Energy-Optimal Control of an NMR Spin Chain for Quantum Information Processing

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ABSTRACT:

Quantum information processing has great theoretical promise but faces daunting technical challenges. In particular, experiments in nuclear magnetic resonance (NMR) quantum computation require extremely precise control of quantum mechanical ensembles. We consider controlled transfer of coherence along a spin chain, one of the proposed topologies for an NMR quantum computer. The problem of time-optimal or energy optimal control of these systems corresponds to finding optimal paths in Lie groups in which evolution in only certain directions can be directly controlled. We obtain necessary conditions for the normal extremal trajectories of a three-spin chain in the continuous time case, and then for its discrete counterpart obtained from a discrete variational scheme. We also obtain expressions for the control inputs, and provide a novel numerical algorithm to carry out accurate numerical simulations. This work is based on research done by the first author under the direction of Dr. Howard Brandt of the Sensors and Electron Devices Directorate during 2005 - 2006.

KEYWORDS: optimal control, sub-Riemannian geometry, quantum computation

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