Statistically Valid Estimation of Limit Velocity in Ballistics Testing with Small Sample Size

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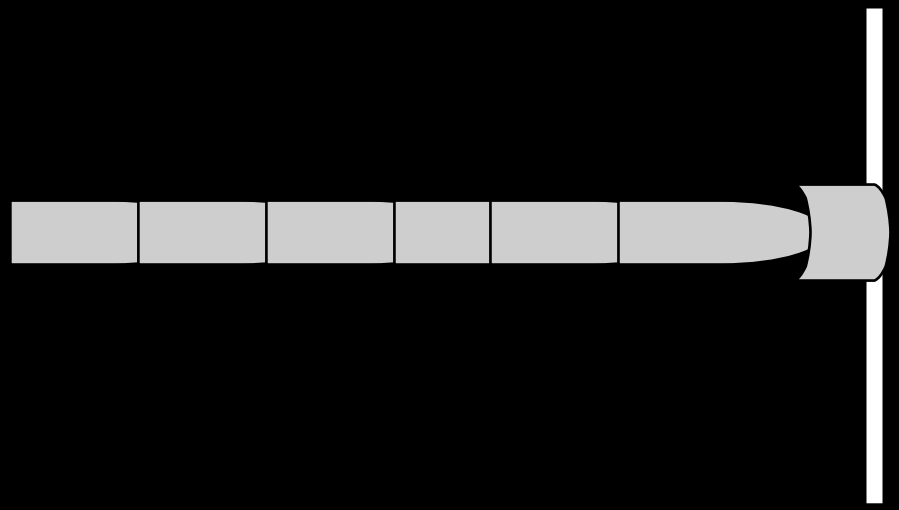
Agenda

- Background
- Current Experimental Methods
- Problem
- Simulations
- Findings
- Conclusions

Background

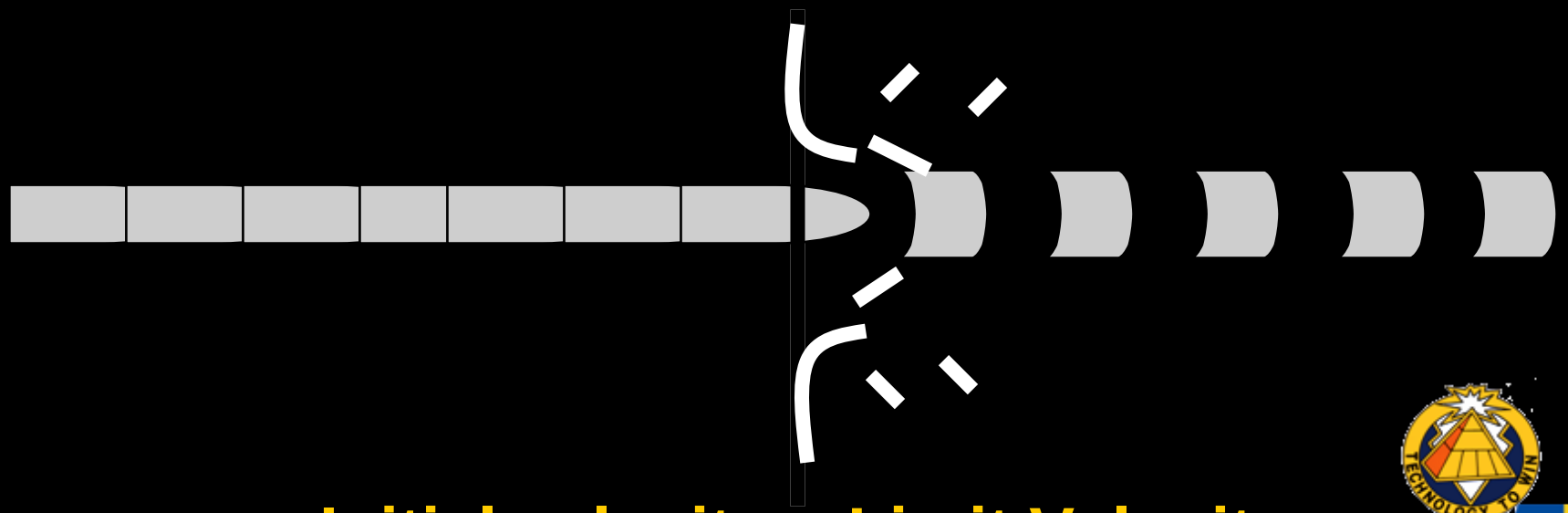
- **Limit velocity is the maximum velocity a projectile can have without penetrating armor.**



Initial velocity < Limit Velocity

Background

- Limit velocity is the maximum velocity a projectile can have without penetrating armor.



Initial velocity > Limit Velocity

Background

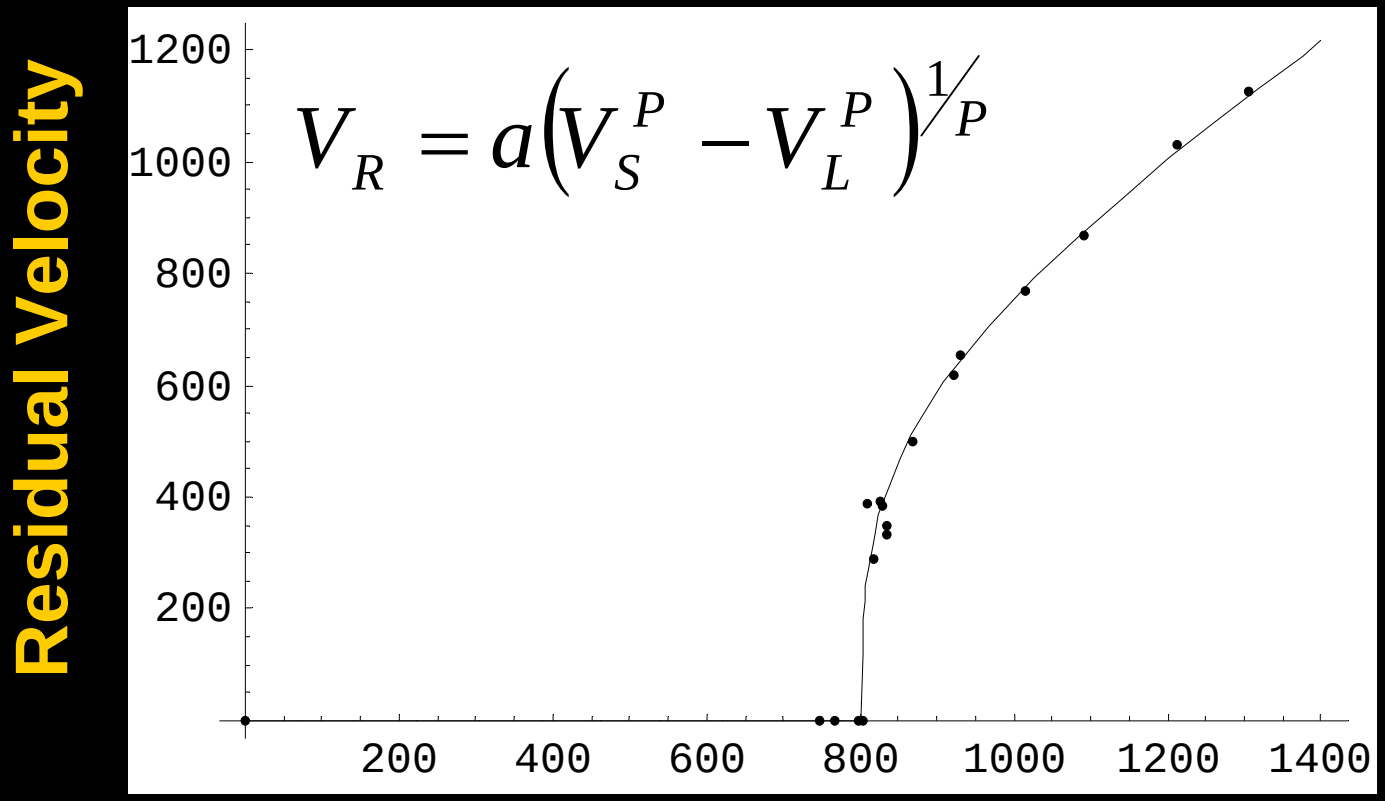
- Government agencies conduct ballistics testing to determine the limit velocity of different armor-penetrator combinations.
- There is no first principles approach to determining limit velocity.
- Ballistics testing is expensive, therefore sample size is small.

Current Experimental Methods

- **V50: Bracket the limit velocity with an upper and lower bound.**

- **Limit Velocity: Use X-ray measurements of residual velocity to parameterize a Jonas-Lambert curve.**

Jonas-Lambert Curve



Problems

- **V50:**
  - **Limited by the quality of the initial estimate of the limit velocity**
  - **Does not estimate well in a zone of mixed results**

Problems

- **Limit Velocity:**
  - **Challenge of estimating 3 parameters and error with a limited number of tests**
    - **Parameter a**
    - **Parameter p**
    - **Limit Velocity**
    - **Error**

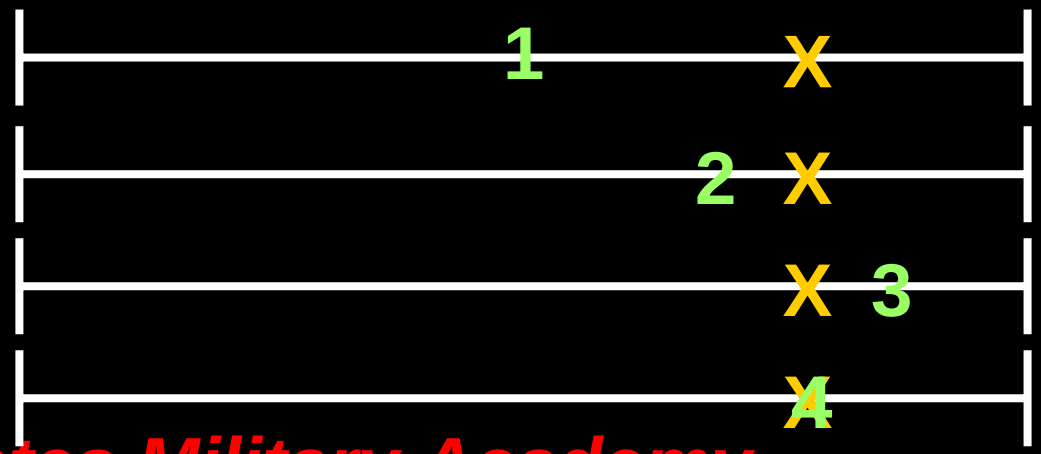
$$V_R = a\left(V_S^P - V_L^P\right)^{1/P}$$

Simulations

- Monte Carlo simulation was conducted for V_{50}, Golden Ratio, and Jonas-Lambert.
- Standard procedures were developed for each technique to optimize the results.
- All three methods were compared to determine the “best” method.

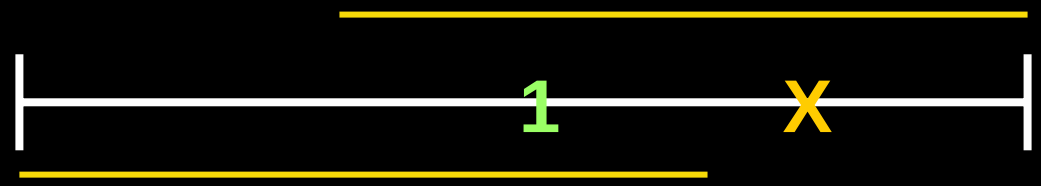
The V_{50} or Bisection Method

- Assume a good ballistician can estimate V_L within ±50 m/s
- V_L is simulated by a uniform rand var between 0 and 100.



Golden Ratio Method

- Similar to V_{50}, only concerned with penetration, thus simulated the same.
- The potential span is divided into upper and lower 61.8% (golden ratio).
- If a shot penetrates, the lower bracket is used, otherwise the upper is used.



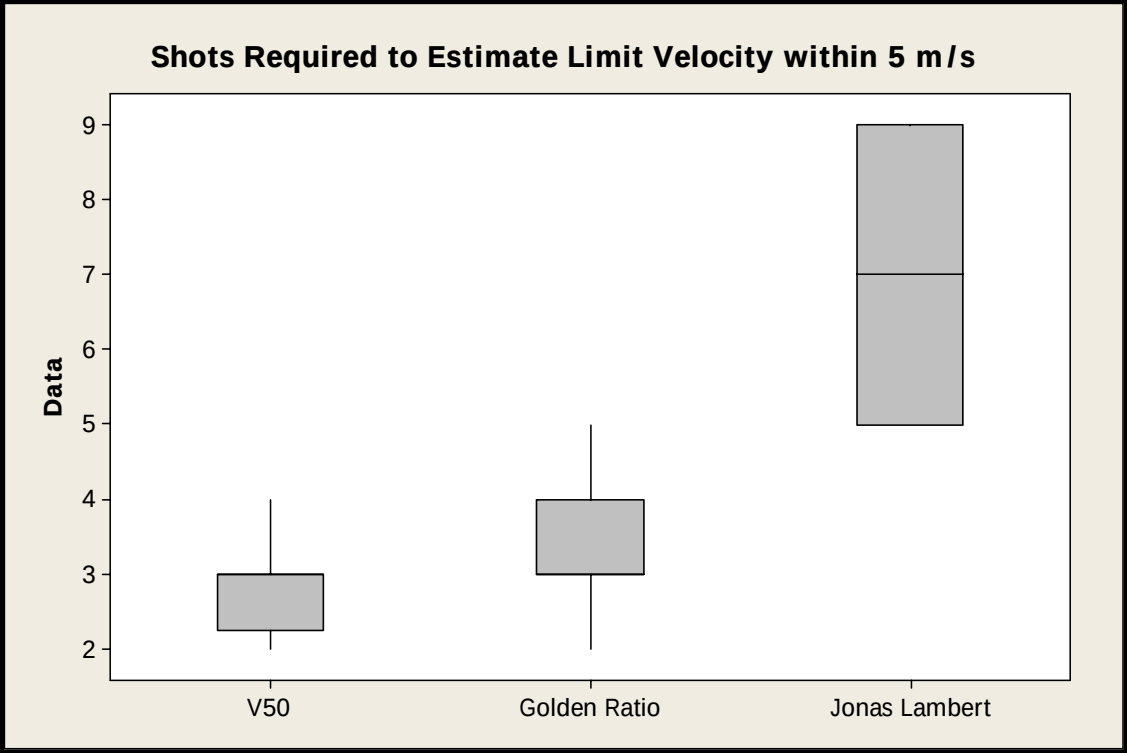
Jonas-Lambert Method

- 7 different data sets were used.
- Assumed that the Jonas-Lambert equation correctly modeled V_L.
- Standard procedure initially assumed *a*=1, *p*=2, *error*=0. With each penetration first estimate *p*, then *a*, then *error*.

Jonas-Lambert Method Example

Shot	V_S	V_R	Est V_R	Sq Err	SSE	Est		Act	
1	1405	416	446	941	0	a	0.89	a	0.89
2	1342	0	0	0	0	p	2.59	p	2.59
3	1380	230	321	8350	0	V_L	1373	V_L	1373
4	1375	142	155	176	0				

Findings



Does not account for zone of mixed results

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Conclusions

- Currently, the Jonas-Lambert or Limit Velocity technique does not outperform the V50 technique.
- Jonas-Lambert requires complete penetrations for useful data points.
- Jonas-Lambert's success assumes that the model is correct.

Future Research

- Jonas-Lambert model is derived from conservation of energy (a=1, p=2).
- Improve the model with the application of PDEs.
- This will create a closer starting point, which may result in improved performance.