Quantum Limits to the Force Sensitivity of a Micromechanical Resonator Observed by an Optical Interferometer and Implications for the Large-Scale Simulation of Quantum Systems

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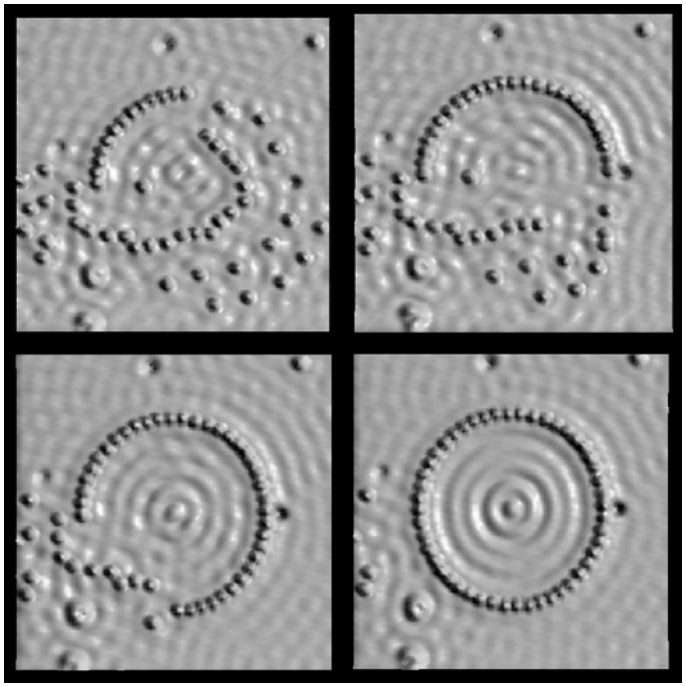
Scanning probe Microscopy

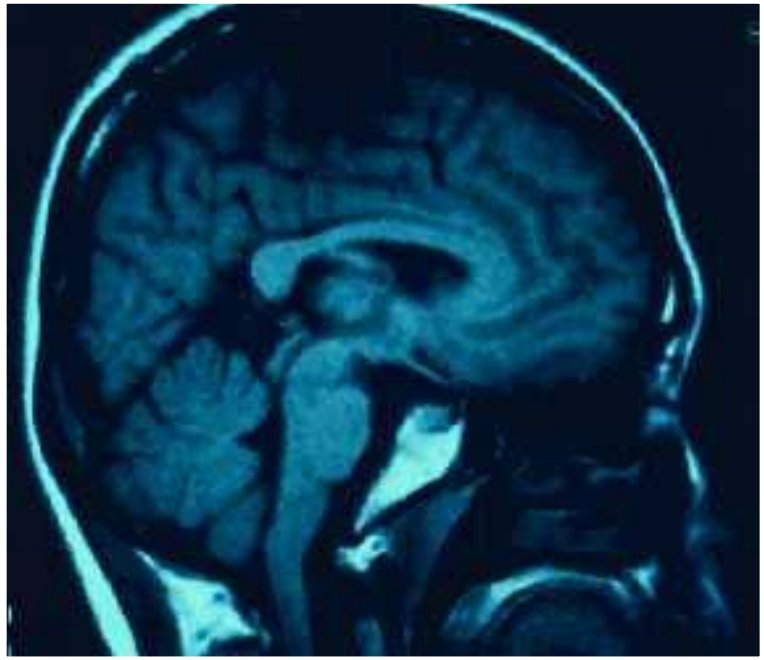


M.F. Crommie, C.P. Lutz, and D.M. Eigler, Science (1993)

Single charges! single atoms!

Only surface features resolved
Little chemical information

Magnetic Resonance Imaging

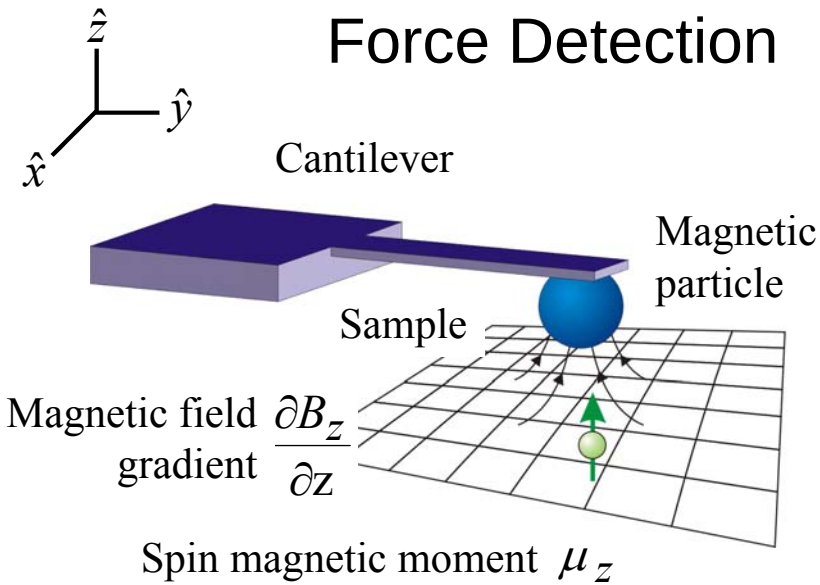


Images of <u>subsurface</u> features (in 3D)
Isotope- and chemical-specific images

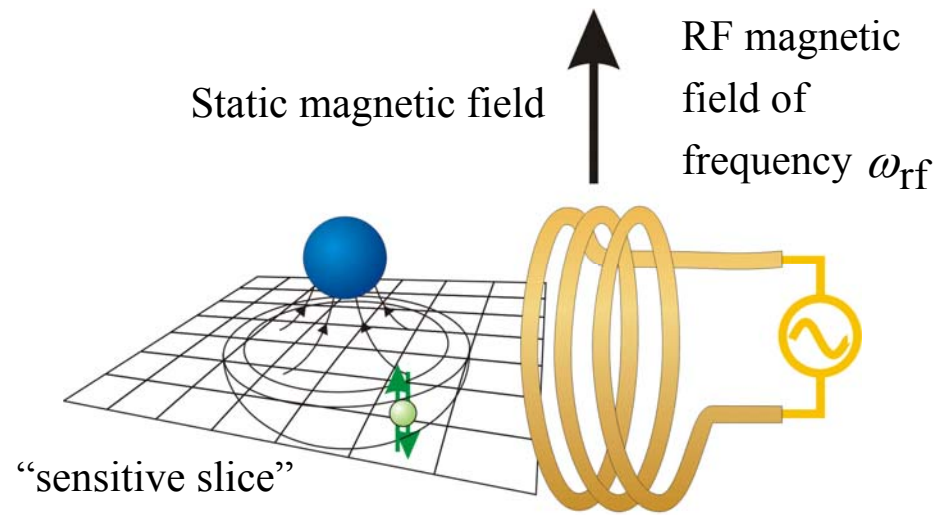
Only ~10-100 micron imaging resolution

Sidles' idea (1991): Detect Magnetic Resonance as a Force: Magnetic Resonance Force Microscopy or MRFM

Force Detection



Magnetic Resonance



Gradient dipole force moves the cantilever

$$F_z = \mu_z \frac{\partial B_z}{\partial z}$$

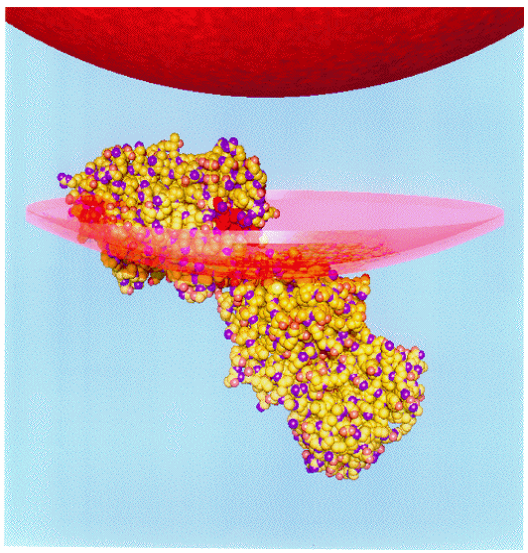
Flip spins, create oscillating force

$$B_z = \frac{\omega_{\mathrm{rf}}}{\gamma} = \frac{350\ \mathrm{MHz}}{42.56\ \mathrm{MHz/T}} = 8.224\ \mathrm{T}$$

Potential MRFM Applications

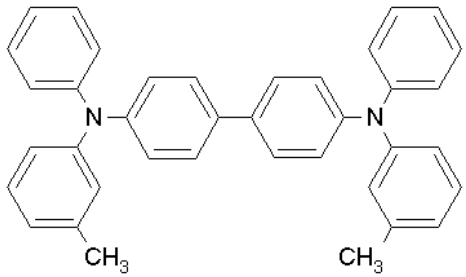
Molecular Biology

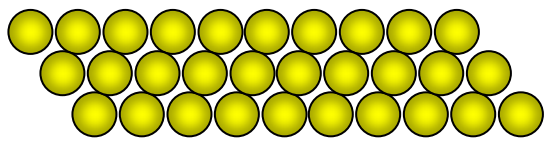
image a <u>single</u> protein



Chemistry Materials Science

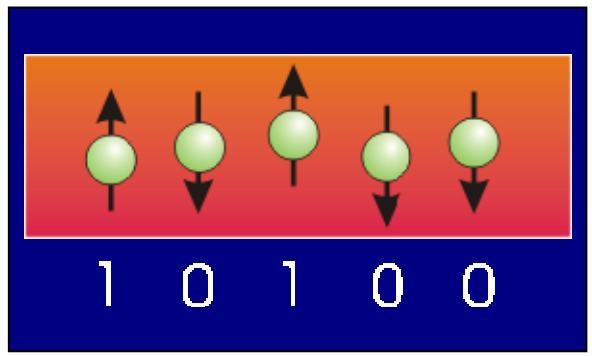
probe the buried organic / inorganic interface



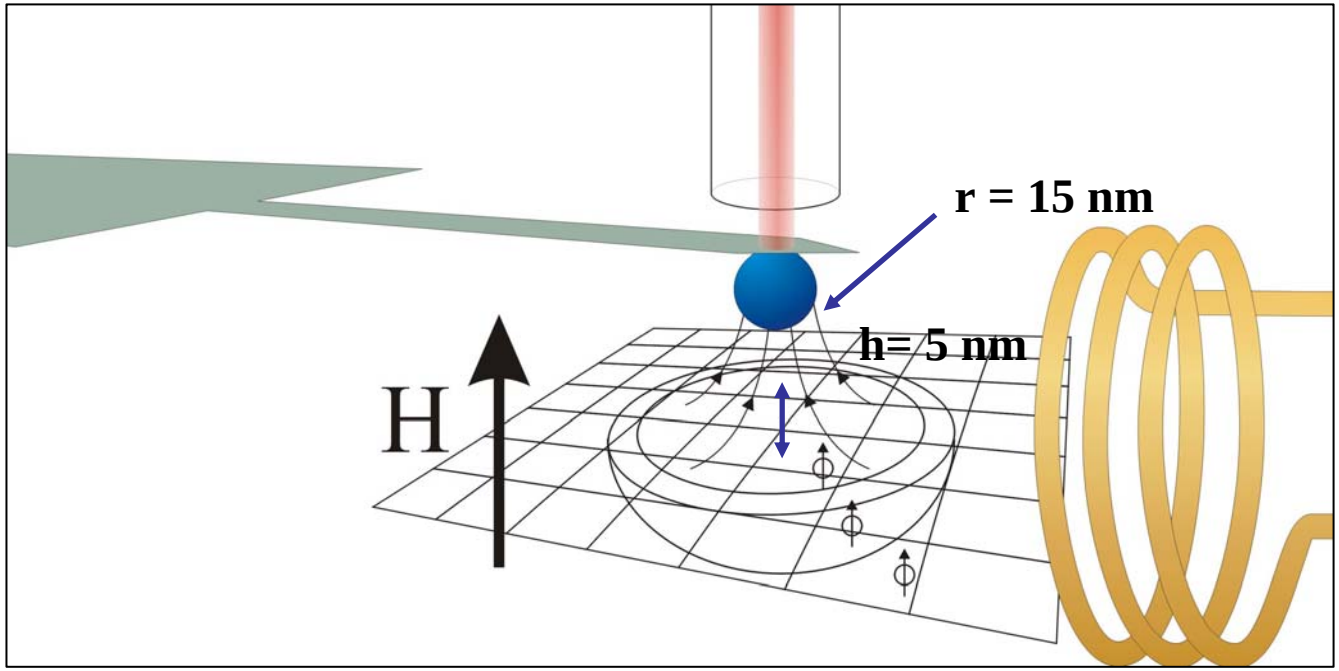


Physics & Information Technology

readout a quantum computer



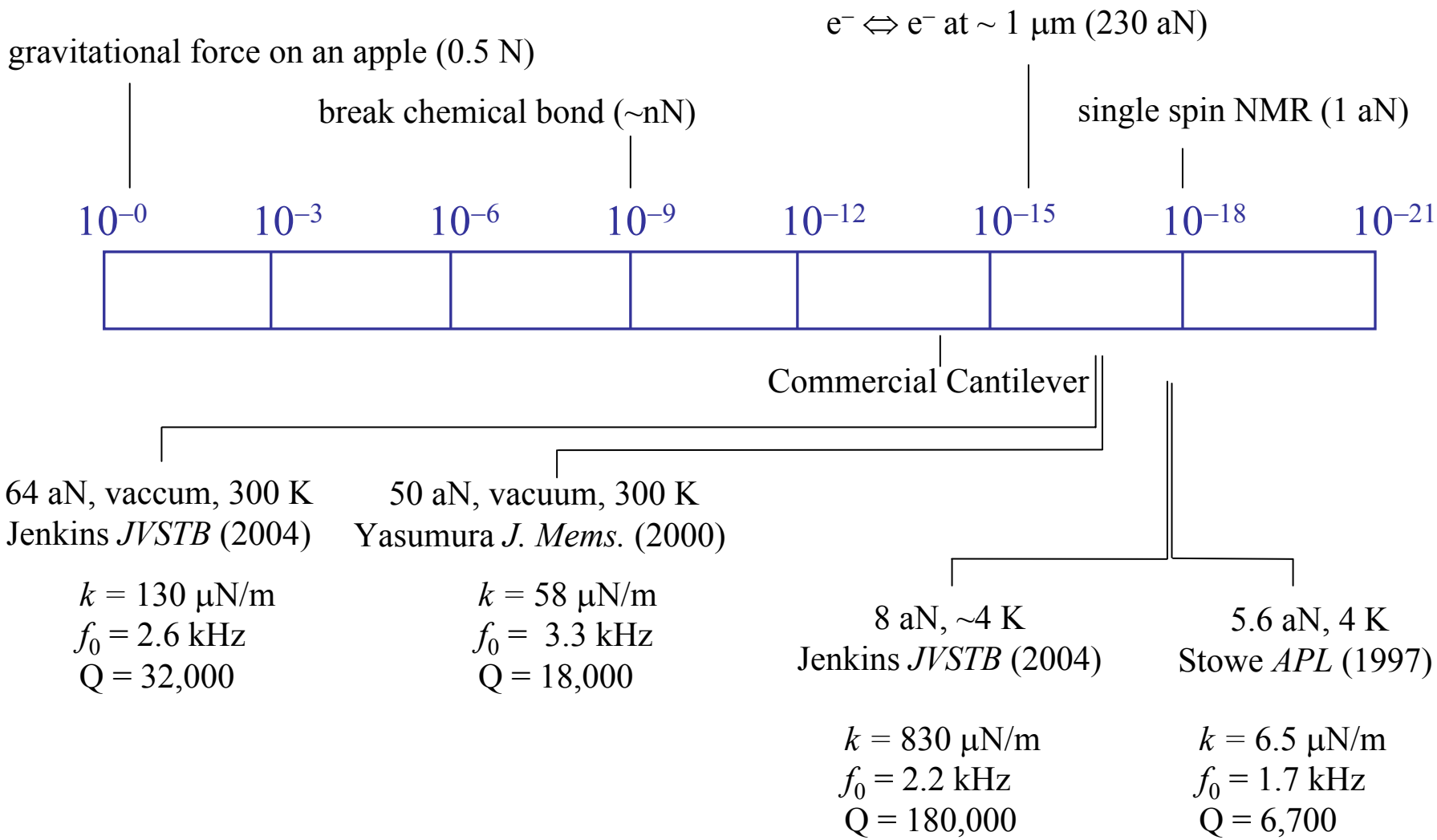
The Force From a Single Proton



$$F_p = \mu_p \frac{\partial B_z}{\partial z} \longrightarrow \frac{\partial B_z}{\partial z} = \frac{2\mu_0 m r^3}{(r+h)^4} = 9.09\times10^7\ T/m$$

$$\boxed{F_p = 1.3 \times 10^{-18}\ \mathrm{N} = 1.3\ \mathrm{aN}}$$

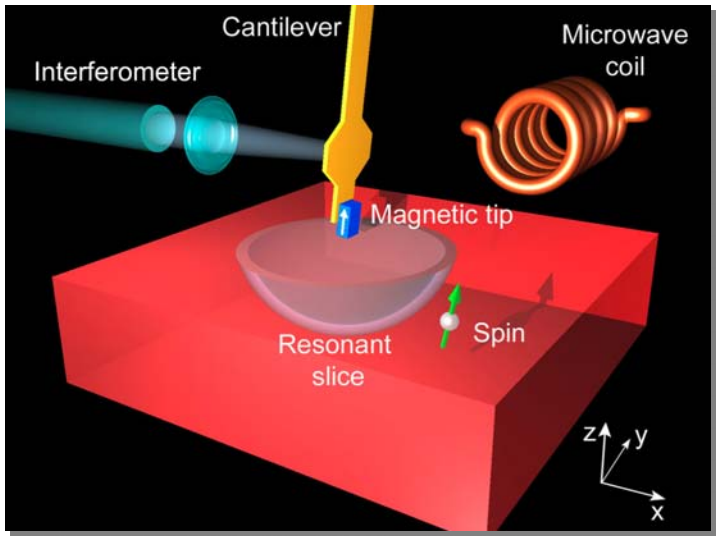
How close are we to detecting attonewton forces?



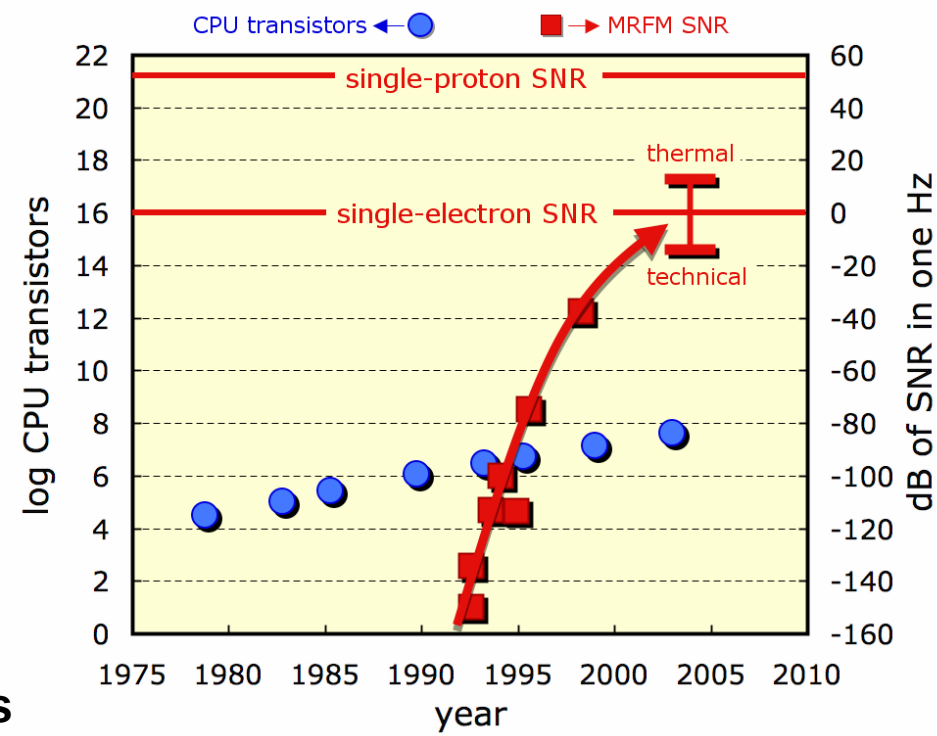
The Path to Single Nuclear Spin Detection: FAQ

Q1: What is a reasonable technical path to single-nuclear-spin detection?

A1: The path is smaller, colder, quieter device development



Moore's Law Progress in MRFM



- **MRFM sensitivity improved by 140 dB in twelve years**
- **Equivalent to doubling sensitivity every 3.1 months for 46 doublings**
- **MRFM has Moore's Scaling: smaller, colder, quieter devices work better**

Moore's Law design rules

$$S_\mu = \frac{m}{g^2\tau} 2k_B T \quad \begin{cases} m & \text{cantilever mass} \\ T & \text{temperature} \\ g & \text{magnetic gradient} \\ \tau & \text{damping time} \end{cases}$$

- *smaller*
- *colder*
- *quieter*

The Heisenberg Uncertainty Principle: Fundamental Limits to Force Sensitivity

$$\sigma_x^2 \sigma_p^2 \geq \frac{\hbar^2}{4}$$

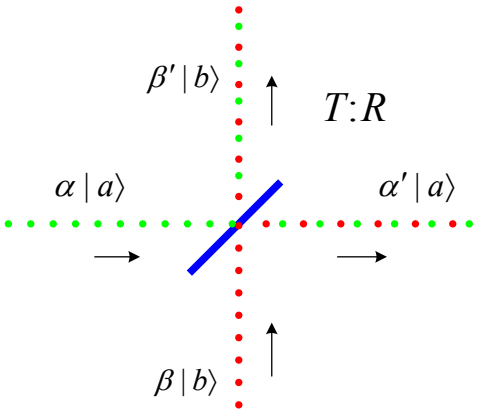
- As time progresses the variance in the position of the cantilever decreases because it is observed by the interferometer.
- The uncertainty principle demands a minimal rate of increase in the variance of the linear moment as the variance of the position decreases:

$$\frac{d}{dt}\sigma_p^2 \geq \frac{d}{dt}\frac{\hbar^2}{4\sigma_x^2}.$$

- An increase in the variance of the linear momentum is indistinguishable from the influence of a random external force with power spectral density:

$$S_{F\ \mathrm{ba}} = \frac{d}{dt}\sigma_p^2.$$

Ideal Symmetric Beam Splitter



Transfer function for a 50:50 beam splitter:

$$| a\rangle \to \frac{1}{\sqrt{2}} | a\rangle + \frac{i}{\sqrt{2}} | b\rangle,$$

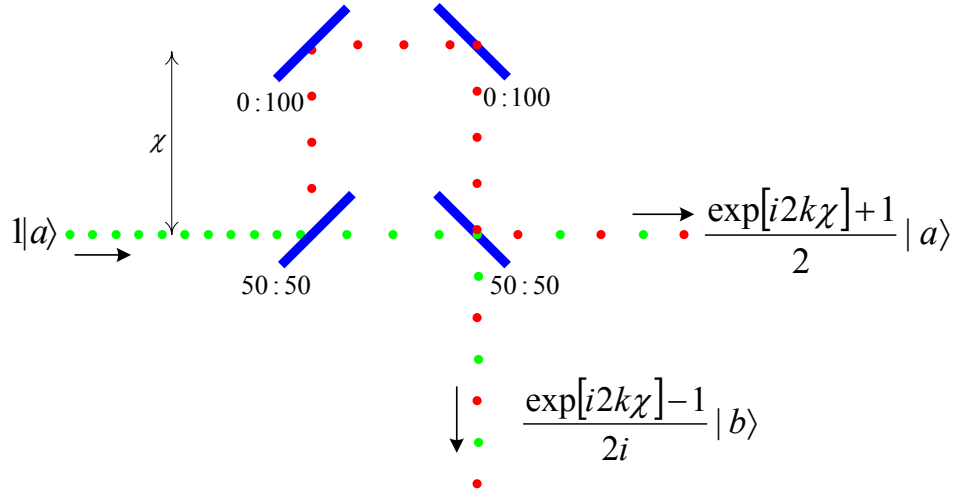
$$| b\rangle \to \frac{i}{\sqrt{2}} | a\rangle + \frac{1}{2} | b\rangle.$$

Transfer function for a 0:1000 beam splitter (mirror):

$$| a\rangle \to i | b\rangle,$$

$$| b\rangle \to i | a\rangle.$$

Ideal Interferometer



Transfer function for a photon entering the *a* channel from the left:

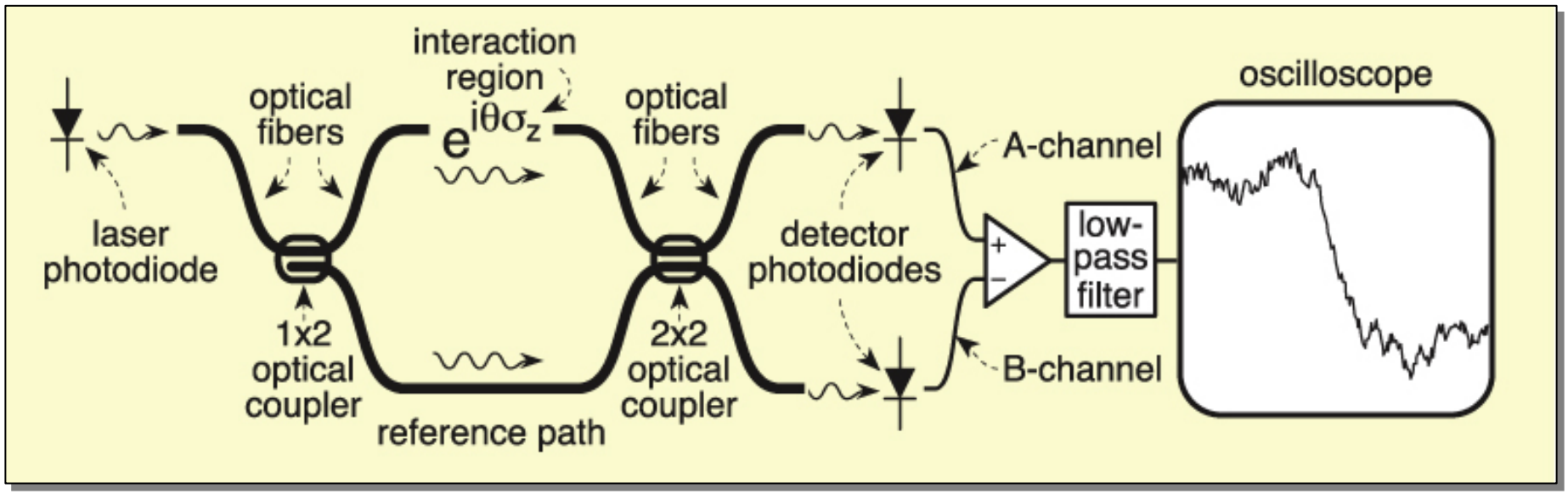
$$| a\rangle \to \frac{\exp(i2k\chi)+1}{2} | a\rangle + \frac{\exp(i2k\chi)-1}{2i} | b\rangle$$

Transfer function for a tuned interferometer:

$$\chi = x + \pi(n+1/4)/k$$

$$\boxed{| a\rangle \to \frac{i\exp(i2kx)+1}{2} | a\rangle + \frac{i\exp(i2kx)-1}{2i} | b\rangle \;\Rightarrow x = \frac{P_a - P_b}{2k}}$$

Fiber Optic Interferometer



Statistical Uncertainty in Interferometer Measurements (Shot Noise)

In a single measurement N photons pass through the interferometer at a rate of ν photons per second for Δt seconds. N_a photons are observed in detector a and N_b photons are detected in detector b.

$$x = \frac{P_a - P_b}{2k} \Rightarrow x_{est} = \frac{N_a - N_b}{2kN}$$

The statistical variance of the measured value of the position is:

$$\sigma^2_{x\ \mathrm{est}} = \frac{1}{4k^2 \nu \Delta t}.$$

Power spectral density of the noise in the position data:

$$S_{x\ \mathrm{est}} = \frac{1}{4k^2 \nu}$$

Minimum power spectral density of the back-action force:

$$S_{F\ \mathrm{ba}} \geq \hbar^2 k^2 \nu$$

Does the photon back action actually achieve the Heisenberg limit?

Quantum Measurement Theory

Prior to the measurement, the oscillator and photon are uncorrelated.

$$|\psi_{\text{initial}}\rangle = \psi_0(x) \otimes |a\rangle$$

After the photon passes through the interferometer, but before it is detected, the photon and oscillator are in an entangled state.

$$|\psi_{\text{initial}}\rangle = A(x)\psi_0(x) \otimes |a\rangle + B(x)\psi_0(x) \otimes |b\rangle,$$

The photon is destroyed by the observation and the oscillator is left in the state

$$\psi(x) = \frac{A(x)\psi_0(x)}{\sqrt{P_A}} \quad \text{or} \quad \psi(x) = \frac{B(x)\psi_0(x)}{\sqrt{P_B}},$$

with probabilities

$$P_A = \int_{-\infty}^{+\infty} \psi^*(x) A^*(x) A(x) \psi_0(x)\, \mathrm{d}x \quad \text{or}$$

$$P_B = \int_{-\infty}^{+\infty} \psi^*(x) B^*(x) B(x) \psi_0(x)\, \mathrm{d}x$$

Ideal Interferometer Performance

$A(x)$ and $B(x)$ are obtained from the previous analysis of the interferometer.

$$A(x) = \frac{i\exp(i2kx)+1}{2}, B(x) = \frac{i\exp(i2kx)-1}{2i}.$$

With these and some reasonable assumptions about the initial state of the oscillator, we find:

$$\frac{\mathrm{d}}{\mathrm{d}\,t}\sigma_p^2 \cong \frac{\langle (p_{2n} - \langle p_{2n}\rangle)^2\rangle - \langle (p_{2(n-1)} - \langle p_{2(n-1)}\rangle)^2\rangle}{2n/\nu - 2(n-1)/\nu}$$

$$\cong 2\hbar^2 k^2 \nu,$$

$$\boxed{S_{F\,\text{ba}} = 2\hbar^2 k^2 \nu,}$$

Optimal Force Sensitivity

The data record will contain the true force signal and fluctuations from both interferometer shot noise and interferometer back action.

$$S_{F\,\text{measured}} = k_s^2 S_x + S_{F\,\text{ba}}$$
$$= k_s^2 \frac{1}{4k^2\nu} + 2\hbar^2 k^2 \nu.$$

For a fixed operating wavelength, the force noise is minimized by adjusting the photon flux. Optimal operation is achieved when

$$P = \frac{c k_s \lambda}{2\pi\sqrt{8}}.$$

At this power

$$\boxed{S_{F\,\text{measured}} = \sqrt{2} k_s \hbar.}$$

Is this good enough?

In a state-of-the-art MRFM experiment:

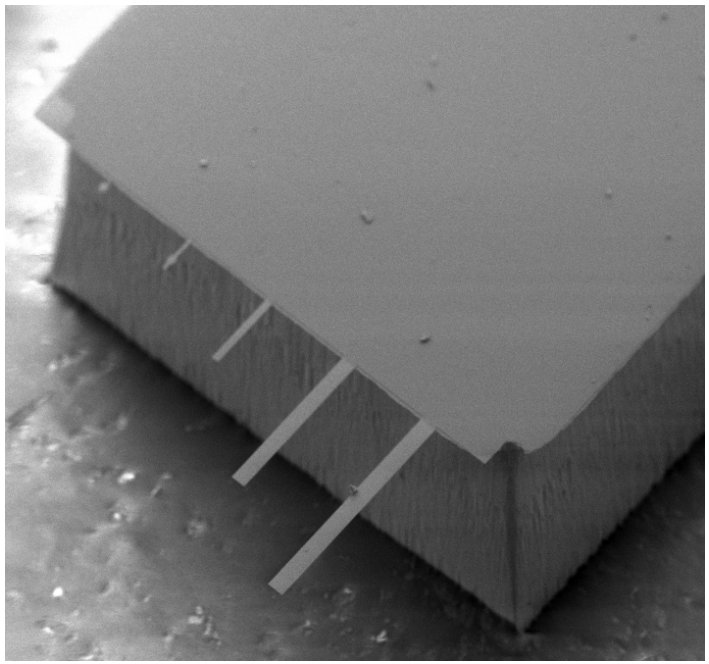
k_s = 10^{-6} N/m,
λ = 10^{-6} m.

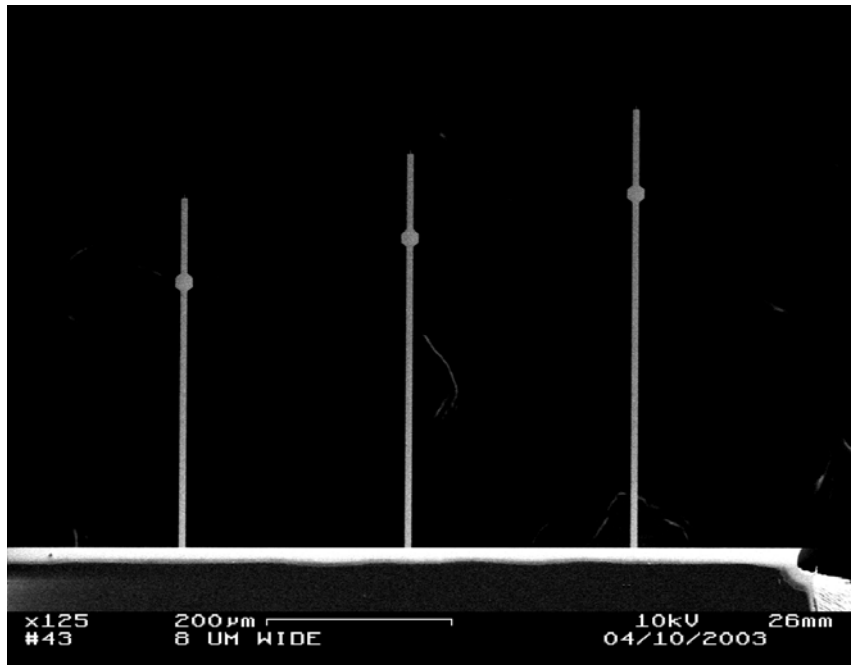
This gives a quantum limited force noise and optimal optical power of:

$$\sqrt{S_F}_{\text{measured}} = 0.01 \text{ aN Hz}^{-1/2},$$
$$P = 20\ \mu\text{W}.$$

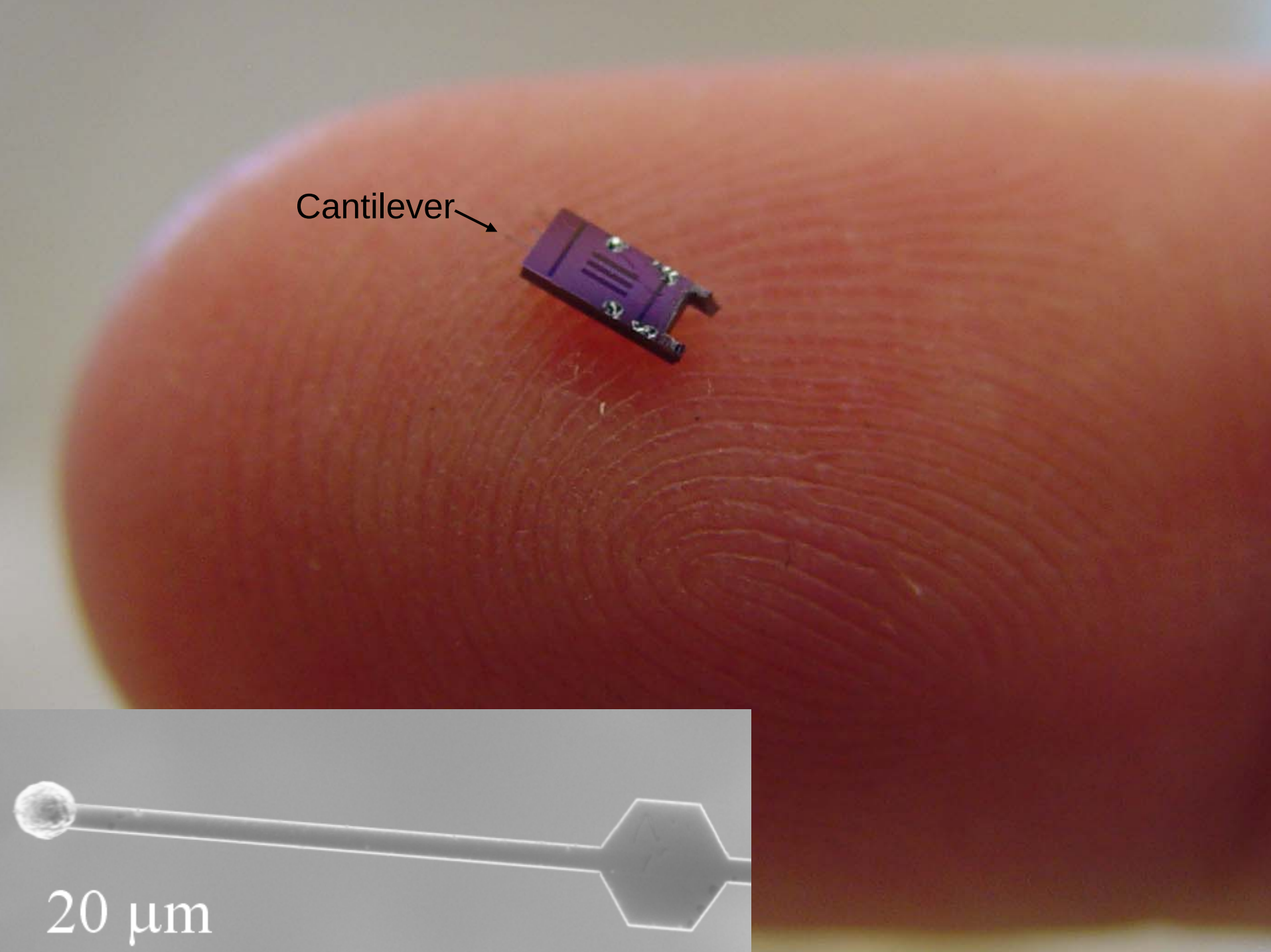
Detectable Forces are Currently Limited by Thermal Fluctuations of the Cantilever

$$F_{\min} = \left(\frac{2 k_B T \, k \, \Delta f}{\pi \, Q \, f_o} \right)^{1/2} \propto \text{thickness} \sqrt{\frac{\text{width}}{\text{length}}} \times \frac{1}{\sqrt{Q}}$$





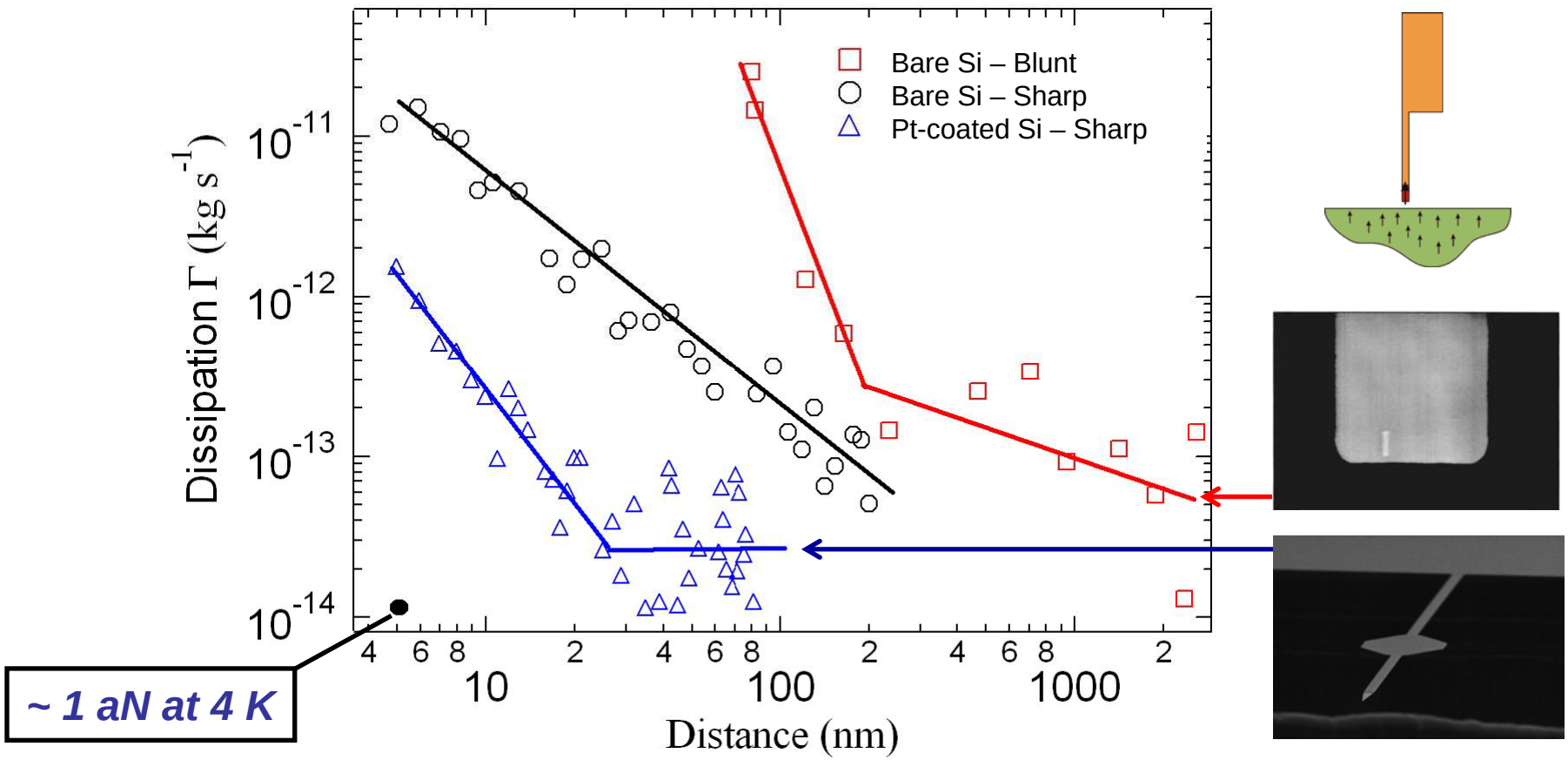
- Fabricated at the Cornell Nanofabrication Facility
- Only 340 nm thick, 5 µm wide, up to 500 µm long
- Spring constants down to $\sim 10^{-5}$ N/m


Cantilever
20 μm

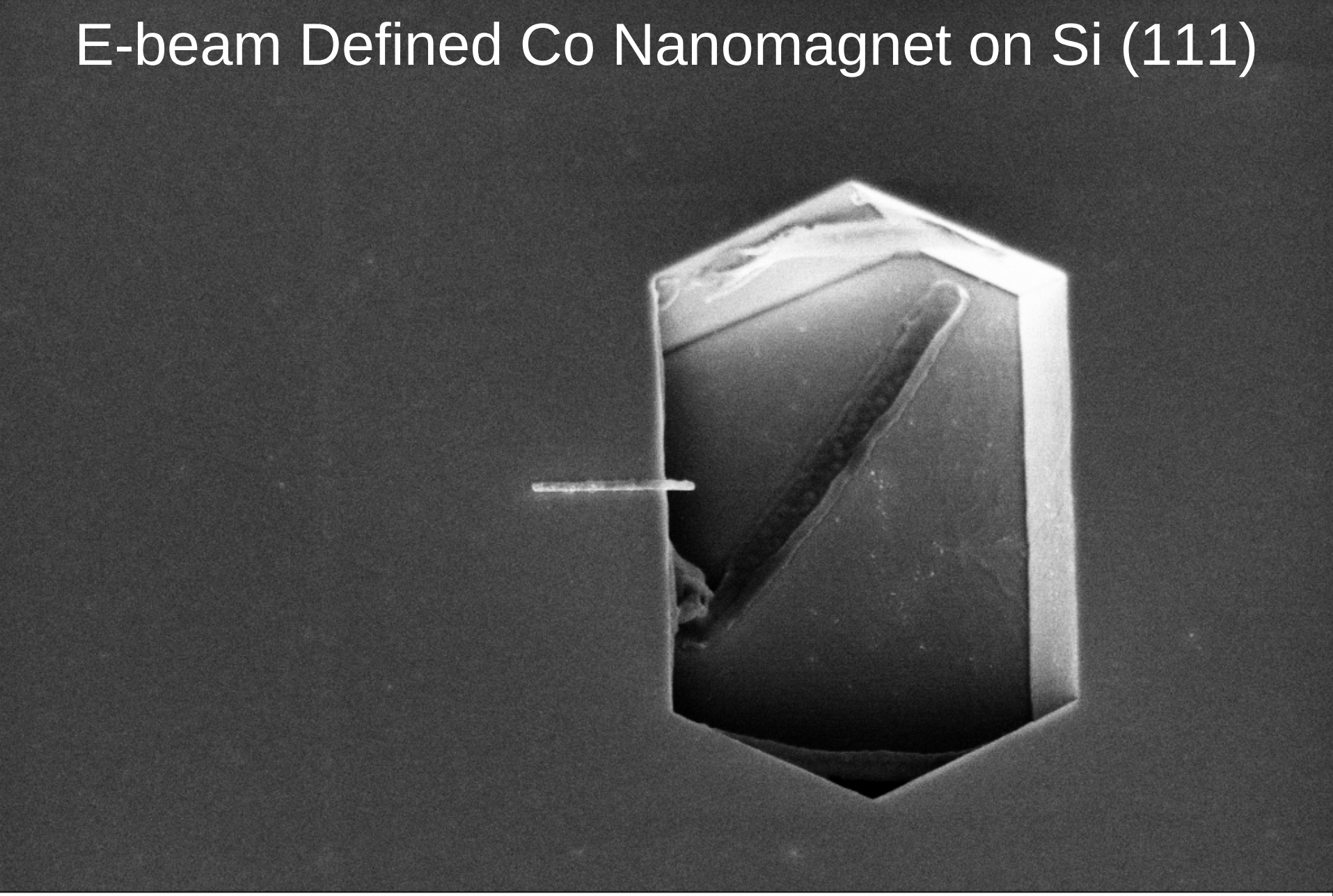
Tip-Surface Interactions – Dissipation at Room Temp

1. Apply voltage to tip to cancel contact potential difference
2. Measure the cantilever resonance freq. f and quality factor Q (ringdown time τ) vs. distance

$$\Gamma = \frac{k}{2\pi f Q} = \frac{k}{4\pi^2 f^2}\frac{1}{\tau} \quad \Rightarrow \quad F_{\min} = \sqrt{4 k_B T \Gamma b}$$



E-beam Defined Co Nanomagnet on Si (111)



1µm

WD = 6 mm
Mag = 43.84 K X

Aperture Size = 30.00 µm
EHT = 2.00 kV
Pixel Size = 8.1 nm

Signal A = InLens
Signal B = InLens

Date :15 Aug 2005
Time :17:40:11

CNF

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