Quantitative relationships between stress distributions, microstructure, and high strain rate performance of advanced ceramics: an interim report

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Project

Advanced Ceramics

Conditions: high strain rate, high stress, large strain
Materials: armor ceramics

Goals

- Contribute to fundamental theory about cracking of advanced ceramics
- Develop quantitative relationships between
  - stress distributions and
  - microstructures (texture, grain size, grain shape)

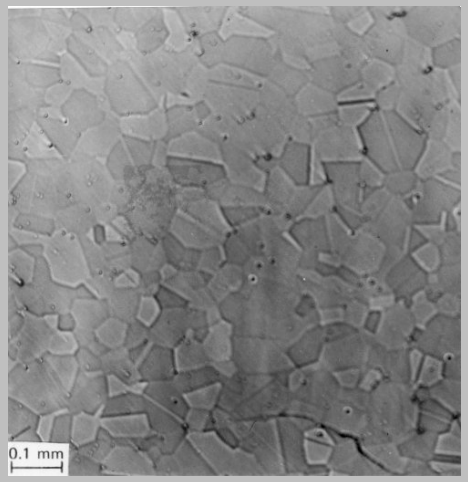
Today's Goals

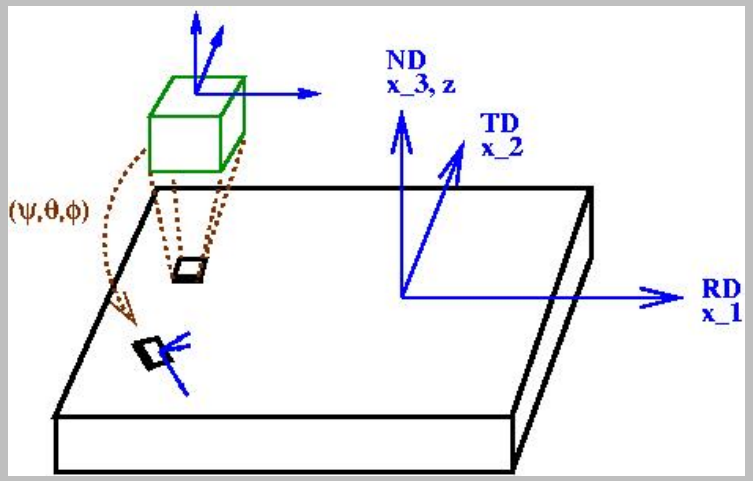
Report on progress made in understanding affect of texture on internal stress and clarifying internal stress distributions

Outline

1. Crystallographic Texture
   - Relationship to Stress
   - Visualization and OOF2
   - Thermo-anisotropic Elasticity
   - In Progress
2. Stress Distributions
   - Papers Reviewed Last Year
   - Research Direction
   - In Progress

Texture





- $\sigma = \mathbb{C}(w)[E]$
- Elastic constants $\mathbb{C}_{ijk\ell}$
- Orientation distribution function

$$w(\psi,\theta,\phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \sum_{n=-\ell}^{\ell} W_{\ell mn} Z_{\ell mn}(\cos\theta) e^{-im\psi} e^{-in\phi}$$

with **texture coefficients** $W_{\ell mn}$

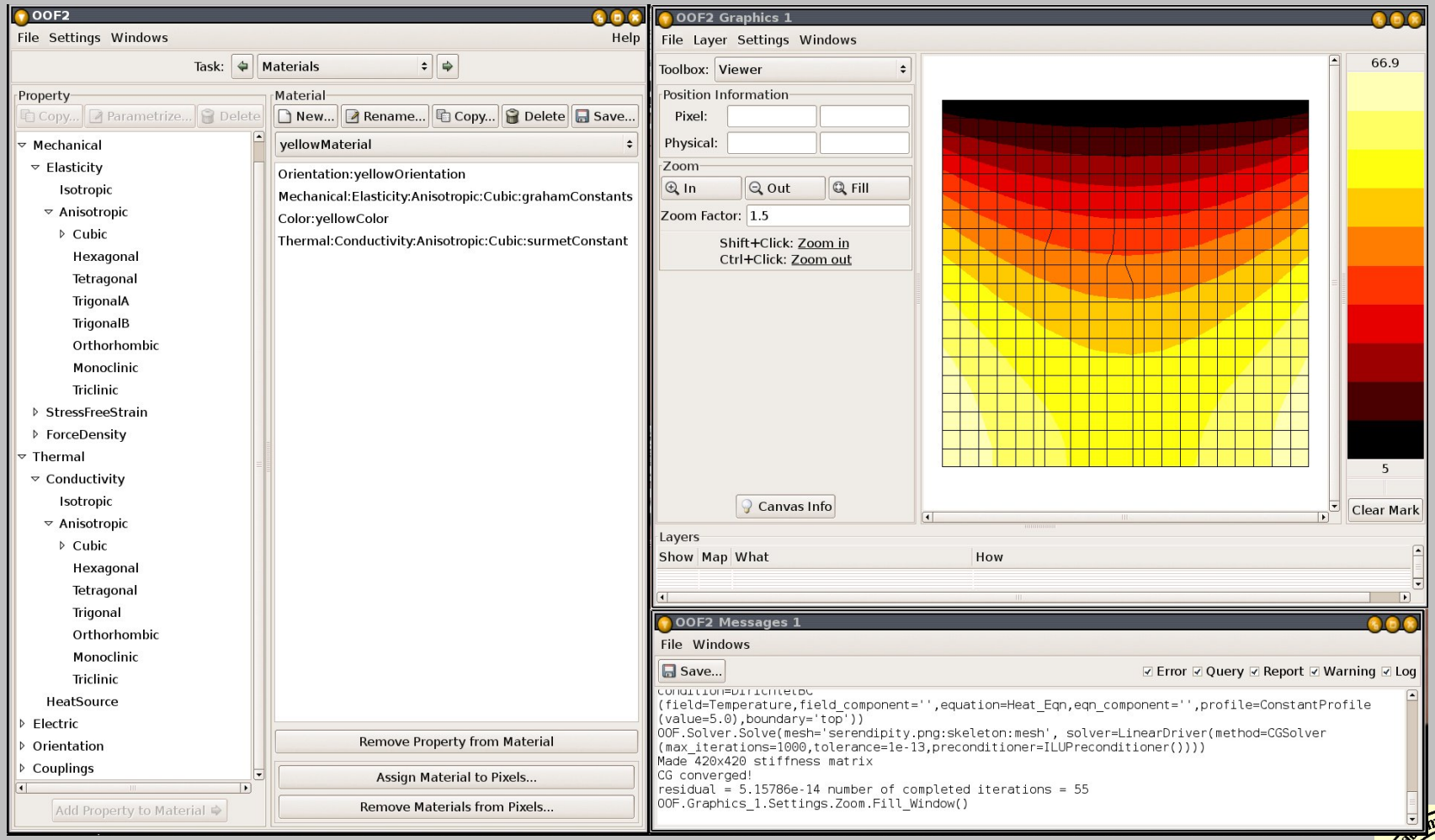
Visualization

What does the internal stress distribution look like?

OOF2

- Object Oriented Finite element method in 2-dimensions
- Specifically designed for use on actual micrographs
- Heat & force balance equations ($\nabla \cdot \text{flux} = \text{applied force}$)
  Plane flux equations ($\text{out of plane components of flux} = 0$)
- Can be extended using `C++` or `Python`

OOF2



available at **http://www.ctcms.nist.gov/oof**

Thermo-anisotropic Elasticity

Thermal contractions lead to fracture in brittle materials

- Equations of thermo-anisotropic elasticity

$$\begin{aligned} \nabla \cdot \sigma &= 0 && \text{equilibrium} \\ \nabla \cdot h &= 0 && \text{balance of energy} \\ \sigma &= \mathbb{C}[E] - \mathcal{B}T && \text{stress-strain law} \\ h &= -\boldsymbol{\kappa}\nabla T && \text{heat conduction} \end{aligned}$$

$\mathcal{B} =$ thermal expansion tensor, $\boldsymbol{\kappa} =$ thermal conductivity tensor, $h =$ heat flux, $T =$ temperature

- Solved in 2-D by T.C.T. Ting (1996) using Stroh's formalism

In Progress

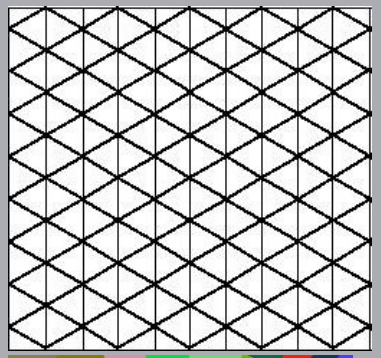
Visualize internal stress in AlON

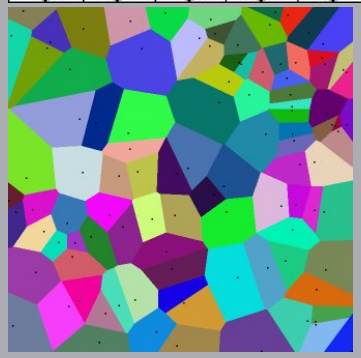
- Implement thermal-anisotropic elasticity equations
- Use material constants, orientation data, and micrograph from actual sample
- Use `OOF2`
- Extend as necessary

Stress Distributions

Papers Reviewed Last Year

1. X.H. Zhang, *et al.* (2004)
2. D. Zhang, *et al.* (2005)
3. Tasdemirci and Hall (2005)





Split Hopkinson Pressure Bar experiments and simulation

Last Year (con't)

Maximum stress alone is not best predictor of damage. Stress inhomogeneity is important too.

- After microcracking but before large cracks: high stresses shift rapidly from one location to another.
- As compressive stress increased: heterogenity of stress states increased.

Research Direction

Hypothesis

- High stress next to low stress is a *critical event*
- Heterogeneity of internal stress states, not simply exceeding max stress, leads to cracking

Idea

The heterogeneity of internal stress can predict imminent crack formation.

$\Longrightarrow$ Use internal stress state to predict response of ceramics to high strain rate, high stress, large strain conditions

In Progress

Characterize stress states using fractal dimension

Definition (fractal dimension: box-counting dimension)

Given a self-similar object of N parts scaled by the ratio r from the whole, its fractal dimension is

$$D = \frac{\log N}{\log (1/r)}.$$

Multifractal formalism

- When different subsets of the object exhibit different fractal dimensions the object is considered to be **multifractal**
- The **singularity spectrum** fully describes a multifractal object

Summary

Effects of Texture

1. Preferred crystallographic orientation and mechanical anisotropy of individual crystals affects stress response
2. Working on numerical solution to thermo-anisotropic equations using **OOF2** and "real" data to visualize affects of anisotropy on internal stresses under compression

Stress Distributions

3. Simulations reported by others show stress heterogeneities prior to cracking
4. Multifractal formalism under investigation as method to characterize heterogeneous stress states prior to cracking.

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Many Others

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Project Goal

To better predict fracture in advanced ceramics under high strain rate, high compressive stress, and large strains by studying microstructure and internal stress distributions

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