13th ARL/USMA Technical Symposium

ANALYTICAL KINEMATIC EXPRESSIONS FOR PSEUDOVELOCITY SHOCK RESPONSE SPECTRA OF GENERALLY DAMPED LINEAR SYSTEMS

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PRESENTATION OUTLINE

INTRODUCTION

Background

SRS definition

SRS responses for local damped harmonic inputs

SRS responses for remote inputs w/ Rayleigh damping

OBJECTIVE

SRS responses for remote inputs w/ general damping

SOLUTION

Remote responses of basic system

Remote responses for SRS

FUTURE WORK

CONCLUDING REMARKS

INTRO: Overview of Vibration Analysis Approach

Geometries of structural model
Coordinates of impact nodes
Equipment mounting locations
Loading impulse levels

BRL-CAD Model
and other sources

Unique for each equipment mounting location

Shock Model

Pseudo-Velocity Shock Response Spectrum

Unique for each impact
location and loading type

Compare
to determine
failure probability

Damage Criteria

Equipment Fragility Parameters

From existing database,
or to be established experimentally

Statistical failure spectrum,
unique for each equipment category

INTRO: FEA Shock Models for Armored Vehicles: Reducing the time for impact calculations

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\} = \{F\}$$

FEA Structural Transient Analysis

- Direct Integration
  - Explicit — Dyna, Abacus
  - Implicit — Nastran, Ansys

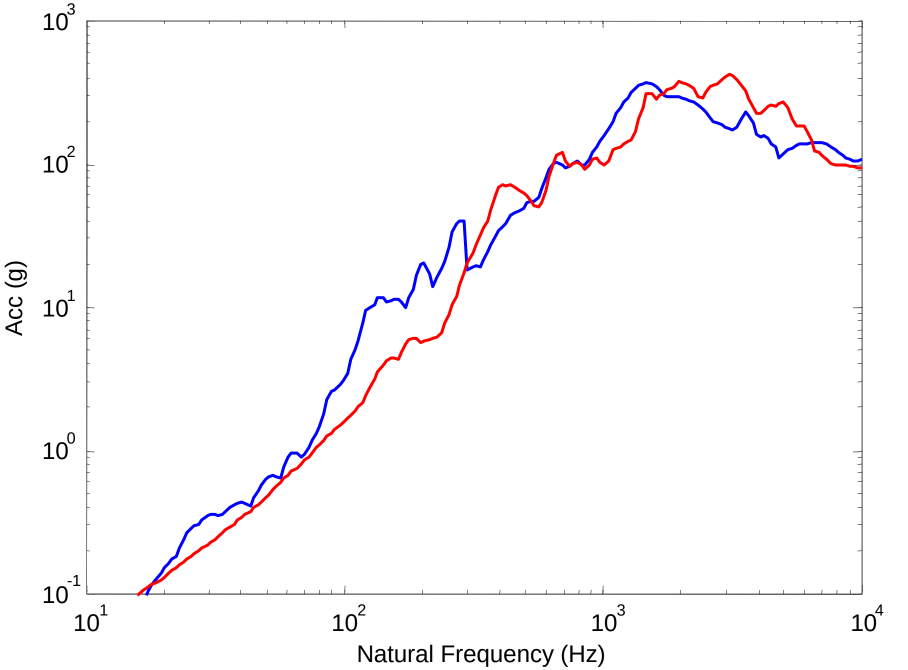
- Used by most analysts
- Current shock model: ~2 days/impact
- Too slow for many impacts
- But can be used for comparison in a few cases

- Modal Superposition — Ansys

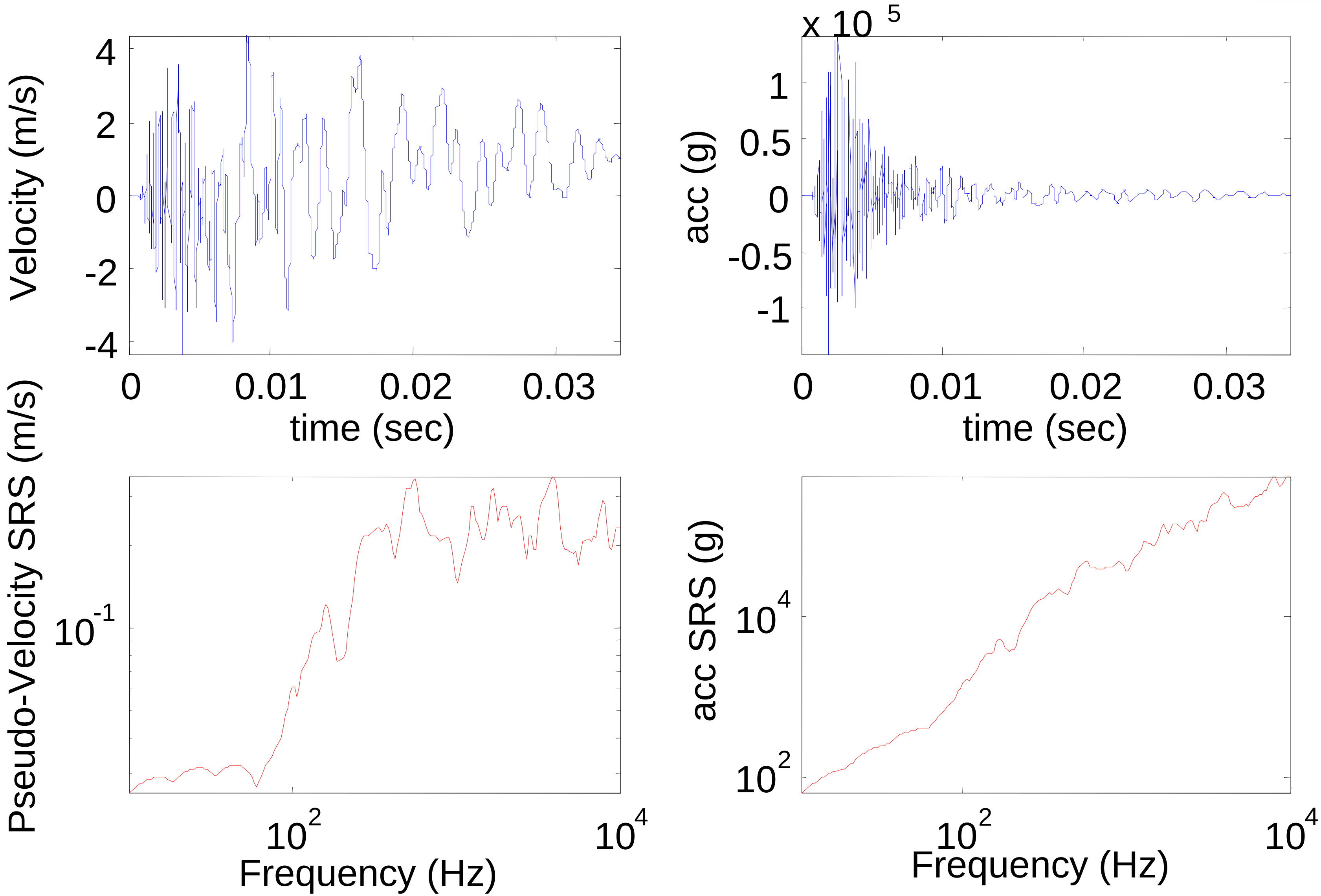
- Very fast for a large number of impacts
- Useful for a linear system only (based on linear superposition of modes)
- Permitting a clear-cut upper frequency, limited by the number of modes; possible in eigen-analysis

INTRO: Typical shock model

Blue: Shock response measurement from a ballistic test
Red: Corresponding response from an FEA shock model



INTRO: Typical SRS Data and Plots



INTRO: FEA Modal Superposition Method

FEA shock model

~99% of total solution time

Modal Analysis

Eigenvalues
Eigenvectors

~1% of total sol'n time for each impact node

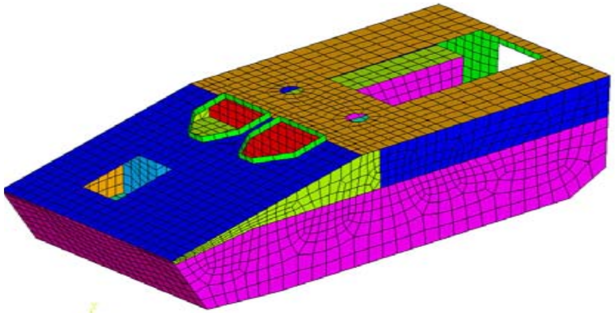
Transient Analysis

New impact node

Time-domain Response

PSRS

Example of solution time
(IHS coarse mesh model)



~4000 shell elements
450 modes

Modal analysis time:

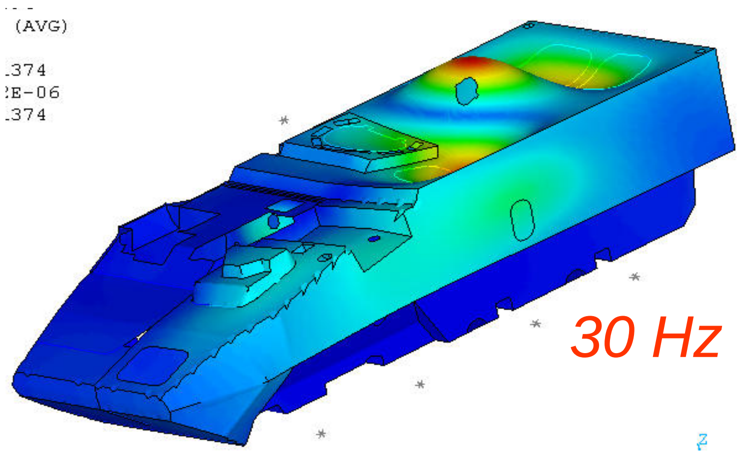
67 hrs

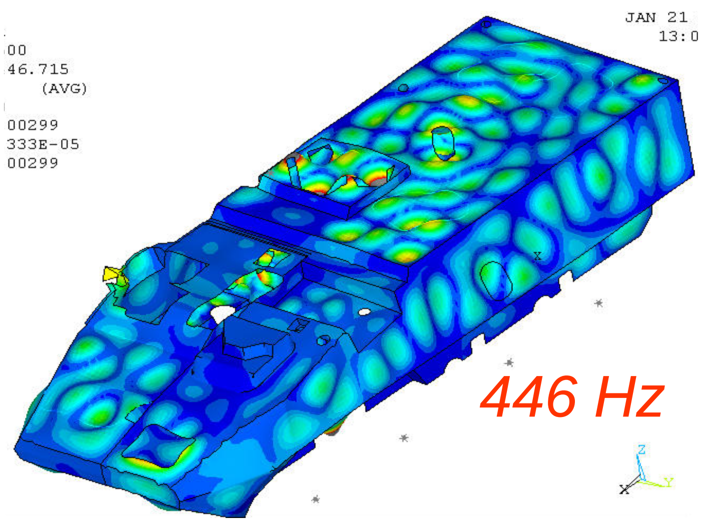
Transient solution time:

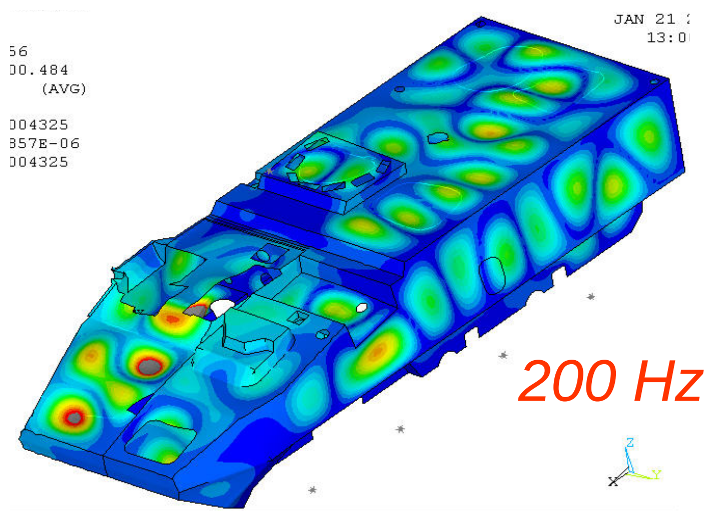
1 impact: 0.17 hr, or 10 min
600 impacts (typical): 100 hrs

INTRO: *Modal Analysis*
Needed for Modal Superposition Method (MSM)

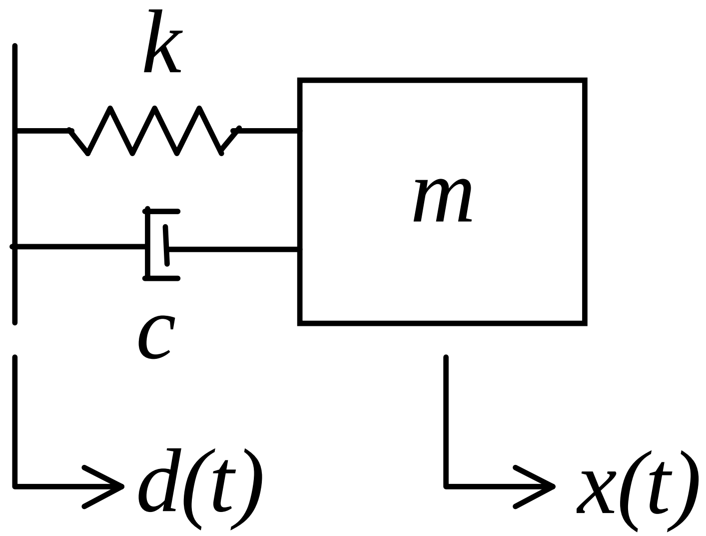
- *Eigen-analysis forms the basis of MSM transient analysis*
- Very fast for a large number of impacts because eigenvectors are used again and again for each impact
- For a linear system only
- Upper frequency is limited by the number of modes accurate and practical in the eigen-analysis





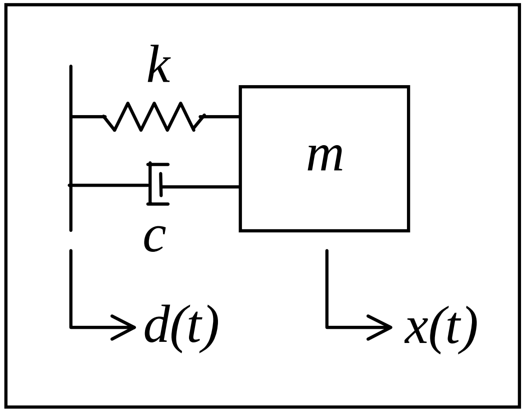


INTRO: SRS Definition



Hypothetical SDOF MSD system,
for SRS determination

<u>SRS Definitions</u>:



Spectral displacement SRS:

$$S_D(\omega_n) := \left|x(t) - d(t)\right|_{\max}$$

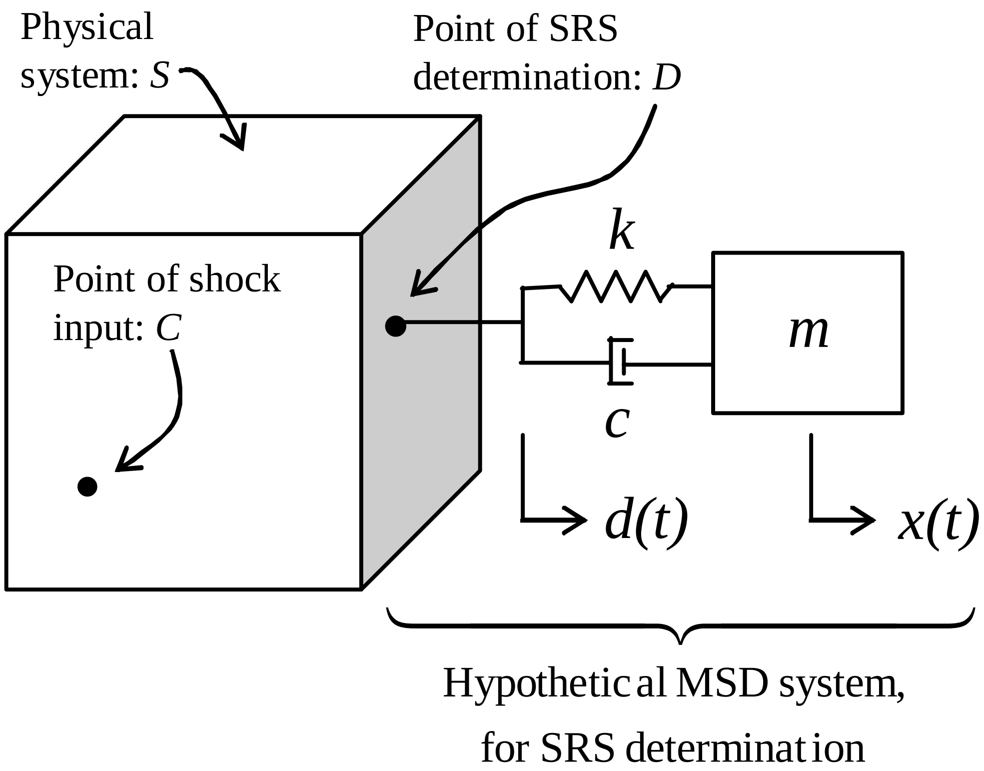
Spectral velocity SRS:

$$S_V(\omega_n) := \omega_n \left|x(t) - d(t)\right|_{\max} = \omega_n S_D(\omega_n)$$

Spectral acceleration SRS:

$$S_A(\omega_n) := \left|\ddot{x}(t)\right|_{\max}$$

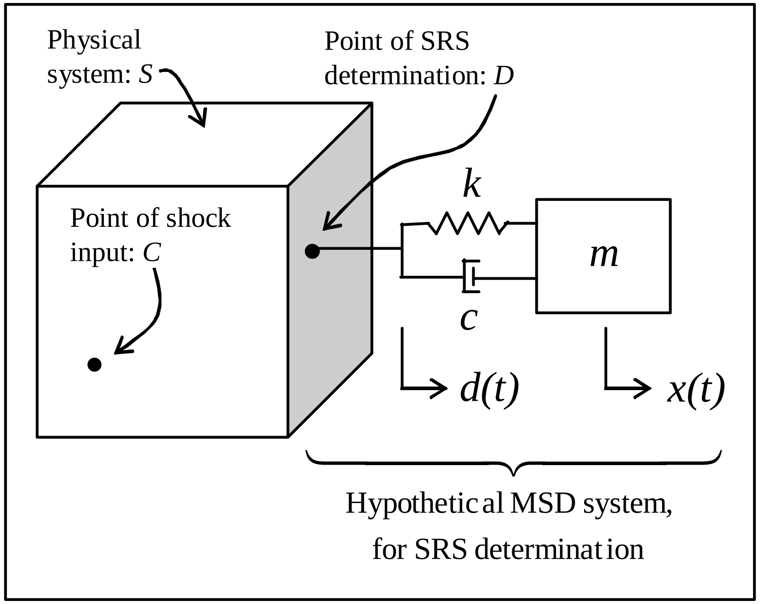
<u>Linear system model</u>:



INTRO: SRS Responses for Local Damped Harmonic Inputs

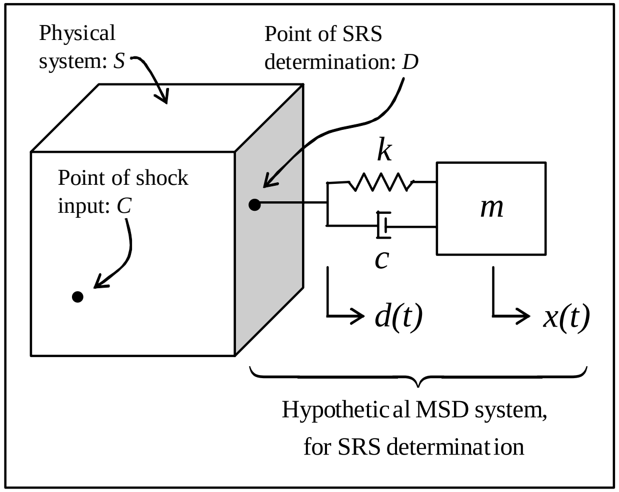
<u>Local Harmonic Disturbance</u>

$$d(t) = \sum_{i=1}^{\nu} d_i(t)$$



$$d_i(t) = D_i e^{-\alpha_i t} \sin(\omega_i t + \phi_i) u_{-1}(t)$$

INTRO: SRS Responses for Local Damped Harmonic Inputs



FOR LOCAL HARMONIC INPUTS

(TS 11, 2003; SAVIAC 75, 2004)

Relative Displacement:

$$\delta(t) = x(t) - d(t)$$

$$= \beta\, e^{-\varsigma\omega_n t} \sin(\omega_d t + \phi)$$

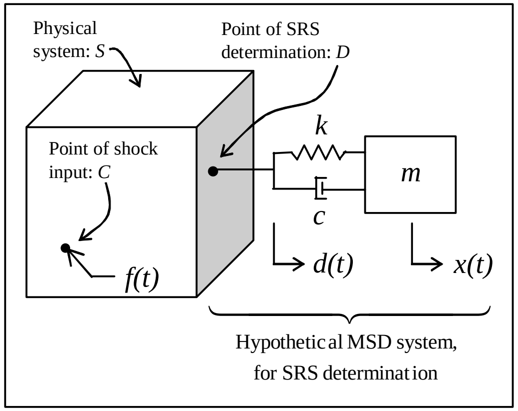
$$+ \sum_{i=1}^{\nu} D_i \Big[e^{-\varsigma\omega_n t} C_{1i} \sin(\omega_d\, t + \varphi_{1i})$$

$$+ e^{-\alpha_i t} C_{2i} \sin(\omega_i t + \varphi_{2i}) \Big]$$

INTRO: SRS Responses for Remote Impulsive Inputs (with Rayleigh Damping)

FOR REMOTE IDEAL IMPULSE

(TS 12, 2004; SAVIAC 76, 2005)



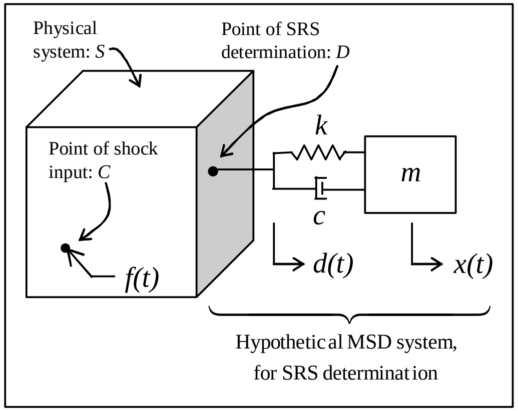
<u>Remote shock input</u>

$$f(t) = \gamma u_0(t)$$

<u>Local displacement</u>

$$d(t) = \sum_{i=1}^{n} D_i e^{-\varsigma_i \omega_{ni} t} \sin(\omega_{di} t + \varphi_i) u_{-1}(t)$$

INTRO: SRS Responses for Remote Impulsive Inputs (with Rayleigh Damping)



FOR REMOTE IDEAL IMPULSE

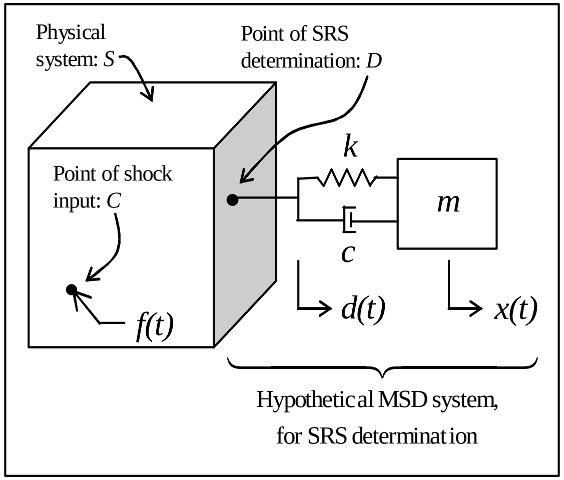
<u>Relative Displacement:</u>

$$\delta(t) = \left[V e^{-\varsigma\omega_n t} \sin\left(\omega_d t + \Phi\right) + \sum_{i=1}^{n} V_i e^{-\varsigma_i \omega_{ni} t} \sin\left(\omega_{di} t + \Phi_i\right) \right] u_{-1}(t)$$

Amplitudes $V_{()}$ and phase shifts $\Phi_{()}$ are <u>known functions of</u>: ICs, <u>the eigen-structure of S</u>, and m, c, and k.
The eigen-structure need be found only once.

OBJECTIVE: Determine SRS Response for Remote Ideal Impulsive Input with General Damping

FOR REMOTE IDEAL IMPULSE
(TS 13, 2005; SAVIAC 76, 2005)



GIVEN: Remote shock input

$$f(t) = \gamma u_0(t)$$

FIND: Relative displacement $\delta(t) = x(t) - d(t)$

in terms of the eigenstructure of physical system S

SOLUTION APPROACH

1. **Determine linearized equations of motion for *S*.**

$$M\,\underline{\ddot{x}} + C\,\underline{\dot{x}} + K\,\underline{x} = E\,\underline{f}$$

2. **Express equations of motion for *S* in state-space form.**

$$\underbrace{\begin{Bmatrix} \underline{\dot{z}}_1 \\ \underline{\dot{z}}_2 \end{Bmatrix}}_{\underline{\dot{\hat{z}}}} = \underbrace{\begin{bmatrix} O & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}}_{A_1} \underbrace{\begin{Bmatrix} \underline{z}_1 \\ \underline{z}_2 \end{Bmatrix}}_{\underline{\hat{z}}} + \underbrace{\begin{bmatrix} O \\ M^{-1}E \end{bmatrix}}_{E_1} \underline{f}$$

SOLUTION APPROACH

3. Express the system matrix in Jordan Canonical Form.

$$A_1 = \underbrace{X_1}_{\text{Eigenvector Matrix}} J_1 X_1^{-1}$$

4. Expand matrices in useful row and column form.

$$E_1 = \left[\underline{\hat{e}}_1, \underline{\hat{e}}_2, \cdots, \underline{\hat{e}}_m\right] \qquad X_1 = \left[\underline{x}_1^c, \underline{x}_2^c, \cdots, \underline{x}_{2\nu}^c\right]$$

$$J_1 = \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_{2\nu} \end{bmatrix} \qquad X_1^{-1} = Y_1 = \begin{bmatrix} \underline{y}_1^r \\ \underline{y}_2^r \\ \vdots \\ \underline{y}_{2\nu}^r \end{bmatrix}$$

SOLUTION APPROACH

5. **Use these forms to determine system response to scalar input f_k.**

$$\underline{x} = \left[I \quad O \right] \sum_{i=1}^{2\nu} \underbrace{\left\langle \left(\underline{y}_i^r \, \hat{\underline{z}}_0 \right) e^{\lambda_i t} + \left(\underline{y}_i^r \, \hat{\underline{e}}_k \right) \left(e^{\lambda_i t} * f_k \right) \right\rangle}_{\text{scalar function}} \underline{x}_i^c$$

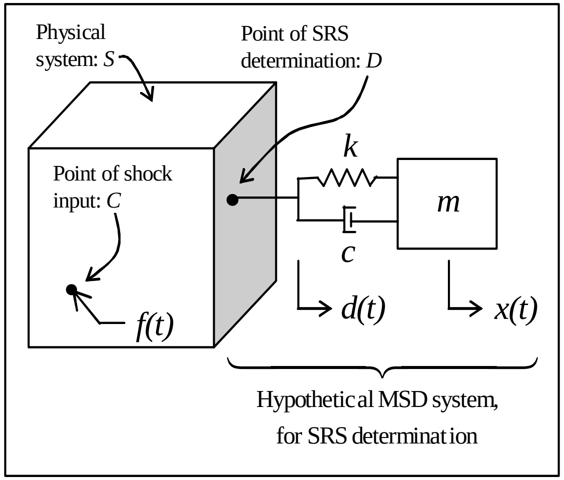
6. **Evaluate convolution to determine local shock response.**

For $f(t) = \gamma u_0(t)$, determine $d(t)$.

7. **Determine SRS- and relative response.**

RESULT: SRS Response for Remote Ideal Impulsive Input with General Damping

FOR REMOTE IDEAL IMPULSE



<u>Remote shock input</u>

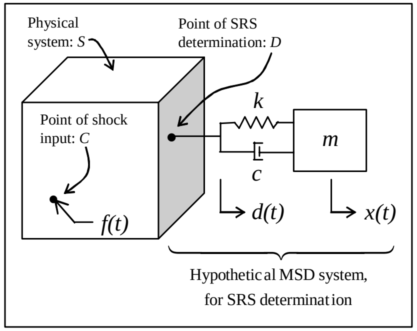
$$f(t) = \gamma u_0(t)$$

<u>Local displacement</u>

$$d(t) = \sum_{i=1}^{n} D_i \, e^{-\varsigma_i \omega_{ni} t} \sin(\omega_{di} t + \varphi_i) u_{-1}(t)$$

RESULT: SRS Response for Remote Ideal Impulsive Input with General Damping

FOR REMOTE IDEAL IMPULSE



Relative Displacement:

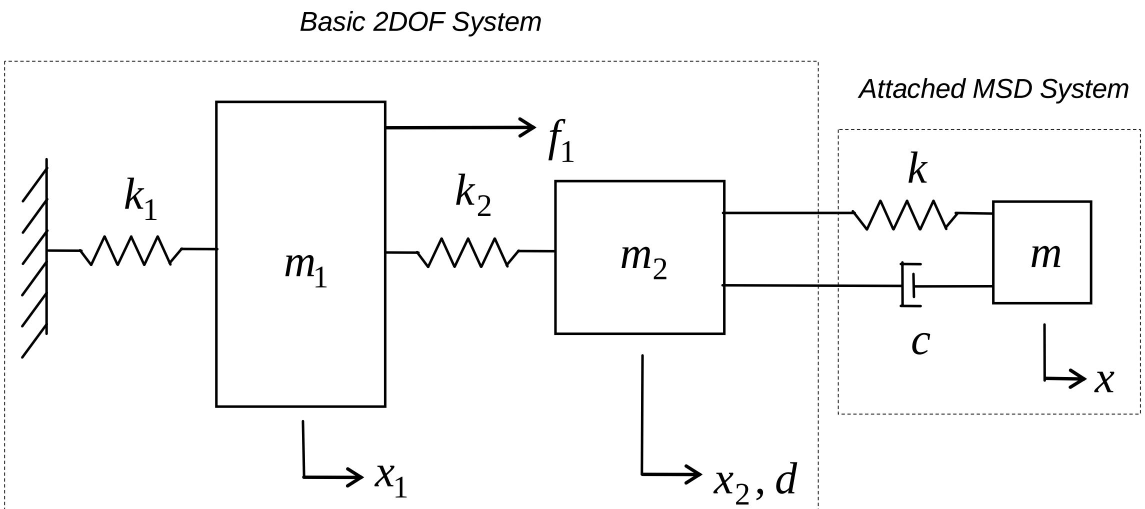
$$\delta(t)=\left[V e^{-\varsigma\omega_n t}\sin\left(\omega_d t+\Phi\right)+\sum_{i=1}^{n} V_i e^{-\varsigma_i\omega_{ni} t}\sin\left(\omega_{di} t+\Phi_i\right)\right]u_{-1}(t)$$

Amplitudes $V_{()}$ and phase shifts $\Phi_{()}$ are known functions of: ICs, the eigenstructure of S, and m, c, and k. The eigenstructure need be found only once.

FUTURE WORK: Verification

- **Perform dimensional analysis for key equations**
- **Benchmark against previous results for Rayleigh damping.**

VERIFICATION: Benchmark Test Case



Proportional damping $D = aM + bK$,
for modal damping ratios ς_1 and ς_2.

CONCLUDING REMARKS

- **SRS quantities determined for general damping**

  --Numerical convolution not required

  --System eigenstructure needed only once

  --Analytical response determined for ideal impulse

  --Method usable for other analytical shock inputs

  --Rectangular pulses

  --Sawtooth pulses

  --Approach valid for linearized system model

- **Results useful**

  --for evaluating other methods of SRS calculation

  --for determining minimum number of modes required
  for SRS of specified accuracy