Center for Data Analysis & Statistics

Response Surface Optimization of Ammunition Manufacturing

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Agenda

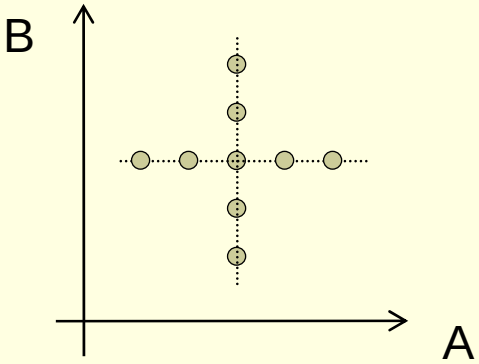
- DOE (Design of Experiments) and the Value of Interactions (Cadet Push-up Example)
- Response Surface Optimization
- Picatinny Arsenal and Lead-Azide
- Efficient Factorial Experiments
- Inclusion into the Cadet’s Curriculum

Design of Experiments

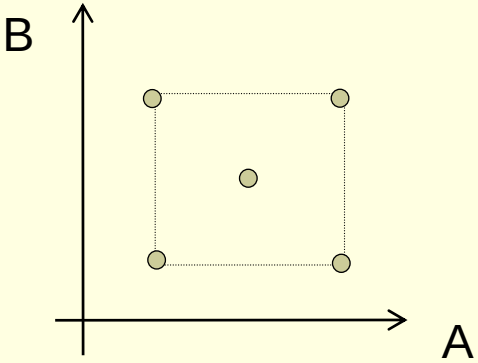
- The primary tool for system understanding and improvement is statistical design and analysis of experiments
- Statistical experimental design provides experimental efficiency and greater system understanding enabling cost reduction, quality improvement, and new approaches to system operation
  - Characterize
  - Optimize
  - Reduce Variability

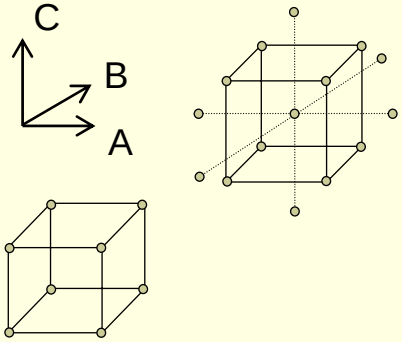
Methods of Experimentation

- Best guess



- One Factor at a Time (OFAT)
- Statistically Designed Experiments (Factorial Designs)





Value of Estimating Interactions

- Fourth-class cadets in MA103 at USMA were tasked to perform push-ups and measure their heart-rate.
- Factors of interest:
  - Number of push-ups 10 or 50
  - Cadence 1pu/s or 0.5pu/s
- 48 Cadets in this study

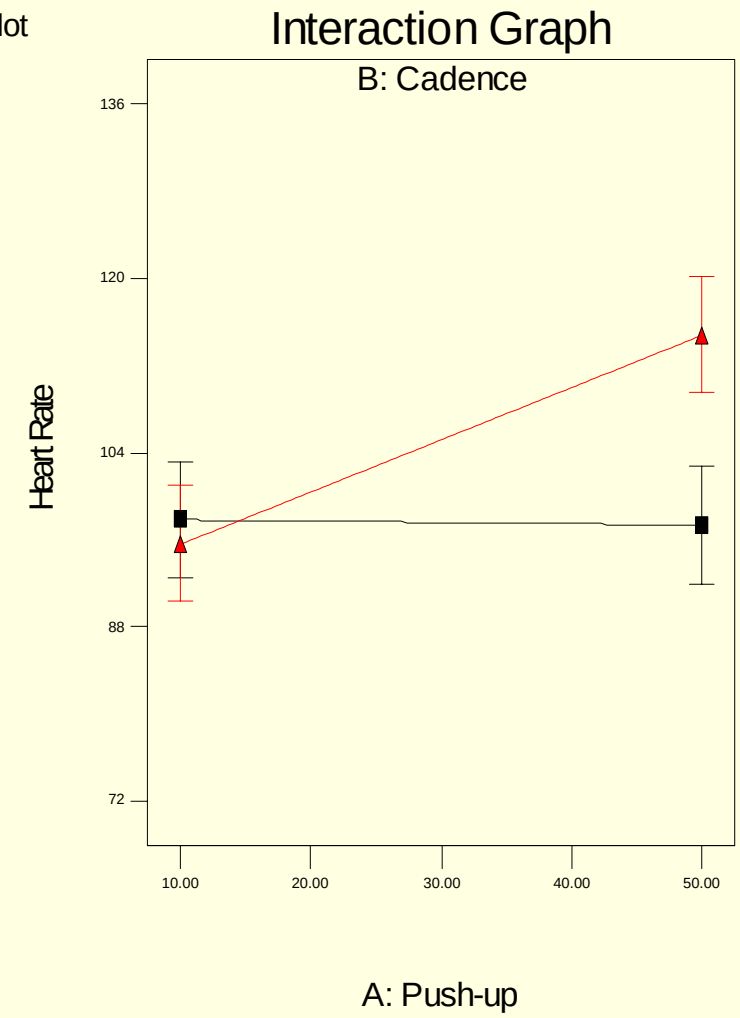
Value of Estimating Interactions

DESIGN-EXPERT Plot

Heart Rate

X = A: Push-up
Y = B: Cadence

■ B- 0.500
▲ B+ 1.000



Value of Estimating Interactions

DESIGN-EXPERT Plot

Heart Rate
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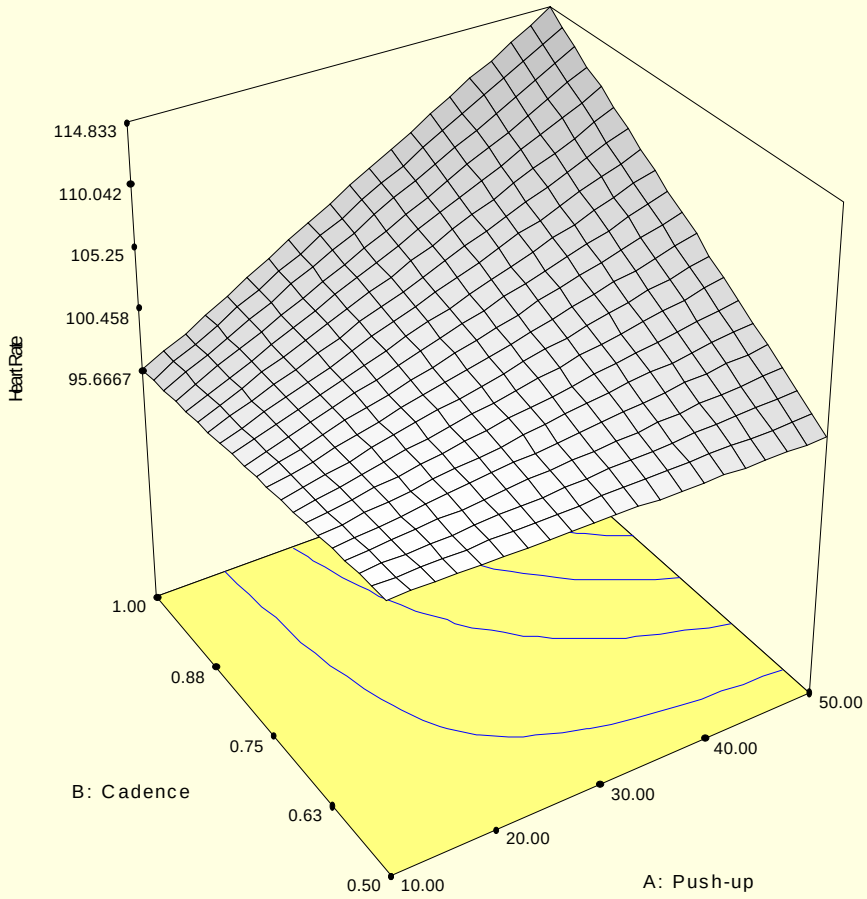
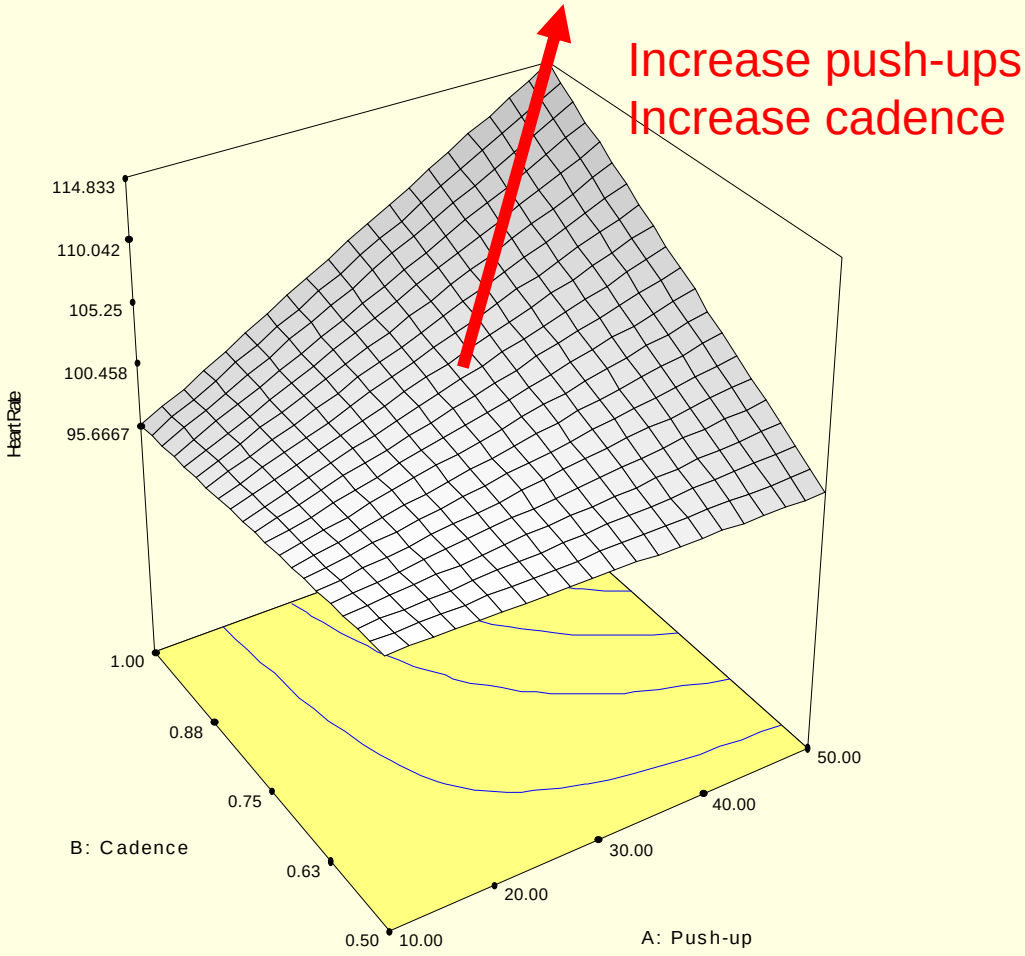


Response Surface Methodology

- Mechanistic Model - Known exactly
- Empirical Model - Approximate the unknown
- Response Surface - 1st or 2d order polynomial
  - Identifying and fitting a response surface (RS) from experimental data requires DOE.
  - Mathematical optimization techniques can be applied to the RS to find an optimal value.
  - The optimum corresponds to system settings that achieve the optimal response.

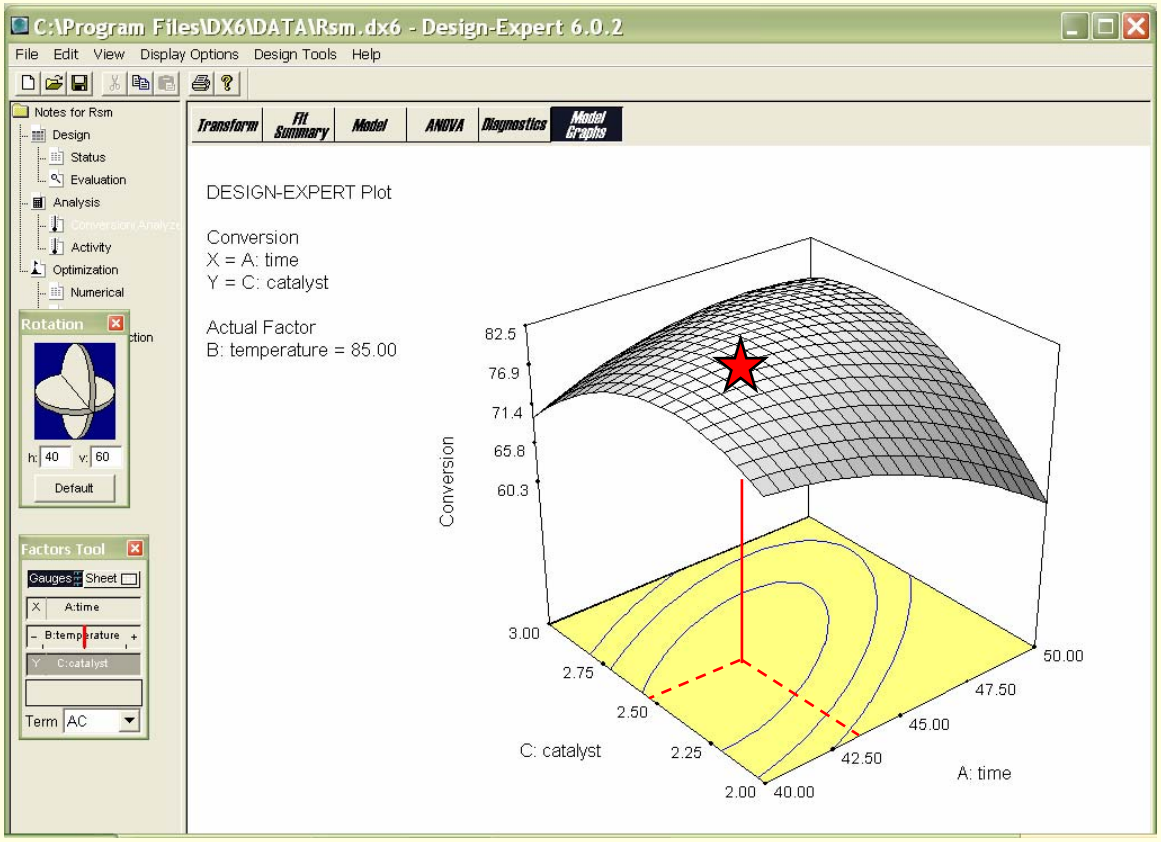
Response Surface Methodology

- Steepest ascent for the push-up problem.

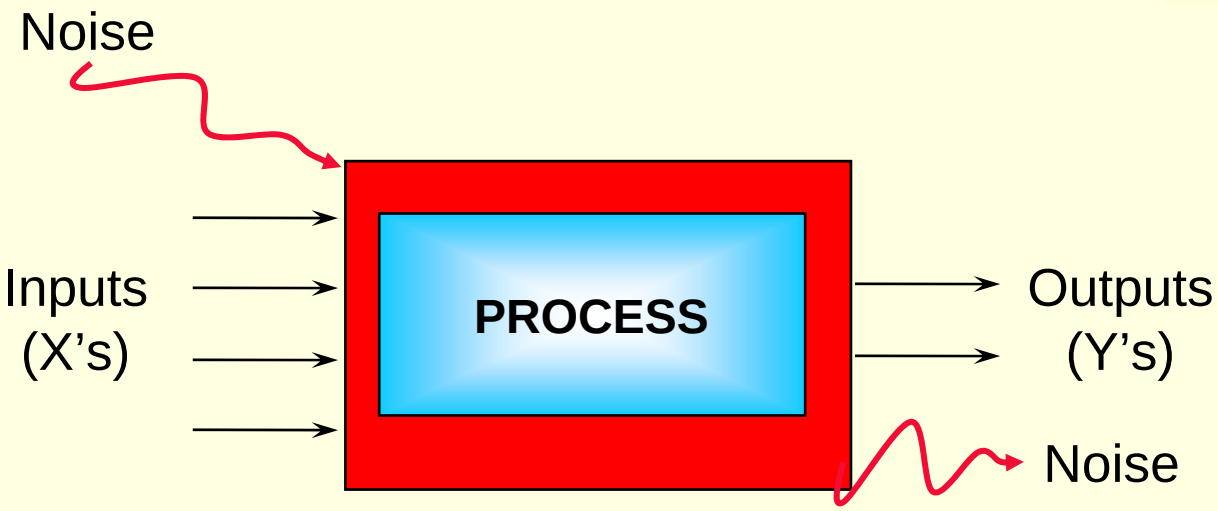


Response Surface Methodology

- Steepest ascent for a chemical engineering application in Design Expert.



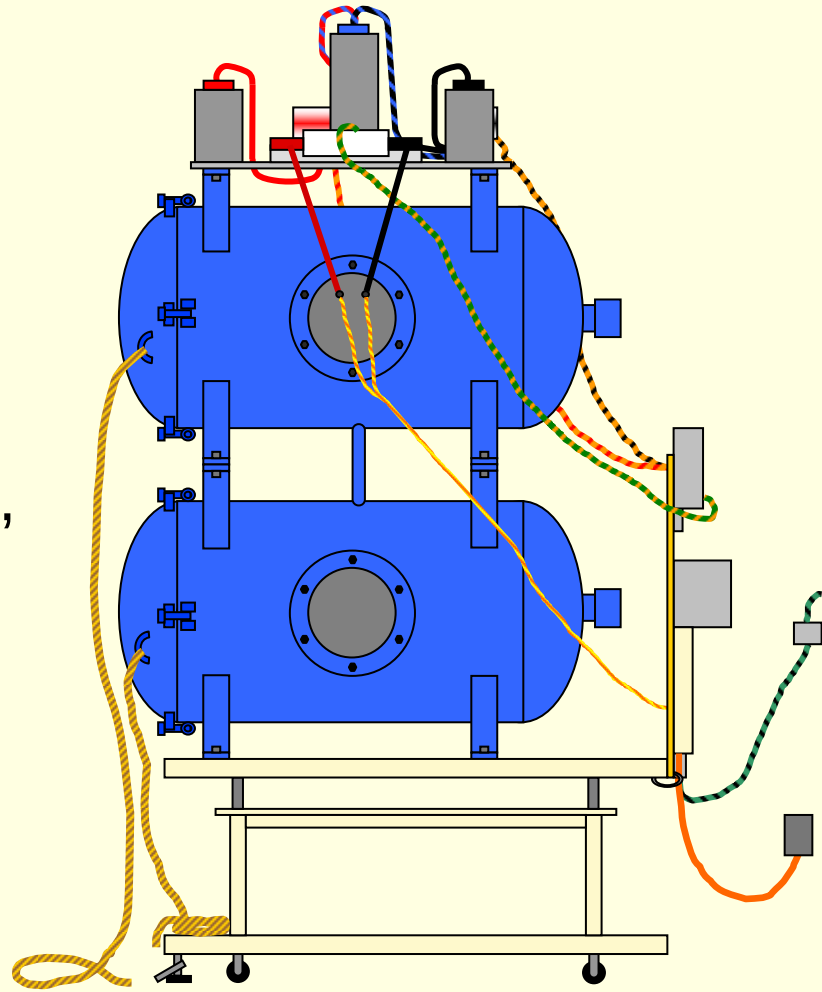
Statistically Designed Experiments



- Purposeful and systematic changes in the inputs (factors) in order to observe corresponding changes in the output (response)
- Results in the development of a mathematical model (response surface) that predicts system responses for specified factor settings

Picatinny Arsenal and Lead-Azide

- Lead-Azide, key ingredient in explosive detonators.
- Previous method was large, dangerous to personnel and the environment.
- New method is small and safe, yet not proven successful.
- Currently, the US is using stockpiles that are going bad.
- Old technology has been sold and is no longer available.

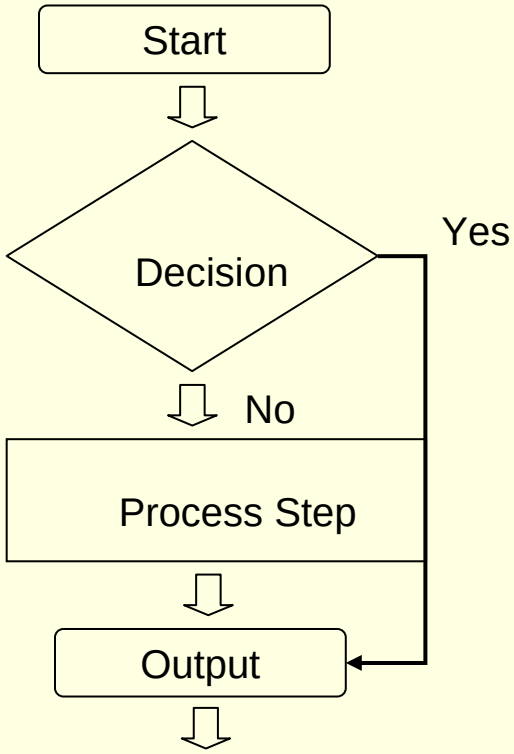


Basic Approach to DOE

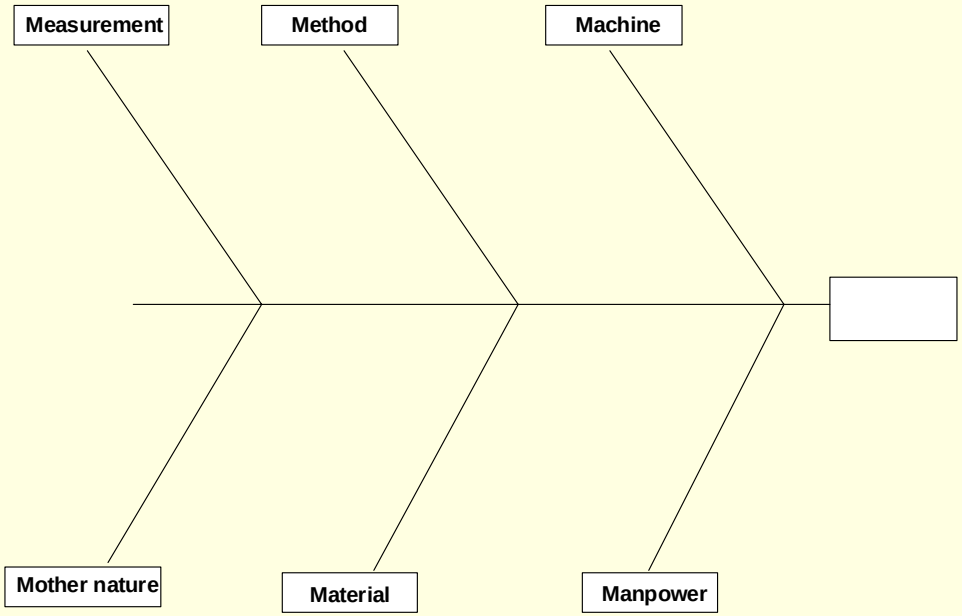
- State the Problem
- Fully Describe the System (Planning)
- Stabilize the System (Apply Controls)
- Simulate the System (if necessary)
- Experiment
- Analyze and Verify
- Link to Savings, Performance Enhancements

Planning a Designed Experiment

System Flow Diagram

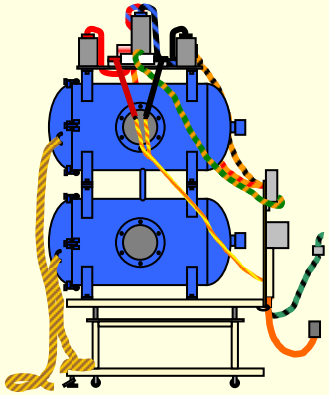


Cause and Effect Diagram



C = Control
H = Hold Constant ⇨ Standard Operating Procedures
N = Noise

Identify Sources of Variability



Measurement

Microscopy *(Hard to measure)*
Thermocouples

Method

Storage Time
Batch Number
Length of Run

Machine

Flow Rate
Process Set-up

Desireability

Environment

Azide Temperature
Lead Temperature
Ambient Room Temp
(Heat transfer prior to mix)
Humidity of Test Room

Material

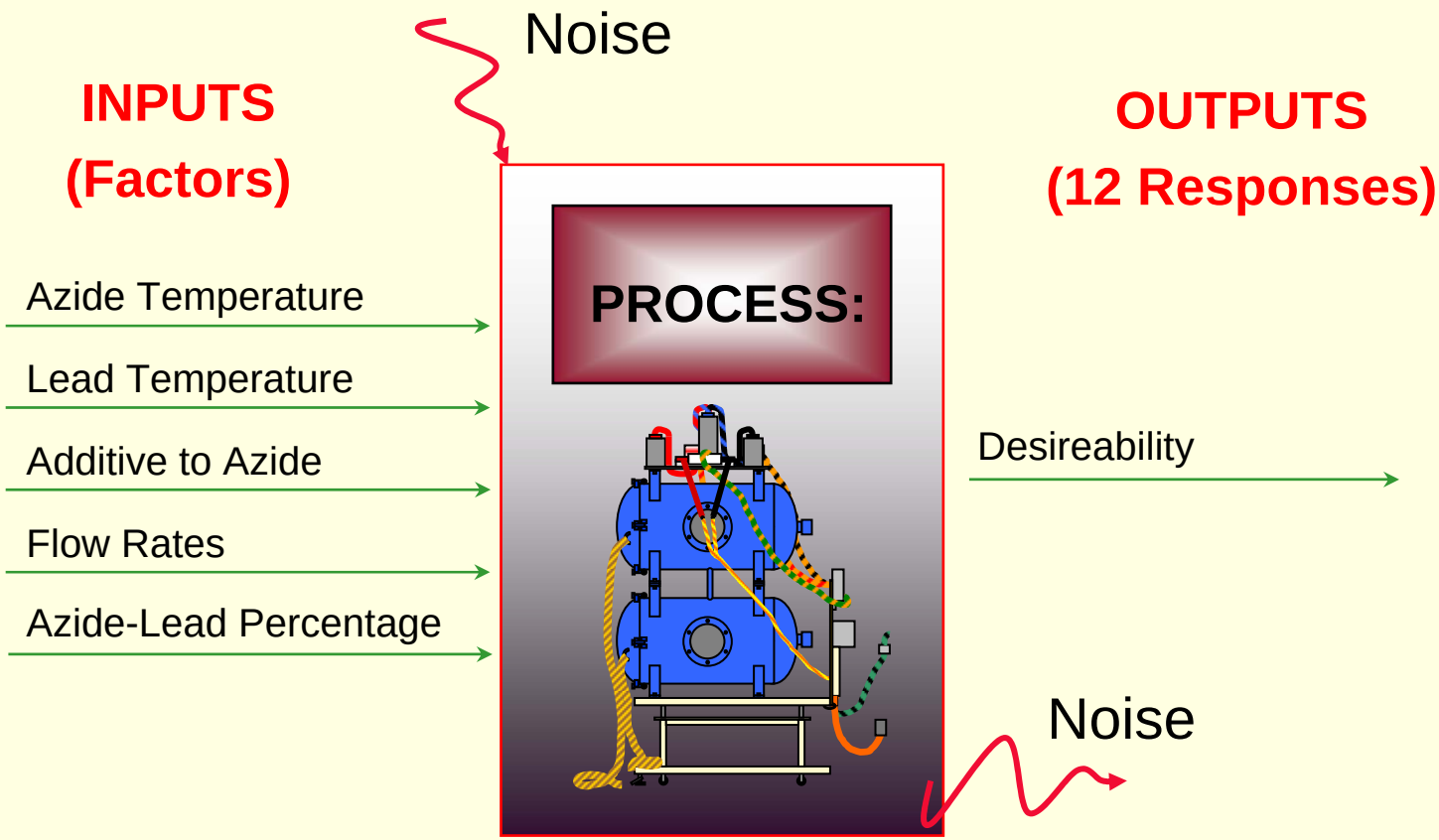
Azide Concentration Lead Concentration
CMC Added to Azide
CMC Added to Lead
Static Mixer
Source of Raw Mat'l
Azide
Lead
CMC

Manpower

Process Control
Ball Drop Manpower
Friction Test Manpower
Electrostatic Test Manpower
Purity Test Manpower
DSC Manpower
Detonate Fn Test Manpower

Lead-Azide Experiment

- How to optimize a chemical process?



Desireability

- Each response is converted into an individual desireability, d_i , such that $0 \le d_i \le 1$.
- Individual desireabilities are combined into an overall desireability that can be optimized,

$$D = \left(d_1 * d_2 * d_3 * \cdots * d_n\right)^{\frac{1}{n}}$$

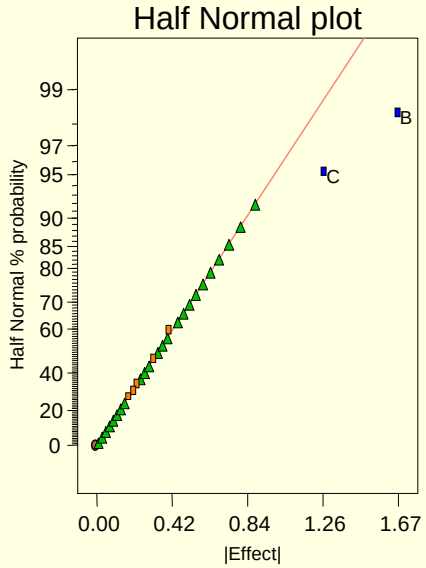
- Example individual desireability for Detonation

$$d_1 = \left(\frac{y}{1.56}\right)^5$$

Factorial Experiments

- Factorial experiment studies the effects of two or more factors
- Simplest type of factorial is the 2-level factorial
- In a “Full Factorial” all combinations are tested, 2^k experiments, where k is the number of factors
- In a “Fractional Factorial” a certain subset is tested
  - Ambiguity (aliasing) in results
  - Sparsity of effects
- Sequential Experimentation

Fractional Factorials



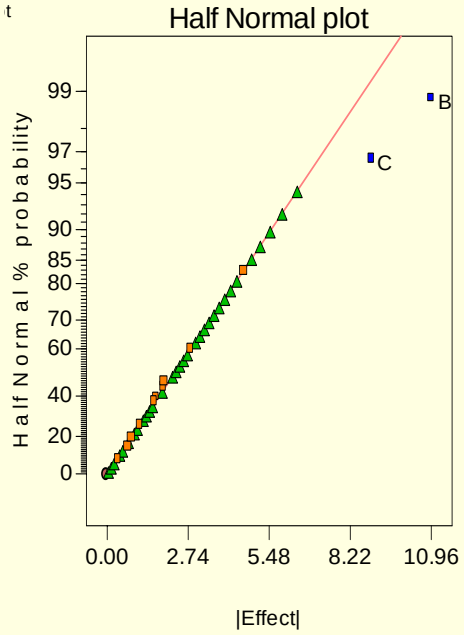
- 5 factors: 2^5 = 32 tests
- ¼ fractional = 8 tests, Res III
- B=AD, C=AE

There is ambiguity as to what is important

Sparsity of effects says B and C are most likely important

- Resolution
  - Res III (Red): Main effects aliased with 2FI
  - Res IV (Yellow): Main effects clear, 2FI aliased with 2FI
  - Res V (Green): Main effects and 2FI clear

Fractional Factorials

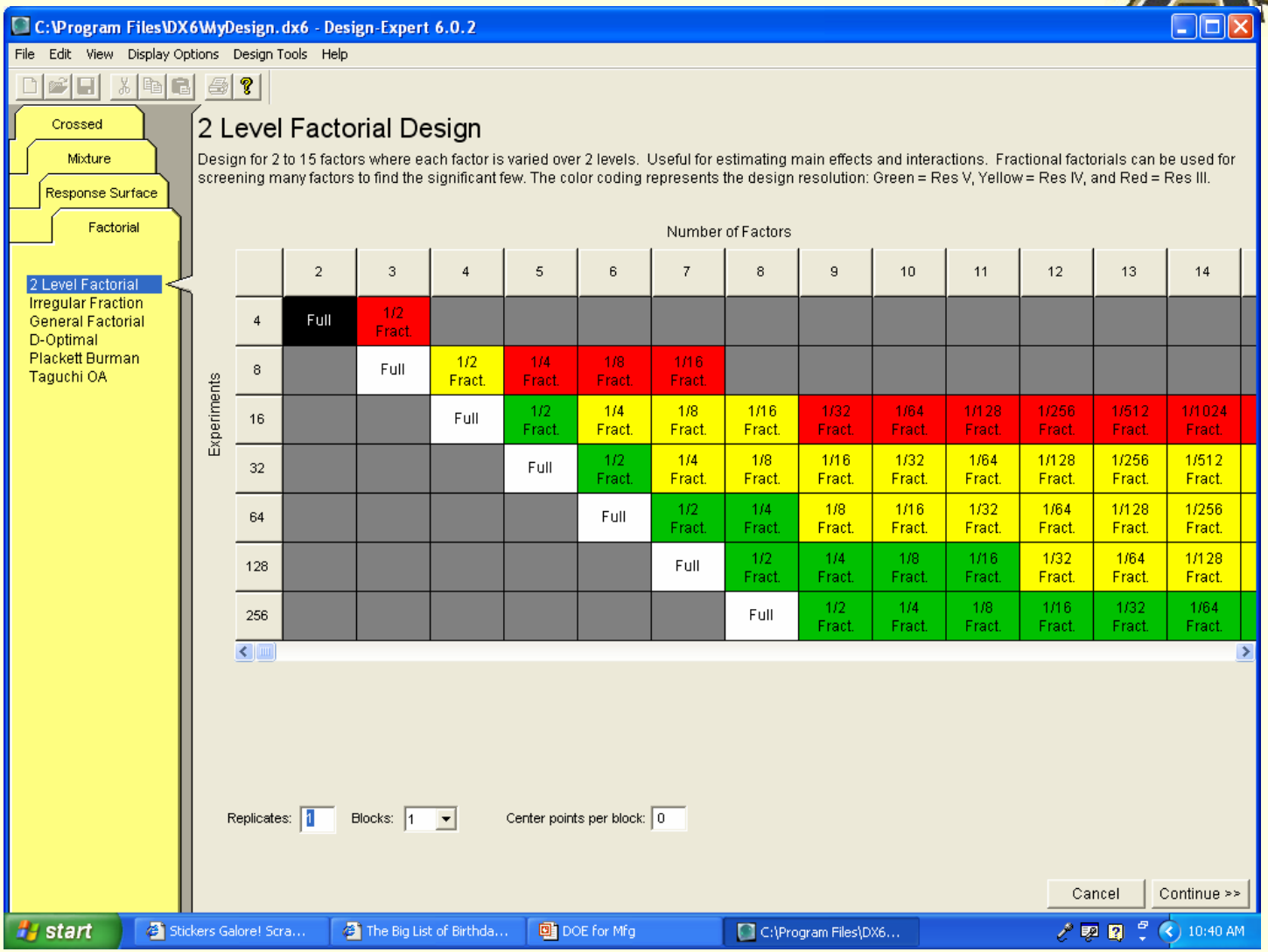


- 8 tests conducted in ¼ fractional
- ½ fractional = 16 tests, Res V
- 8 more tests

Sometimes factors can be dropped in the fold-over experiment, reducing tests

- Fold-Over Experiment
  - Use another certain subset of tests to “break” the alias
- With 2 factors important, $2^2 = 4$ unique tests
- Sequential experimentation provides 4 replicates/test

Design Expert



Experiments	2	3	4	5	6	7	8	9	10	11	12	13	14
4	Full	1/2 Fract.											
8		Full	1/2 Fract.	1/4 Fract.	1/8 Fract.	1/16 Fract.							
16			Full	1/2 Fract.	1/4 Fract.	1/8 Fract.	1/16 Fract.	1/32 Fract.	1/64 Fract.	1/128 Fract.	1/256 Fract.	1/512 Fract.	1/1024 Fract.
32				Full	1/2 Fract.	1/4 Fract.	1/8 Fract.	1/16 Fract.	1/32 Fract.	1/64 Fract.	1/128 Fract.	1/256 Fract.	1/512 Fract.
64					Full	1/2 Fract.	1/4 Fract.	1/8 Fract.	1/16 Fract.	1/32 Fract.	1/64 Fract.	1/128 Fract.	1/256 Fract.
128						Full	1/2 Fract.	1/4 Fract.	1/8 Fract.	1/16 Fract.	1/32 Fract.	1/64 Fract.	1/128 Fract.
256							Full	1/2 Fract.	1/4 Fract.	1/8 Fract.	1/16 Fract.	1/32 Fract.	1/64 Fract.

Design Expert

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File Edit View Display Options Design Tools Help

Crossed
Mixture
Response Surface
Factorial

2 Level Factorial
Irregular Fraction
General Factorial
D-Optimal
Plackett Burman
Taguchi OA

2 Level Factorial Design

Design for 2 to 15 factors where each factor is varied over 2 levels. Useful for estimating main effects and interactions. Fractional factorials can be used for screening many factors to find the significant few. The color coding represents the design resolution: Green = Res V, Yellow = Res IV, and Red = Res III.

5 Factors: A, B, C, D, E

Design Matrix Evaluation for Factorial Reduced 2FI Model

Factorial Effects Aliases

[Est. Terms] Aliased Terms

[Intercept] = Intercept + ABD + ACE

[A] = A + BD + CE

[B] = B + AD + CDE

[C] = C + AE + BDE

[D] = D + AB + BCE

[E] = E + AC + BCD

[BC] = BC + DE + ABE + ACD

[BE] = BE + CD + ABC + ADE

Factor Generator

D = AB

E = AC

Factorial Effects Defining Contrast

I = ABD = ACE = BCDE

<< Back Continue >>

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Design Expert

Factors

	Name	Units	Type	Low	High
A:	A		Numeric	-1	1
B:	B		Numeric	-1	1
C:	C		Numeric	-1	1
D:	D		Numeric	-1	1
E:	E		Numeric	-1	1

Responses: 1

	Name	Units
	Response 1	

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File Edit View Display Options Design Tools Help

- Notes for MyDesign
  - Design
    - Status
    - Evaluation
  - Analysis
    - Response 1(Empty)
  - Optimization
    - Numerical
    - Graphical
    - Point Prediction

	Std	Run	Block	Factor 1 A:A	Factor 2 B:B	Factor 3 C:C	Factor 4 D:D	Factor 5 E:E	Response 1 Response 1
	3	1	Block 1	-1.00	1.00	-1.00	-1.00	1.00	
	5	2	Block 1	-1.00	-1.00	1.00	1.00	-1.00	
	4	3	Block 1	1.00	1.00	-1.00	1.00	-1.00	
	6	4	Block 1	1.00	-1.00	1.00	-1.00	1.00	
	1	5	Block 1	-1.00	-1.00	-1.00	1.00	1.00	
	2	6	Block 1	1.00	-1.00	-1.00	-1.00	-1.00	
	7	7	Block 1	-1.00	1.00	1.00	-1.00	-1.00	
	8	8	Block 1	1.00	1.00	1.00	1.00	1.00	

Cadet Curriculum

- Cadet Trip Section to Picatinny Arsenal, 7 Oct 05

Questions?