13th ARL/USMA Technical Symposium

ANALYTICAL SHOCK RESPONSE OF A TRANSVERSELY POINT-LOADED LINEAR RECTANGULAR PLATE

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PRESENTATION OUTLINE

INTRODUCTION

Motivation

Context

OBJECTIVES

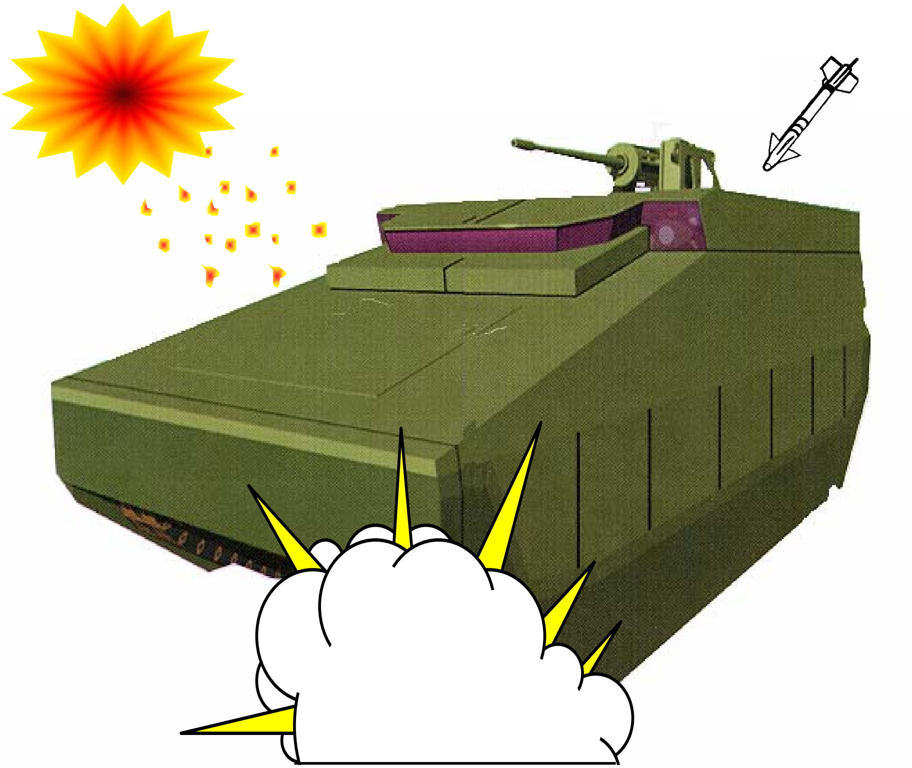
- Develop matrix equations for same-plate shock analysis
- Solve equations for idealized impulsive inputs
- Verify solution

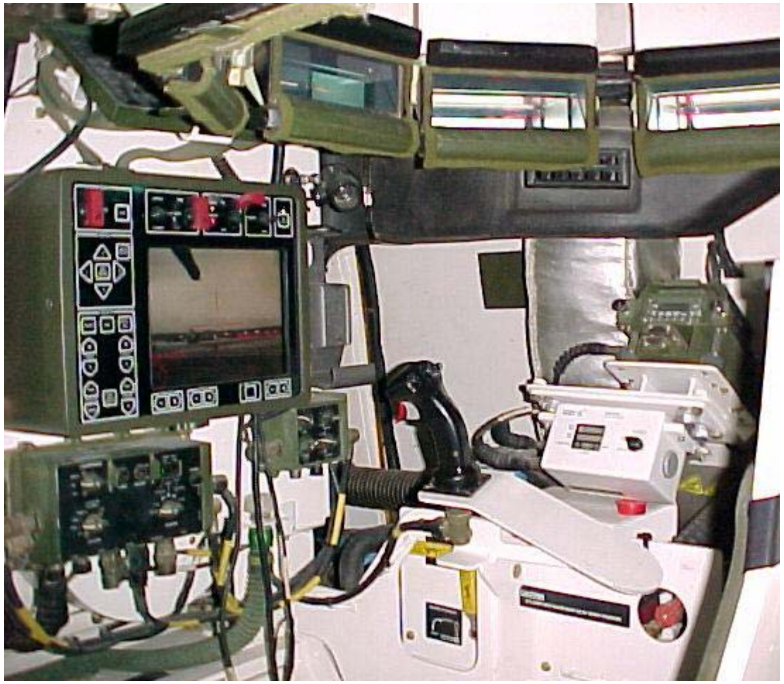
MODEL DEVELOPMENT & SOLUTION

IMPLEMENTATION & VERIFICATION

REMARKS & FUTURE WORK

MOTIVATION: Reduce Damage to Electronics





INTRO: Overview of Vibration Analysis Approach

Geometries of structural model
Coordinates of impact nodes
Equipment mounting locations
Loading impulse levels

BRL-CAD Model
and other sources

Unique for each equipment mounting location

Shock Model

Pseudo-Velocity Shock Response Spectrum

Unique for each impact
location and loading type

Compare
to determine
failure probability

Damage Criteria

Equipment Fragility Parameters

From existing database,
or to be established experimentally

Statistical failure spectrum,
unique for each equipment category

INTRO: FEA Shock Models for Armored Vehicles: Reducing the time for impact calculations

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\} = \{F\}$$

FEA Structural Transient Analysis

- Direct Integration
  - Explicit: Dyna, Abacus
  - Implicit: Nastran, Ansys

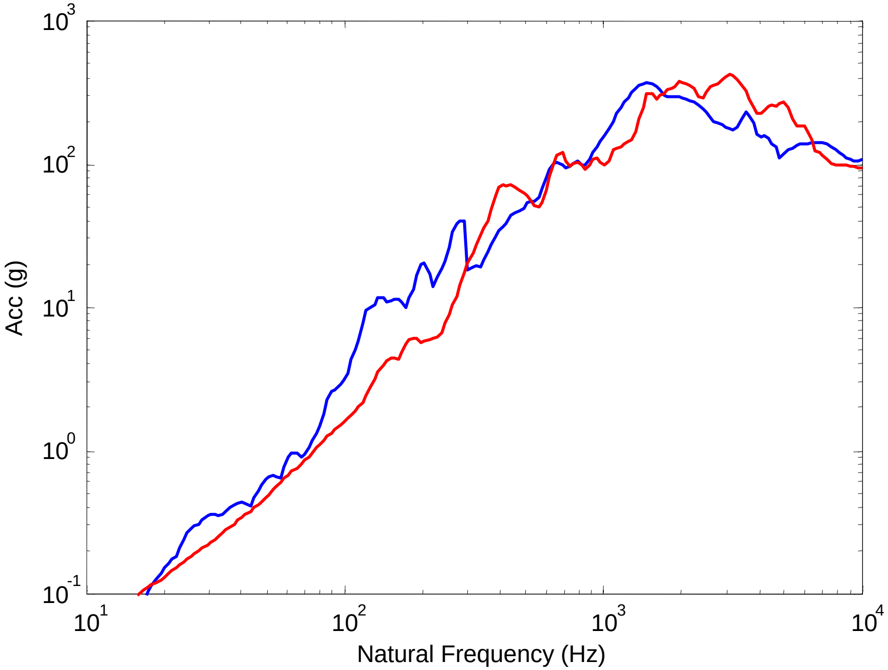
- Used by most analysts
- Current shock model: ~2 days/impact
- Too slow for many impacts
- But can be used for comparison in a few cases

- Modal Superposition: Ansys

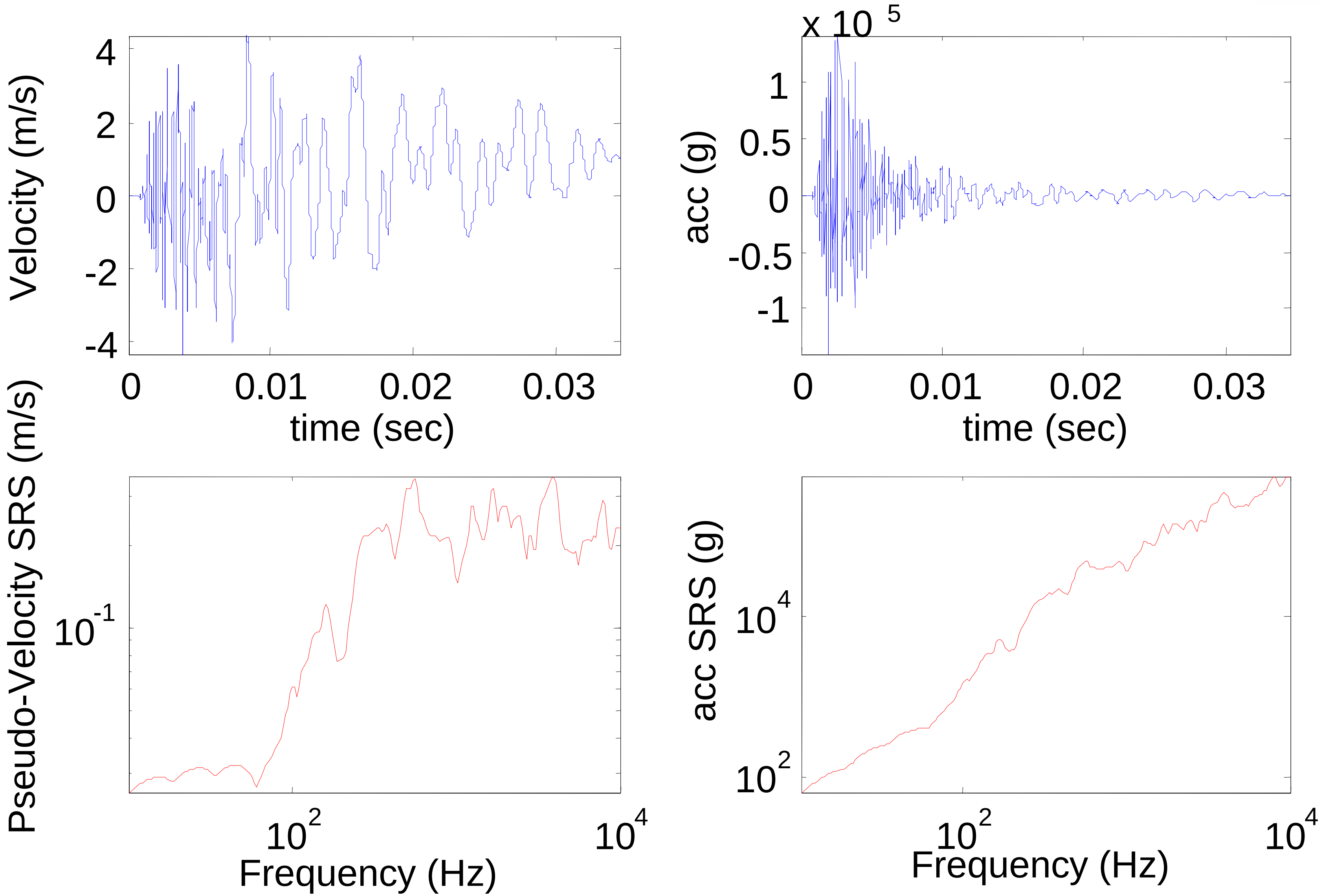
- Very fast for a large number of impacts
- Useful for a linear system only (based on linear superposition of modes)
- Permitting a clear-cut upper frequency, limited by the number of modes; possible in eigen-analysis

INTRO: Typical shock model

Blue: Shock response measurement from a ballistic test
Red: Corresponding response from an FEA shock model



INTRO: Typical SRS Data and Plots



INTRO: FEA Modal Superposition Method

FEA shock model

~99% of total solution time

Modal Analysis

Eigenvalues
Eigenvectors

~1% of total sol'n time for each impact node

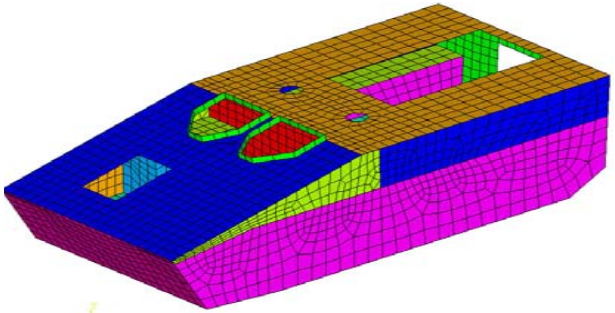
Transient Analysis

New impact node

Time-domain Response

PSRS

Example of solution time
(IHS coarse mesh model)



~4000 shell elements
450 modes

Modal analysis time:

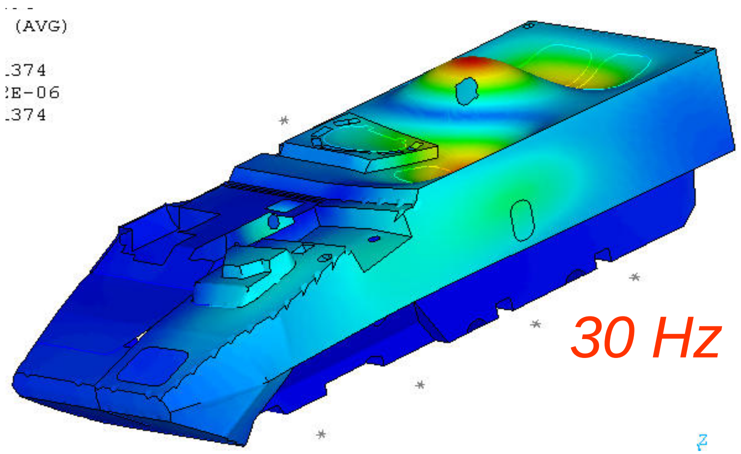
67 hrs

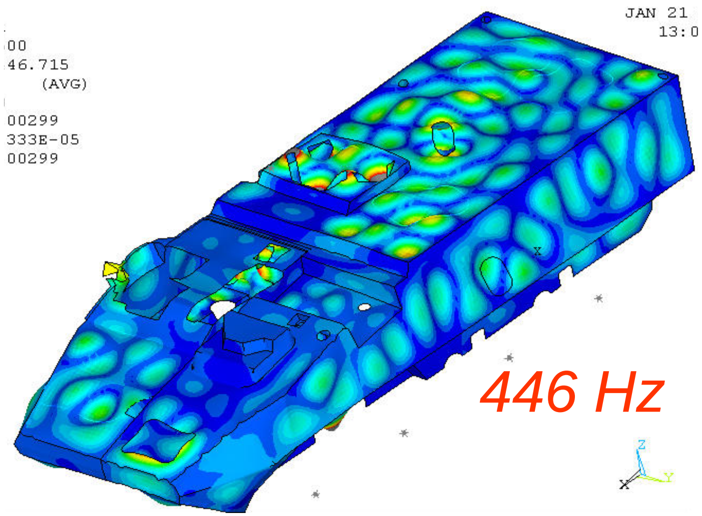
Transient solution time:

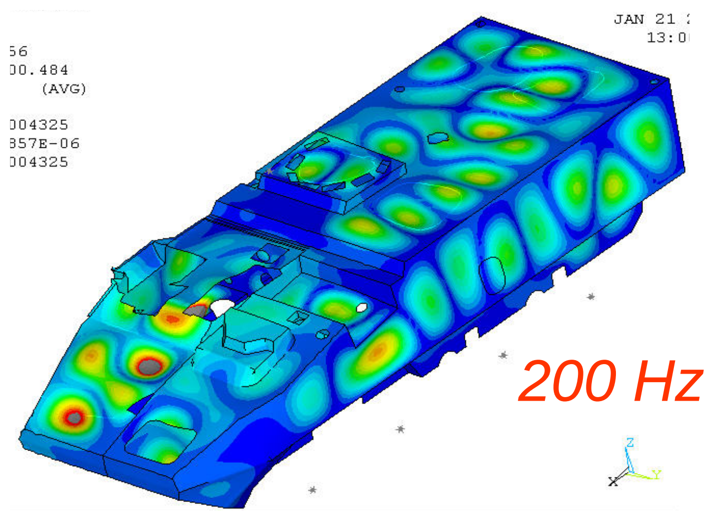
1 impact: 0.17 hr, or 10 min
600 impacts (typical): 100 hrs

***INTRO*: Modal Analysis**
Needed for Modal Superposition Method (MSM)

- *Eigen-analysis forms the basis of MSM transient analysis*
- Very fast for a large number of impacts because eigenvectors are used again and again for each impact
- For a linear system only
- Upper frequency is limited by the number of modes accurate and practical in the eigen-analysis

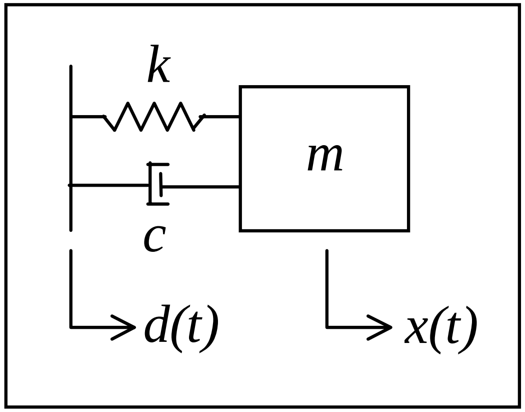






SRS Definitions:

Spectral displacement SRS:



$$S_D(\omega_n) := \left| x(t) - d(t) \right|_{\max}$$

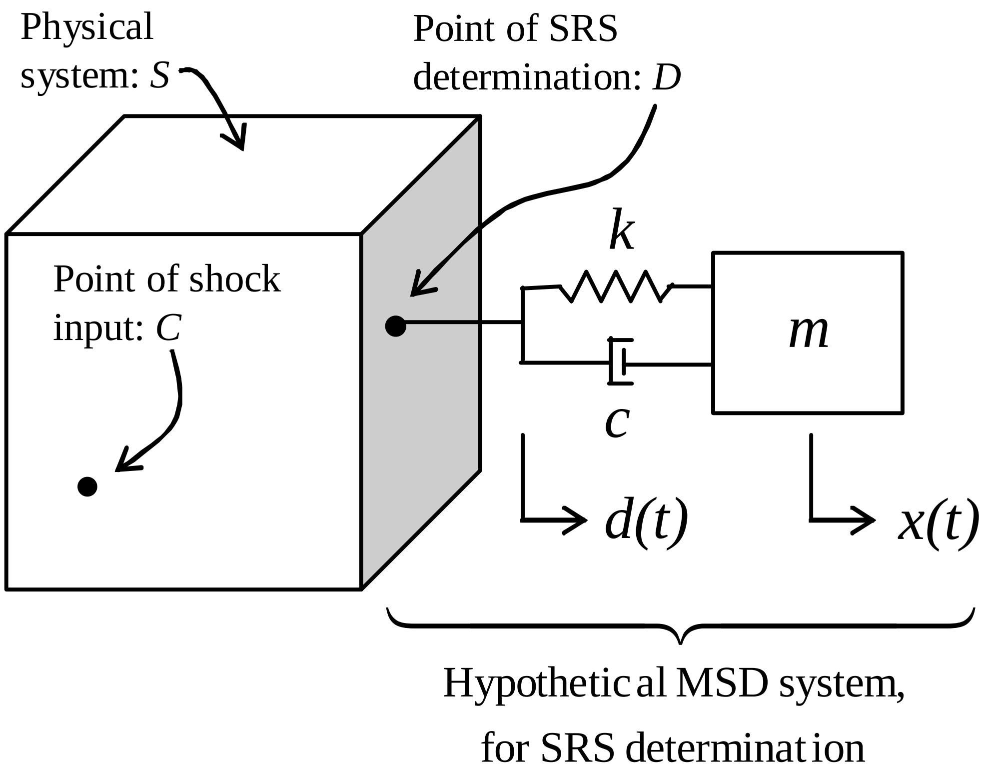
Spectral velocity SRS:

$$S_V(\omega_n) := \omega_n \left| x(t) - d(t) \right|_{\max} = \omega_n S_D(\omega_n)$$

Spectral acceleration SRS:

$$S_A(\omega_n) := \left| \ddot{x}(t) \right|_{\max}$$

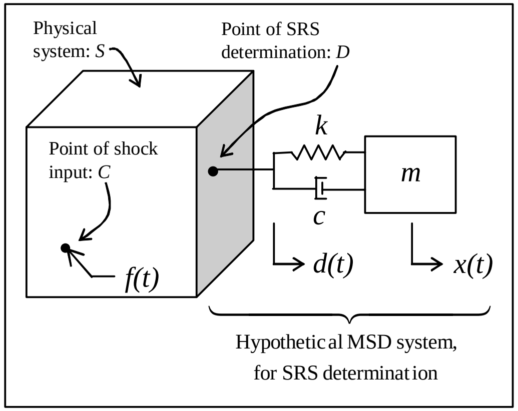
<u>Linear system model</u>:



INTRO: SRS Responses for Remote Impulsive Inputs (with Rayleigh or General Damping)

FOR REMOTE IDEAL IMPULSE

(TS 12, 2004; TS 13, 2005; SAVIAC 76, 2005)



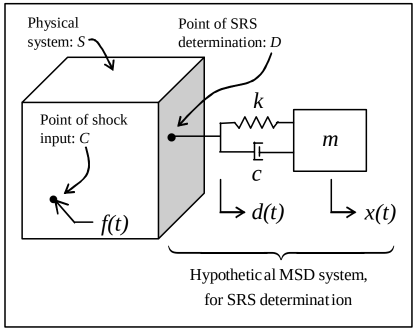
<u>Remote shock input</u>

$$f(t) = \gamma u_0(t)$$

<u>Local displacement</u>

$$d(t) = \sum_{i=1}^{n} D_i e^{-\varsigma_i \omega_{ni} t} \sin(\omega_{di} t + \varphi_i) u_{-1}(t)$$

INTRO: SRS Response for Remote Ideal Impulsive Input with Rayleigh or General Damping



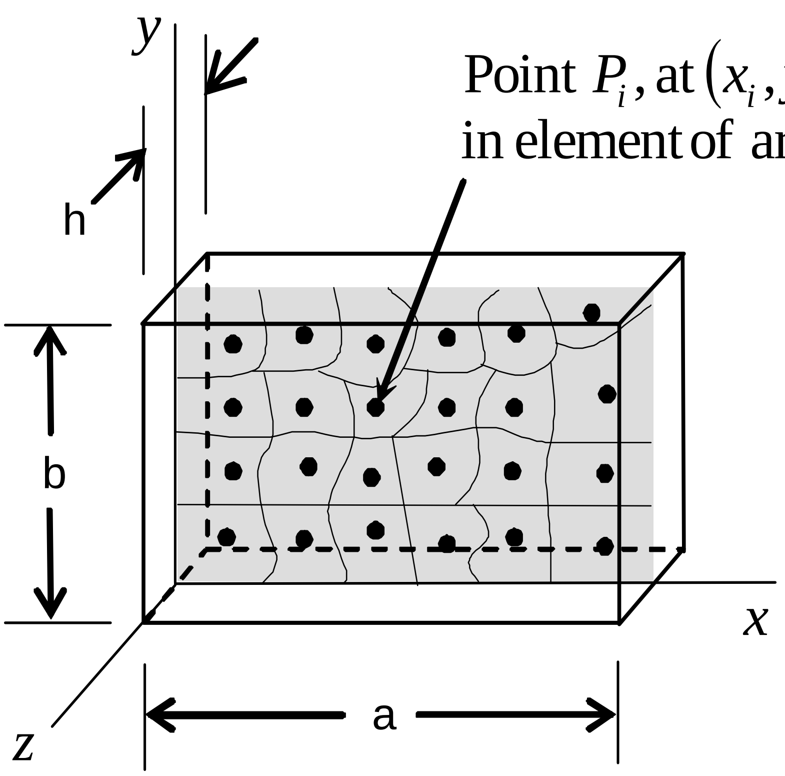
FOR REMOTE IDEAL IMPULSE

<u>Relative Displacement:</u>

$$\delta(t)=\left[V e^{-\varsigma\omega_n t}\sin\left(\omega_d t+\Phi\right)+\sum_{i=1}^{n}V_i e^{-\varsigma_i\omega_{ni}t}\sin\left(\omega_{di}t+\Phi_i\right)\right]u_{-1}(t)$$

Amplitudes $V_{()}$ and phase shifts $\Phi_{()}$ are <u>known functions of</u>: ICs, <u>the eigen-structure of S</u>, and m, c, and k.
The eigen-structure need be found only once.

OBJECTIVE: Same-Plate Response



- *Develop matrix equations for same-plate shock analysis…*
- *Solve equations for idealized impulsive inputs…*

  in terms of plate eigenstructure.
- *Verify solution.*

MODEL DEVELOPMENT

1. **Express PDE w/ modal solution for undamped continuous plate (simply supported on all edges).**

Equation of motion: $$\rho u_{tt} + D\nabla^4 u = q(x, y)$$

Modal Solution:

$$u(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \underbrace{B_{mn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)}_{U_{mn}(x,y)} \underbrace{\sin(\omega_{mn} t + \phi_{mn})}_{\eta_{mn}(t)}$$

Continuous Modeshape → $U_{mn}(x, y)$

$\eta_{mn}(t)$ ← Modal Coordinate

Modal Frequency → $$\omega_{mn} = \sqrt{\frac{D}{\rho}}\,\pi^2\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]$$

MODEL DEVELOPMENT

2. **Discretize modeshapes for undamped lumped-parameter plate (simply supported).**

Continuous Modeshapes: $U_k = B_k \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$

Discretized Modeshapes and Modeshape Matrix:

$$\underline{u}_k = B_k \begin{Bmatrix} \sin\left(\frac{m\pi x_1}{a}\right)\sin\left(\frac{n\pi y_1}{b}\right) \\ \vdots \\ \sin\left(\frac{m\pi x_r}{a}\right)\sin\left(\frac{n\pi y_r}{b}\right) \end{Bmatrix}$$

$$U = \left[\frac{\underline{u}_1}{\left\|\underline{u}_1\right\|}, \ldots, \frac{\underline{u}_p}{\left\|\underline{u}_p\right\|} \right]$$

3. **Express discretized, linearized PDE in matrix form. Include damping. (Modal damping shown below.)**

$$M\ddot{\underline{u}} + C\dot{\underline{u}} + K\underline{u} = \underline{f}$$

$$K = MU\Omega^2U^+$$

$$\Omega = \begin{bmatrix} \omega_1 & & \\ & \ddots & \\ & & \omega_p \end{bmatrix}$$

$$C = MU\Gamma U^+$$

$$\Gamma = \begin{bmatrix} 2\varsigma_1\omega_1 & & \\ & \ddots & \\ & & 2\varsigma_p\omega_p \end{bmatrix}$$

MODEL DEVELOPMENT

4. **Determine displacement at P_j to impulse of strength γ applied at P_i , for p modes.**

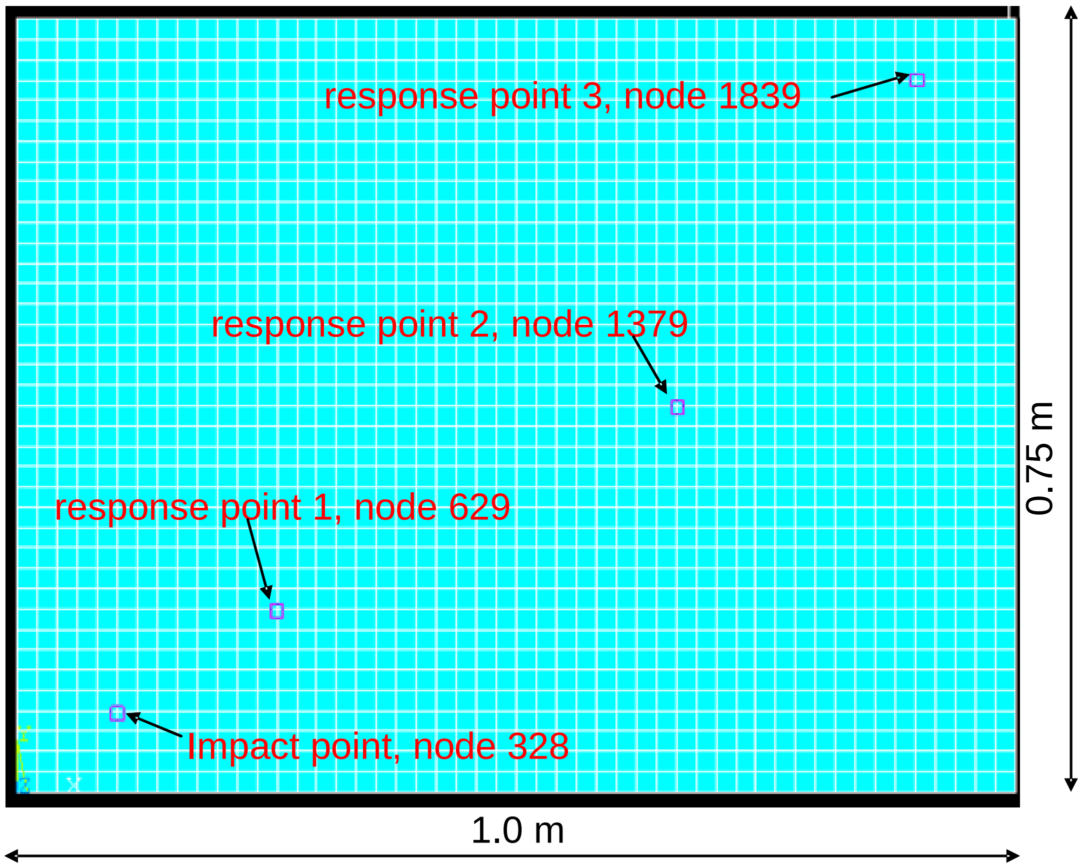
$$w_j(t) = \frac{\gamma}{m_i} \sum_{k=1}^{p} \left(\frac{u_{jk} v_{ki}}{\omega_{dk}} \right) e^{-\varsigma_k \omega_{nk} t} \sin \omega_{dk} t$$

where $U = \left[u_{ij} \right]$ (modeshape matrix)

$U^{+} = \left[v_{ij} \right]$ (pseudoinverse of modeshape matrix)

VERIFICATION: FEA vs. Matlab

Impacted Aluminum Plate Descriptions
Boundary conditions: simply supported on all sides



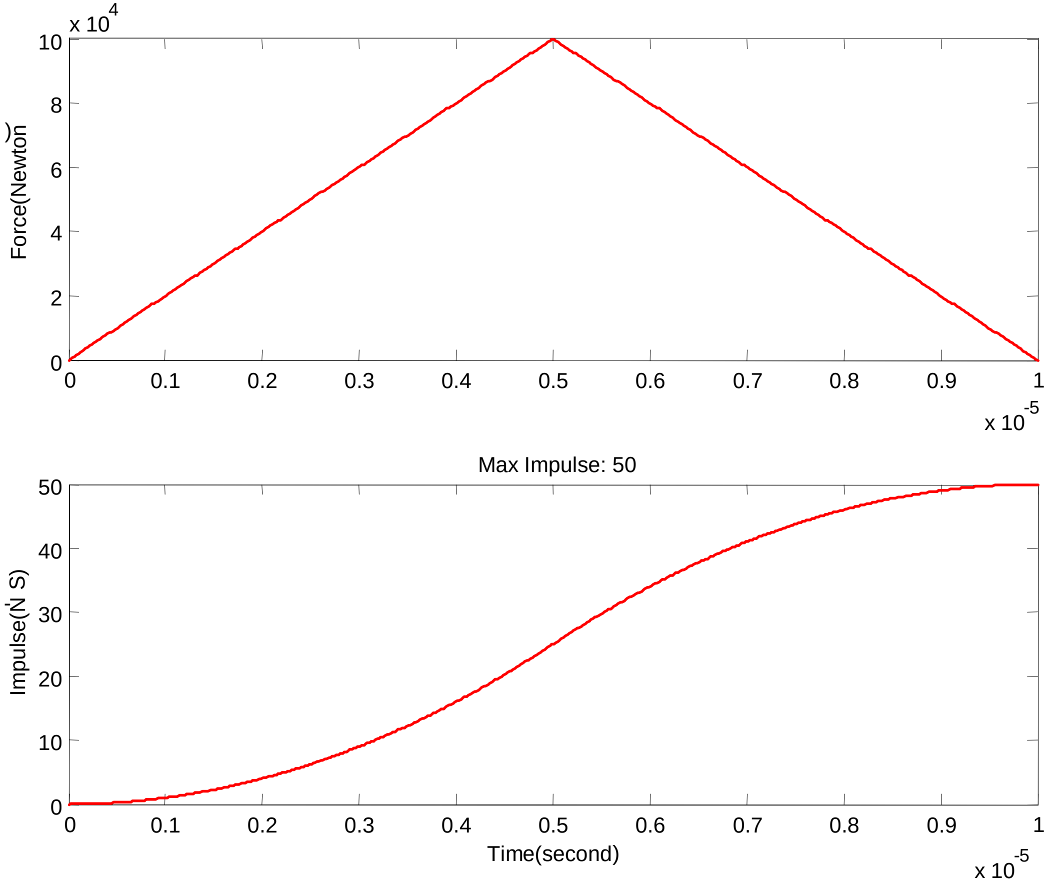
Thickness: 25 mm
Damping: 0.02

Material: al 6061-T6
E: 69e9
Mass Density: 2700 kg/m^3
Poisson's ratio: 0.33

Node Coordinates (m)

Node	x	y
328	0.1	0.0789
629	0.26	0.178
1379	0.66	0.375
1839	0.9	0.691

VERIFICATION: FEA vs. Matlab



Loading function for FEA model

VERIFICATION: FEA vs. Matlab

Modal Frequencies (Hz) Comparison: First 20 Modes

FEA – Ansys	Analytical	Difference (%)
168.6	168.6	0.015
350.6	350.7	0.030
492.2	492.4	0.027
654.0	654.3	0.041
674.1	674.5	0.061
977.1	978.0	0.093
1031.6	1032.0	0.042
1078.7	1079.3	0.052
1213.1	1214.2	0.085
1401.4	1403.0	0.119
1515.6	1517.7	0.137
1624.6	1625.6	0.065
1786.4	1787.5	0.061
1939.1	1942.7	0.185
1946.6	1949.4	0.142
1967.5	1969.6	0.108
2269.3	2273.2	0.173
2291.6	2293.4	0.079
2483.4	2489.0	0.228
2612.9	2617.2	0.163

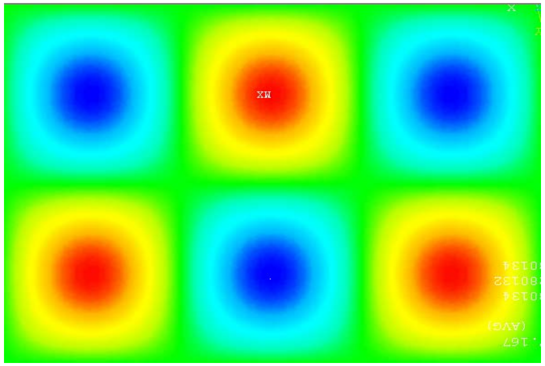
VERIFICATION: FEA vs. Matlab

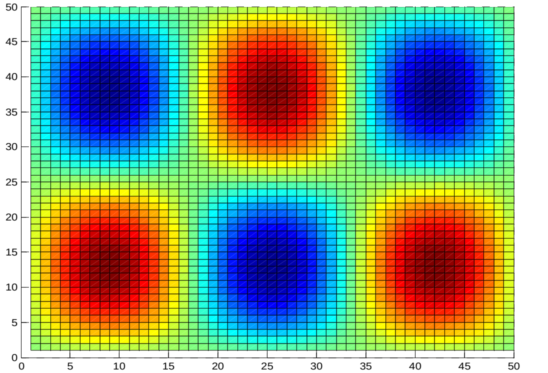
Mode shape comparison

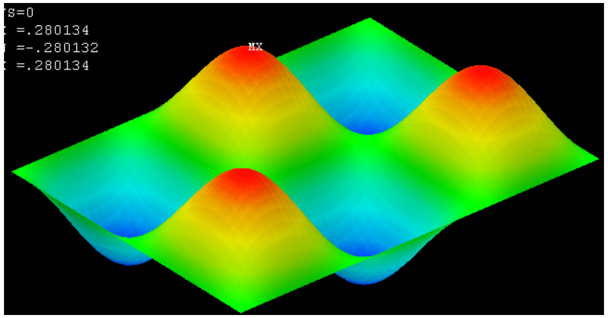
(6th mode, m = 3, n = 2)

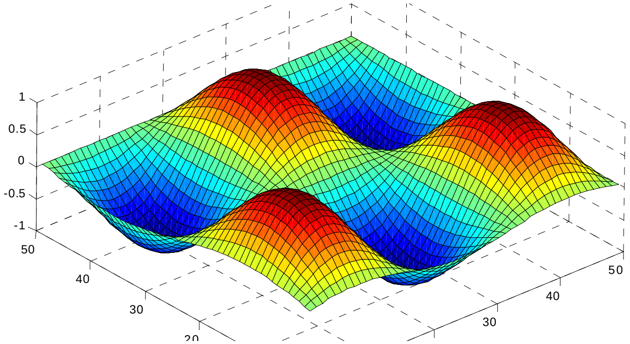
FEA - ANSYS

Analytical - Matlab









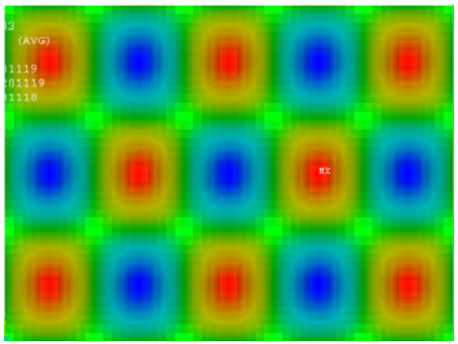
Modeshape comparison, 6th mode

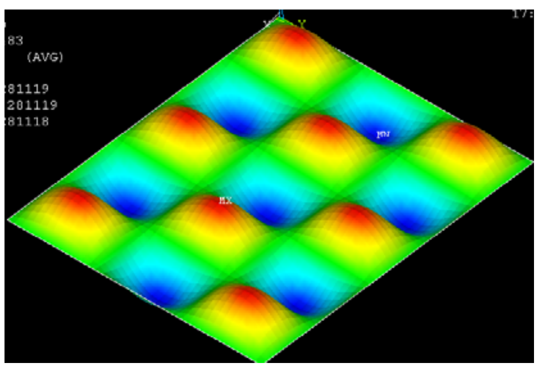
VERIFICATION: FEA vs. Matlab

Mode shape comparison

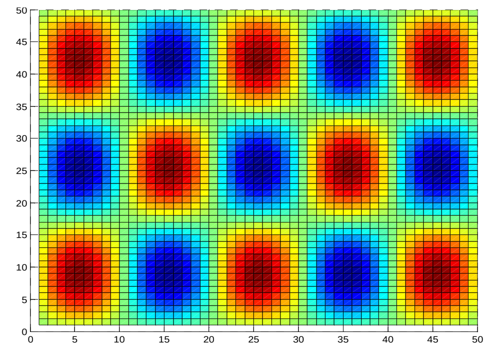
(19^{th} mode, m = 5, n = 3)

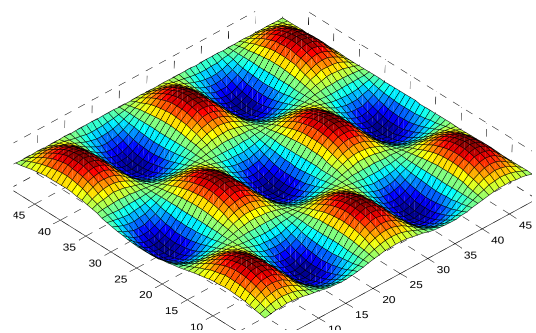
FEA - ANSYS





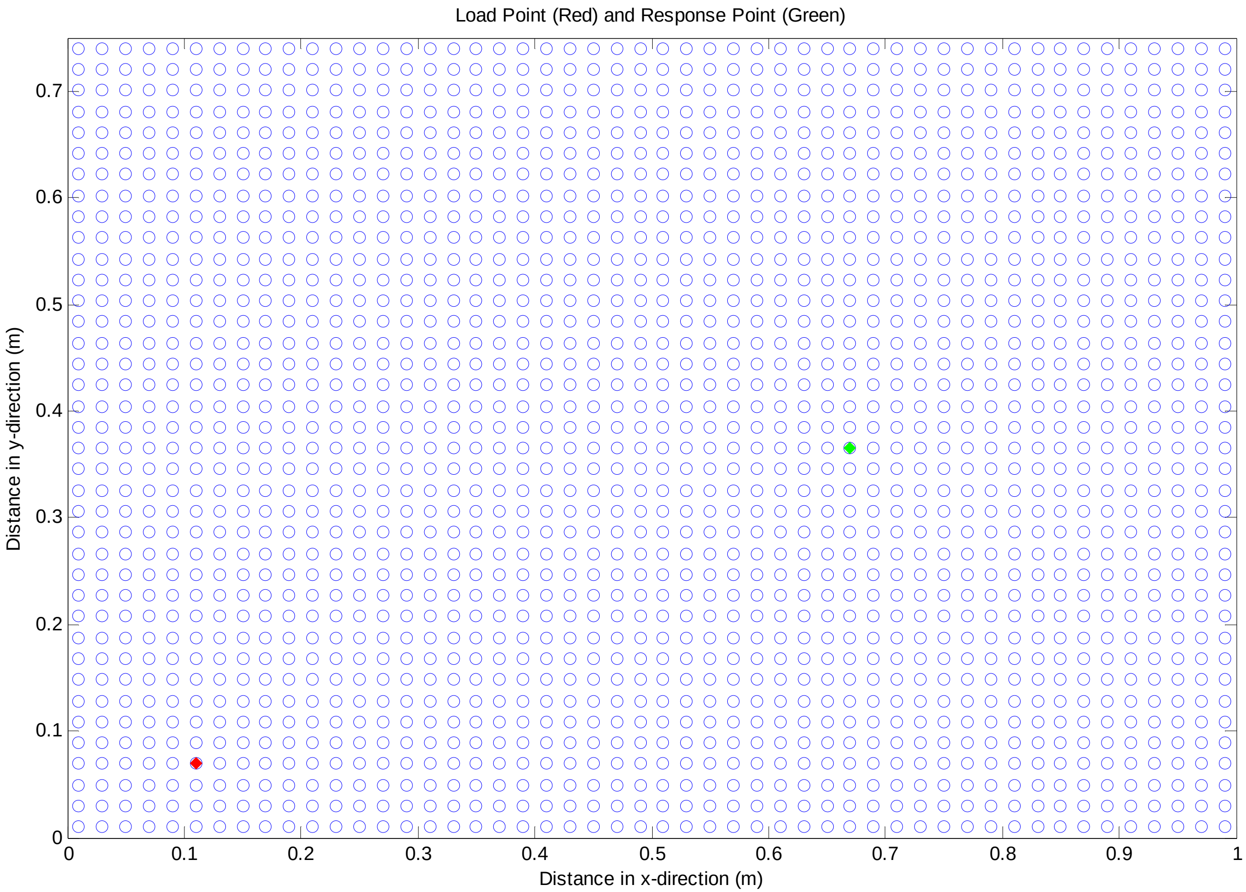
Analytical - Matlab





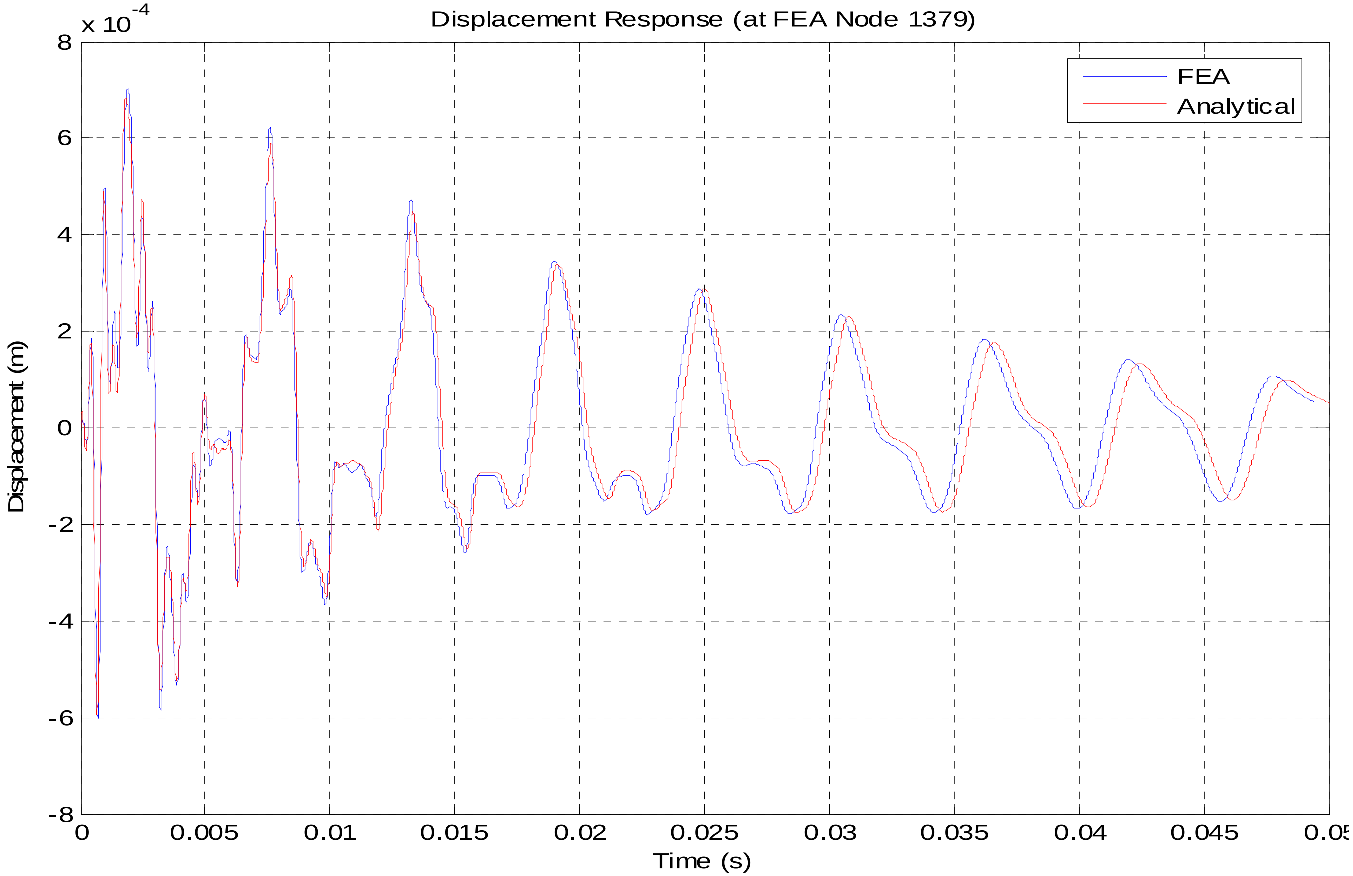
Modeshape comparison, 19th mode

VERIFICATION: FEA vs. Matlab



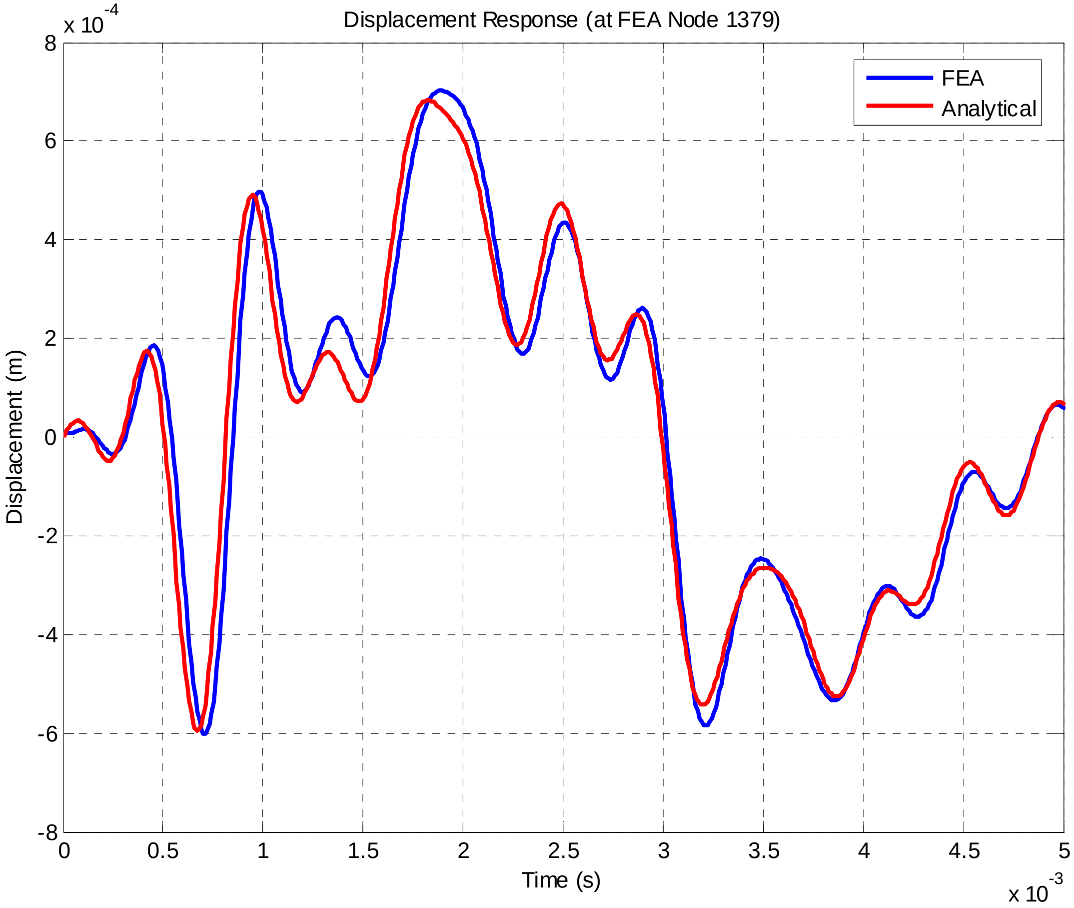
Loading and response points for comparison without damping.

VERIFICATION: FEA vs. Matlab



Response-point displacement comparison, mass element 629 (FEA node 1379), for an equal no. of elements (1900) and of modal components (20) in both models. Modal damping ratio: 0.02 for all modes.

VERIFICATION: FEA vs. Matlab



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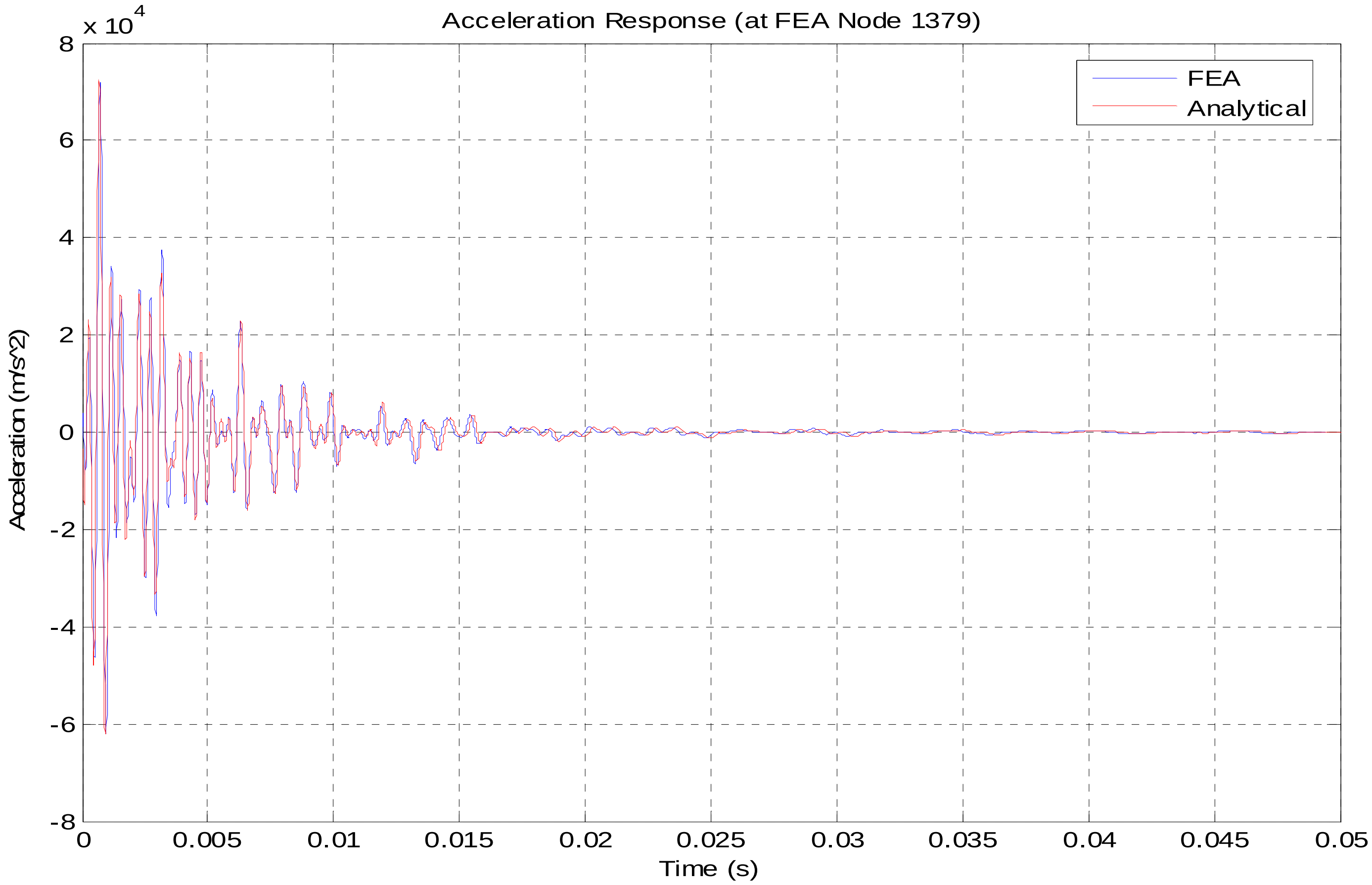
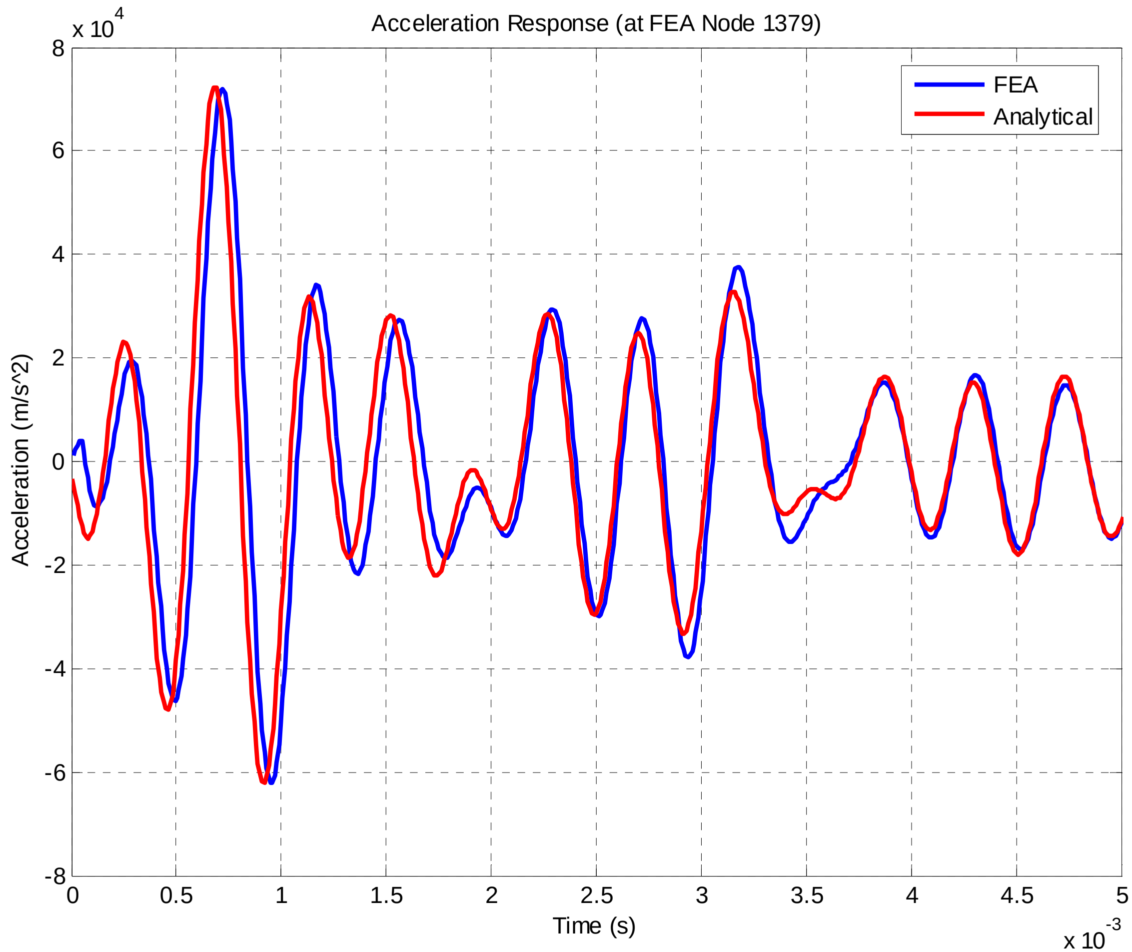


Response-point acceleration comparison, FEA node 1379, for an equal no. of elements (1900) and of modal components (20) in both models. Modal damping ratio: 0.02 for all modes.

VERIFICATION: FEA vs. Matlab



Response-point acceleration comparison, FEA node 1379, for an equal no. of elements (1900) and of modal components (20) in both models. Modal damping ratio: 0.02 for all modes.

REMARKS

- FEA model is incorrect for initial time if impulse duration is too short (<0.1 ms).
- Analytical model is much faster than FEA.
- Models show good correlation of frequencies modeshapes, and shock response.
- Approach is useful for other boundary conditions.

FUTURE WORK

- **Complete current FEA validation**
- **Conduct experimental validation**
- **Extend work to include other BCs**