=9.7

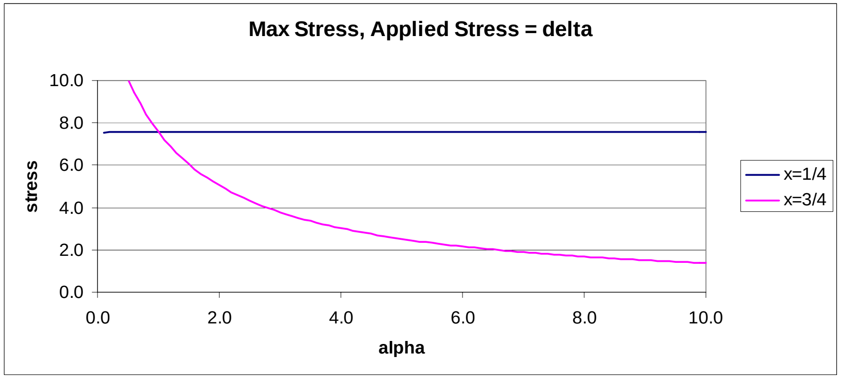



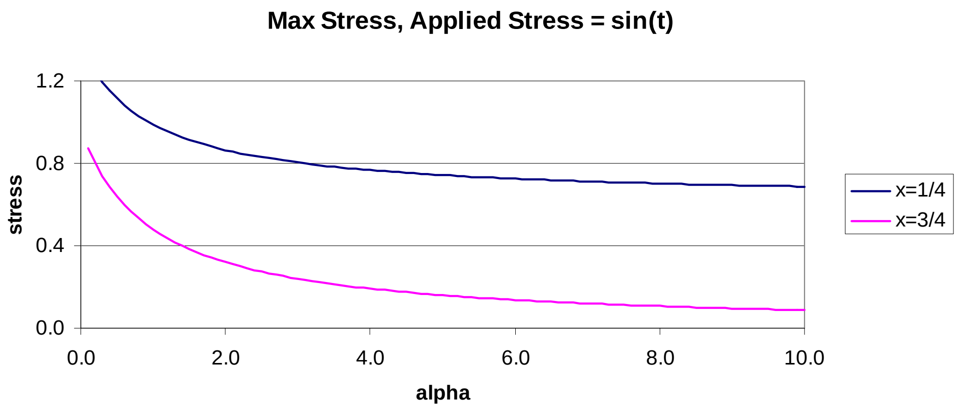
Max Stress, Applied Stress = sin(t)
stress
alpha
x=1/4
x=3/4
alpha=3.25
resonance

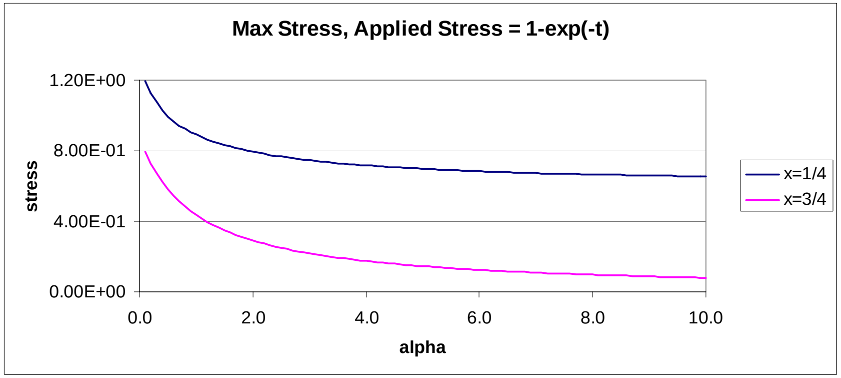



Max Stress, Applied Stress = 1-exp(-t)
stress
alpha
x=1/4
x=3/4
alpha=1.0


Composite Strip
Layer 1
Layer 2
$\sigma_{app} = \sigma_0(t)$
$\sigma_1 = E_1\varepsilon_1$
$\sigma_2 = E_2\varepsilon_2$
FREE
L/2
L


Max Stress, Applied Stress = delta
stress
alpha
x=1/4
x=3/4
10.0
8.0
6.0
4.0
2.0
0.0
0.0 2.0 4.0 6.0 8.0 10.0


Max Stress, Applied Stress = sin(t)
stress
alpha
x=1/4
x=3/4
1.2
0.8
0.4
0.0
0.0 2.0 4.0 6.0 8.0 10.0


Max Stress, Applied Stress = 1-exp(-t)
stress
alpha
x=1/4
x=3/4
1.20E+00
8.00E-01
4.00E-01
0.00E+00
0.0 2.0 4.0 6.0 8.0 10.0

# $2^{nd}$ Summary

Some physical models admit optimal designs. We seek to extend our library of such models to verify larger, more realistic codes.

Far end conditions appear to have largest contribution to determining "optimizability."

We have developed robust Laplace transform inversion algorithm and a modeling framework that handles:

- N Layers
- Arbitrary elastic/visco solids
- Arbitrary applied stresses (time formulas, impacts)
- Arbitrary far end conditions

Problems that <u>WILL</u> be considered:

- Elastic striker
- Robin/mixed far end conditions
- Extend time on impact problems to include free vibrations

# Questions

# Back ups

# Computations

For the elastic/viscoleastic strip:

$$G(t) = G_\infty + (G_0 - G_\infty)e^{-\beta t} \qquad \hat{g} = \sqrt{\frac{\alpha s \hat{G}}{\rho}}$$

$$\hat{\sigma}_2(x) = -\frac{\sigma_0}{s}\left[\left(\frac{s^2\hat{G}}{\hat{g}}\right)\frac{\cosh\left(\frac{s}{\hat{g}}(L-x)\right)}{\frac{s^2\hat{G}}{\hat{g}}\cosh\left(\frac{sL}{2c}\right)\cosh\left(\frac{sL}{2\hat{g}}\right)+\frac{sE}{c}\sinh\left(\frac{sL}{2c}\right)\sinh\left(\frac{sL}{2\hat{g}}\right)}\right]$$

Inversion via Bromwich Integral is possible, but… will an impenetrably complex “exact” formula more useful?

# The DAC Algorithm

Laplace Transform Pair: $\hat{f}(s) = \int_0^{\infty} f(t) e^{-st}\, dt \qquad f(t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \hat{f}(s) e^{st}\, ds$

When $f(t)$ is real and $s = c + i\omega$

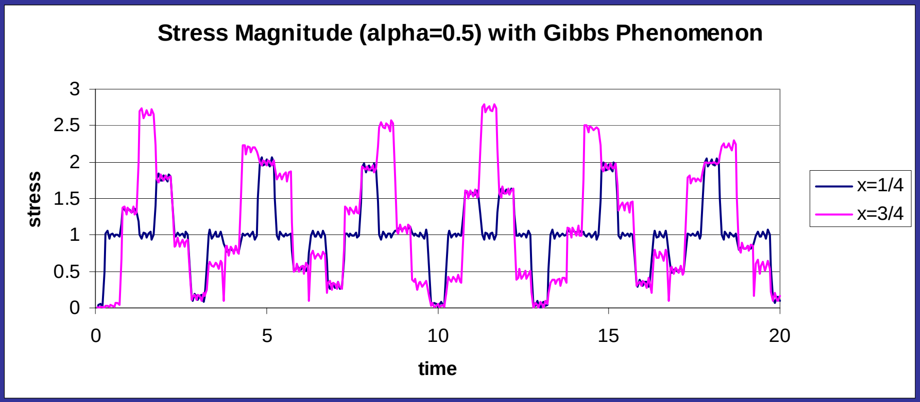
$$f(t) = \frac{e^{ct}}{\pi} \int_0^{\infty} \left[ \mathrm{Re}\left[\hat{f}(s)\right] \cos(\omega t) - \mathrm{Im}\left[\hat{f}(s)\right] \sin(\omega t) \right] d\omega$$

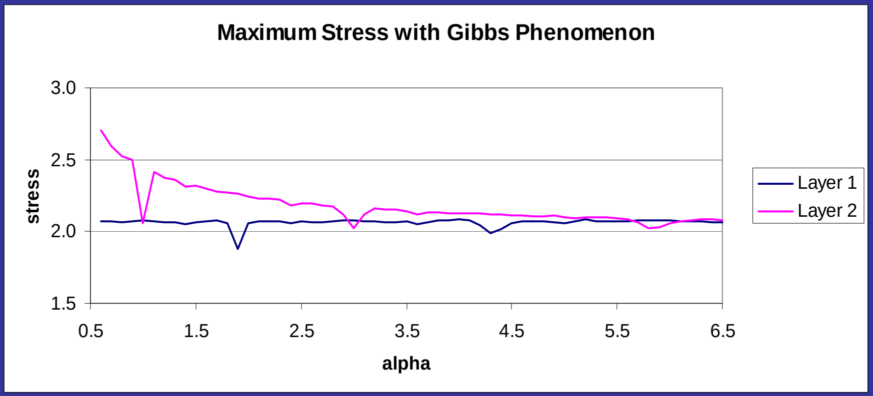
Using Simpson's rule with step size $\Delta\omega = \frac{\pi}{T}$

$$f(t) \approx \frac{e^{ct}}{\pi} \left[ \frac{\hat{f}(a)}{2} + \sum_{k=1}^{\infty} \mathrm{Re}\left[ \hat{f}\left( a + \frac{k\pi i}{T} \right) \right] \cos\left( \frac{k\pi t}{T} \right) - \mathrm{Im}\left[ \hat{f}\left( a + \frac{k\pi i}{T} \right) \right] \sin\left( \frac{k\pi t}{T} \right) \right]$$

The DAC Algorithm generates a Fourier Series on $(0, 2T)$

It is susceptible to Gibbs' Phenomenon at discontinuities:




We mitigate via Lanczos' sigma-factors, a "filter" method.

$$g_N(t) = \frac{a_0}{2} + \sum_{k=1}^{N} \left[ \sigma_k a_k \cos\left(\frac{k\pi t}{T}\right) + \sigma_k b_k \sin\left(\frac{k\pi t}{T}\right) \right] \quad \text{where} \quad \sigma_k = \frac{\sin\left(\frac{k\pi}{N}\right)}{\frac{k\pi}{N}}$$

# Corroboration with DYNA3D

Elastic/Viscoelastic Strip
N=256
tol=0.001