Modeling Diagrammatic Reasoning: Route Planning on a Grid Based Map

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Abstract: We discuss the issues involved in building a cognitive model of diagrammatic reasoning. Specifically, this is a model of route planning on a grid based map, such as a subway map. This model is built in the Adaptive Control of Thought -Rational (ACT-R) cognitive modeling architecture. Maps are a specific type of diagram that are frequently used to describe geospatial information for use in military planning.

Introduction

Maps are diagrammatic abstractions of geospatial information and as such, maps display geospatial information more efficiently than textual or verbal descriptions. A text based description of geospatial information would have to explicitly list each relationship indicated in the map. Even a simple list would need a fairly long list to describe it fully. Our common experience with maps, however, is that geospatial relationships are easily and almost immediately grasped. One aspect of diagrammatic reasoning is that a text based description lacks the inherent organizing schema of a diagram. Instead, the relevant information is in a list and to "construct" the relationships, the information has to be located in the list and reassembled mentally--a time consuming and difficult process.

Two pervasive military examples of problem solving processes that rely on diagrammatic reasoning are route planning and situational awareness updating. Route planning and situational awareness updating would be very difficult with only a text based description of the geospatial information. For this research, we have chosen to build a cognitive model of route planning in order to investigate diagrammatic reasoning. Using a computational approach to describing and predicting diagrammatic reasoning has two benefits: It advances our understanding of the human process of diagrammatic reasoning and lays the foundations for an efficient, robust, and cognitively congruent method of automated reasoning.

The cognitive architecture, Adaptive Control of Thought - Rational (ACT-R), can be used to simulate diagrammatic reasoning - the perceptual process of obtaining information from a diagram and the problem solving process of interpreting the symbolic relationships. The ACT-R cognitive architecture is especially well developed with respect to memory, learning, and higher level cognition such as problem solving [1, 2]. The part of the architecture that invokes the perceptual process is less-well developed. ACT-R incorporates aspects of visual search and attention but not the individual perception per se. The cognitive model of route planning developed as part of this research exposes issues of interest in the area of diagrammatic reasoning. Particular issues that emerge during this process are the following. What scan path does an observer use to obtain information from a map? How is information translated from the visual to symbolic "chunks" that can be readily manipulated as a part of the route planning process? In what way does an observer rely on the diagram to act *as short-term memory*? To what extent does a human observer employ learned algorithms during route planning?

Specifically, we discuss ongoing research in the area of diagrammatic reasoning and construction of a cognitive model of route planning. This cognitive model performs route planning on a grid consisting of named locations connected by regular paths. This simplified version of a map is within the capabilities of the perceptual architecture of ACT-R. This cognitive model does not employ a computational algorithm (as would be used in computer programming) but instead models the cognitive processes that a human uses while executing this task. This work is the initial part of a three year research effort that will be linked with on-going work conducted by ARL-HRED Cognitive Sciences Branch and the Advanced Decision Architectures Collaborative Technology Alliance

Diagrammatic Reasoning

To say that diagrammatic reasoning is defined by the use of diagrams greatly oversimplifies the situation. For example, if someone wanted to determine if there were red balloons at the birthday party, then they could examine photographs of the birthday party and conclude whether or not there were red balloons at the party. But this problem solving process is not diagrammatic reasoning; it is simply a read-out of the feature of a perceptual image.

Diagrammatic reasoning then involves an abstraction that creates symbolic chunks of information in memory, that is, it iconizes key elements. This process eliminates inessential information. Following the research of Chandrasekaran and Josephson [4, 5], diagrams are constructed from primitive objects: lines, points, and regions. The placement of these objects in the diagram results in relationships among them. The diagrammatic reasoning process consists of *perceptual routines* (for example, observing that one region is inside another) and *action routines* that involve the creation, modification, or deletion of objects in the diagram. The creation of a new object need not be the literal addition of a new point, line, or region to the diagram, but could simply involve realizing the existence of a compound object.

The process of diagrammatic reasoning can be facilitated by explicitly taught, intentional knowledge [6]. In geometry, theorem proving is greatly facilitated by formally taught knowledge, to the point where experienced users can rapidly answer questions about geometric figures [8], and in fact predetermine the question from the provided information. However, explicitly taught information is not necessarily essential to diagrammatic reasoning. Experiential knowledge can also be used to infer information from a diagram. For example, free body diagrams can be interpreted using experiential knowledge gained by physically constructing such systems [7]. Sufficient experience in diagrammatic reasoning can possibly result in the development of a proceduralized level of information processing and reasoning where sub-steps of the process become aggregated and therefore, simpler and faster. In this case, the possible interpretations of a diagram have been practiced to the extent that the user need not apply to non-specific intentional or experiential knowledge during the reasoning process.

Diagrams possess both intrinsic and extrinsic properties, in contrast to a verbal description of geospatial information. The extrinsic properties of a diagram are determined by the abstracted geospatial information. There is a direct correspondence between the properties of the geospatial information and the properties of the diagram. Although the intrinsic geometric and topology properties of the flat plane of a piece of paper or the computer screen impose additional restrictions on the diagram, these additional restrictions may not hinder the problem solving process, but instead may facilitate it. That is, diagrams, by eliminating inessential information, reduce both the perceptual processing requirements and the reasoning or problem solving requirements. In the military route planning example, diagrams guide, or essentially limit, the cognitive processing requirements and thereby make it “simpler” to determine the best routes.

Modeling diagrammatic reasoning in a cognitive architecture is a challenging problem. The cognitive architecture must be sufficiently developed to provide methods simulation for some visual/perceptual/attentional and the purely cognitive activities. ACT-R organizes cognitive processes into a series of modules (see Figure 1). The perceptual module can be utilized to request a shift in visual attention to a specific location and then observe the location if it contains a visual object. The memory element representation (or chunk or diagram information) arrived at through the visual/perceptual module can be coordinated with the goal module and the memory module to conduct a search for a specific (terminal) location along a scan path. Finally, by matching what is perceived with the goal and memory elements, ACT-R can solve the problem of determining the shortest path from an initial location to a terminal location. Note that currently ACT-R does not include the initial encoding of the diagram itself; the cognitive model of diagrammatic reasoning must add this encoding.

Route Planning Models

To build a cognitive model specifically of route planning on a map, as stated earlier, we take the viewpoint that diagrams are constructed from three types of primitive objects: points, lines and regions. We obtain information from a diagram by observing the relationships between the objects and recognizing the objects that emerge from these

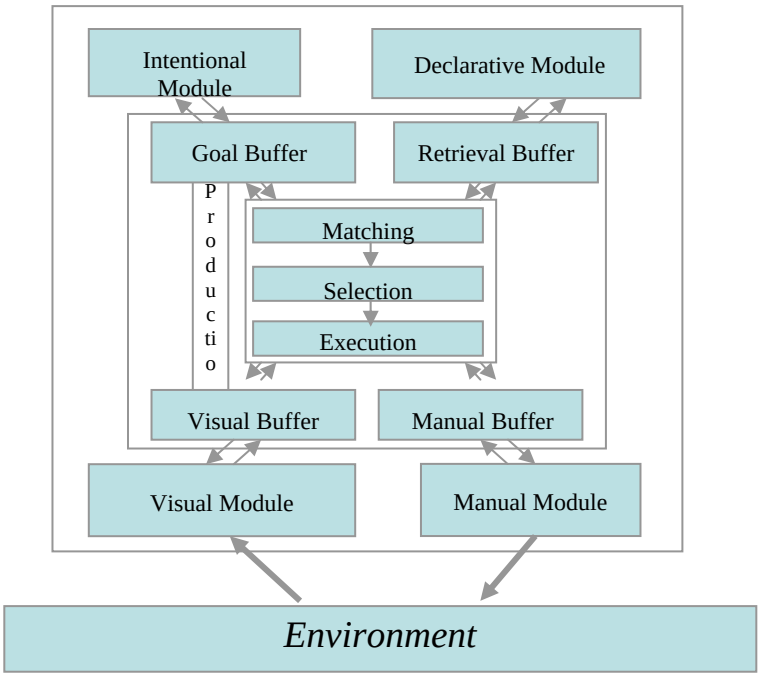

Figure 1. The main elements of ACT-R, version 5 1, 2]

relationships [4, 5]. Information about objects in a diagram is largely information in terms of relationships. Stated verbally this could be: "the region north of this line," "the edge that intersects this edge perpendicularly," or "the point two inches from the center of the
diagram." Diagrams also contain emergent objects - objects that were not explicitly added to the map, but arise from the relative placement of the objects. For example, the intersection of two lines produces a point. This information allows us to use the diagram as an abstracted representation of the environment, and use the diagram as a tool to solve problems.

Route planning on a map is a common problem solving process that utilizes a diagram. A famous historical problem of this type is the Koenigsburg bridge problem [3]. In the problem, the residents of Koenigsburg were asked to plan a route that traversed each of the city's seven bridges exactly once. The mathematician Euler abstracted the arrangement of the bridges into a graph and was able to prove that no such route existed. This graph contained one edge for each bridge and recorded only information about bridge accessibility. If two edges, representing bridge A and Bridge B, shared a common point, then bridge A could be accessed from bridge B, and vice versa. All proposed bridge traversals could be represented on this graph, allowing Euler to demonstrate that the bridge arrangement did not meet the necessary conditions for such a traversal.

Euler's solution used two types of knowledge: experiential knowledge (obtained through experience) and intentional knowledge (formally taught and learned information that can be extended to new situations) [6]. Euler's mathematical knowledge allowed him

to construct a map that could be effectively used to solve the problem. Mathematics provides a schema for organizing and interpreting knowledge obtained from diagrams. Relevant areas of mathematics include graph theory, knot theory, and algebraic topology. These areas provide a descriptive language, including terms such as connected, homotopy, shortest path, and adjacency. This terminology focuses on the properties of the diagrams. This intentional knowledge allows a problem solver to realize the implications of a diagram's properties and a framework for problem solving (algorithms). However, this knowledge can not be accessed unless experiential knowledge obtained through images, personal observation, sensors, etc. is converted to a diagrammatic abstraction.

In a cognitive model, a balance must be struck between this formal knowledge (especially algorithms) and the human tendency of satisficing. The decision to use a formal problem solving technique or simply get a *good enough* answer depends on the scale and complexity of the diagram, human knowledge, and the amount of accuracy required by the task.

Building our cognitive model requires: implementing methods of perception, perceptual routines (as outlined in [4, 5]), and the problem solving activity of matching the perceptions to the goals and, recursively, determining the shortest route. Wanting to keep our initial model small, our diagram is straightforward and so there is no need to construct new diagram elements. As a result, we have not included any “action routines” in the sense of Bannerjee and Chandrasekaran [4] in our model.

The Current Model

The ACT-R model of diagrammatic reasoning plans a shortest route between two locations, A and B, on a 5 X 5 grid based map as shown in Figure 2.

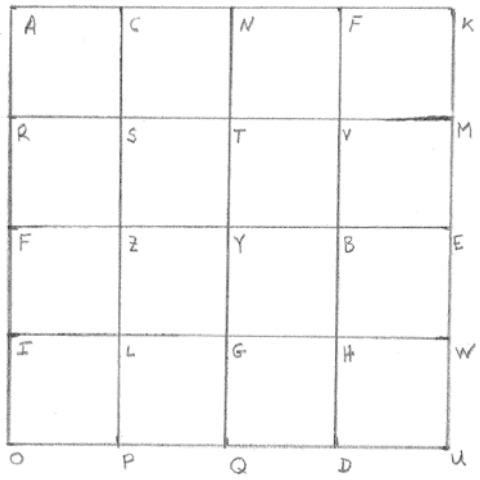

Figure 2: A 5 X 5 Grid Based Map

This map consists of 25 points and 40 edges that define 17 regions. Locations are indicated by labeled points on the map and the edges are described by their endpoints. An edge between two points: A and C (for example) indicates that location C is accessible from location A and vice versa. If two points form the endpoints of a single edge then we refer to the points as adjacent. Note that there is at most one edge between two points. This fact allows us to completely specify a route from location A to location B (that consists of edges) by listing the sequence of locations along the route. The grid is connected, that is, there is a sequence of edges connecting any two locations on the map. The maximum number of edges in a shortest path between any two points is eight.

We make the following assumptions in the construction of the model. First, a maximal foveal sweep or scan of the diagram will reveal the regularity of the grid. This causes the viewer to realize that locations occur at regular intervals and that some path exists between any two locations. The diagram will be used to directly feed short-term or declarative memory, since the user would not attempt to memorize information about all forty edges and would refer to the map prior to selecting an edge in the route.

The cognitive model consists of three distinct activities: methods of perception, perception routines, and problem solving. The ACT-R cognitive architecture consists of modules that simulate various aspects of cognition [2]. These activities must be implemented in the context of these modules. The goal or intentional module holds information about the initial location, A, and the terminal location, B, of the path, and the last observed location in the map. The visual/perceptual module is utilized to observe the diagram following a regular scan path. We model the scan path as proceeding from left to right and then moving from right to left in the row below and then repeating this process. When the productions fire, that is, when the problem solving process executes, the observed information (the name of the location) is used to determine when the terminal location is found and record its relative position on the graph. ACT-R then uses this information to determine a shortest route, choosing edges based on the relative position of the terminal location.

The model will record information about the time required to find the terminal location and plan the path. Finally the model will record and output information about the planned path.

Future Plans

This preliminary model and its results are being used to design a small scale experiment. The experiment will consist of route planning on two types of maps: the type of grid shown in Figure 2 and subway-type maps. Subway maps are presented as series of locations connected by edges, as illustrated by the map of the Chicago Elevated (http://www.transitchicago.com/maps/maps/fwebmaptrain.gif). These maps are slightly more complicated than a grid-based map, as they lack the regularity of a grid. In particular, they require the user to perform a higher level of advance planning prior to choosing the next edge. This type of map is closely related to the grid maps in our

current model. The regions delineated by the lines and points of a subway map are not traversable. The experience of riding on the subway minimizes the amount of geospatial information and the only geographical markers are the station names. The rider is dependent on remembering the name of the terminal station and perhaps the names of some intermediate stops.

This experiment utilizes the preliminary model, in that a reasonable expectation of human performance requires that the participant perform the cognitive steps specified earlier. The participants in this experiment will be given the relative position of an initial point of the map and instructed to first find a terminal location, and then plan a shortest route from location A to location B.

From this experiment, the following information will be obtained: the length of time required to find the terminal location, the length of time required to plan the shortest route, the planned path, and the scan path used to obtain information. The results of this experiment will be compared to the results predicted by the current cognitive model. This model can then be adjusted to model the actual experimental results - reflecting difference between the actual scan path and the expected scan path.

In the trials involving the subway maps, the results will be analyzed using a variety of techniques including comparing scan paths and timing with measures of complexity: girth, circumference, and diameter. Girth indicates the length smallest closed loop in the graph, diameter indicates the length of the longest closed loop, and diameter indicates the length of the longest possible shortest route between any two points. These results will then be used to scale up the initial cognitive model to one that adequately describes the cognitive processes of route planning on a subway.

Additionally, it is expected that more advanced perceptual routines, that is the model code that translates the diagram into symbolic memory chunks, will be required for route planning on the subway compared to route planning on the grid based map. Thus, some of the long term implications of this work is for computational approaches to diagrammatic reasoning generally, over and above the descriptive and predicative power of a particular cognitive model.

Bibliography:

[1] The ACT-R Webpage, http://act-r.psy.cmu.edu/.

[2] Anderson, J. R., Bothell, D., Byrne, M. D., Douglass, S., Lebiere, C., and Qin, Y. (2004). An integrated theory of the mind. Psychological Review 111, (4). p. 1036-1060.

[3] Arlinghaus, Sandra L.; Arlinghaus, William C.; Harary, Frank (2002). Graph theory and geography. An interactive view. Wiley-Interscience Series in Discrete Mathematics and Optimization., John Wiley and Sons, Inc., New York.

[4] Bannerjee, Bonny andChandrasekaran, B. (2005). Perceptual and Action Routines in Diagrammatic Reasoning for Entity-Reidentification. Unpublished manuscript, Laboratory for Artificial Intelligence, The Ohio State University.

[5] Chandrasekaran, B., Josephson, J.R., Bannerjee, B., Kurup, U., and Winkler, R. 2002. Diagrammatic Reasoning in Support of Situation Understanding and Planning. Proceedings of the Army Science Conference, Dec 2-4, Orlando, FL.

[6] Golledge, Reginald G. (1999). Human Wayfinding and Cognitive Maps, Wayfinding Behavior: Cognitive Mapping and other Spatial Processes, Chap 1. John Hopkins University Press, Baltimore, MD.

[7] Larkin, Jill H. and Simon, Herbert A. (1995). *Why a Diagram is (Sometimes) worth Ten Thousand Words*, Diagrammatic Reasoning - Cognitive and Computational Perspectives, AAAI Press/MIT Press. p. 69-109.

[8] McDougal, Thomas F. and Hammond, Kristian J. (1995). *Using Diagrammatic Features to Index Plans for Geometry Theorem-Proving*, Diagrammatic Reasoning - Cognitive and Computational Perspectives, AAAI Press/MIT Press, p. 691-709.