Intro
Domination in graphs
Colorings in graphs
Dominator Colorings in graphs
Results
Dominator Chromatic Number
Safe Clique Partition

On Dominator Colorings in Graphs

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Domination in graphs

Definition

A dominating set S of a graph G is a set of vertices of G so that each vertex is either in S or adjacent to a vertex of S.

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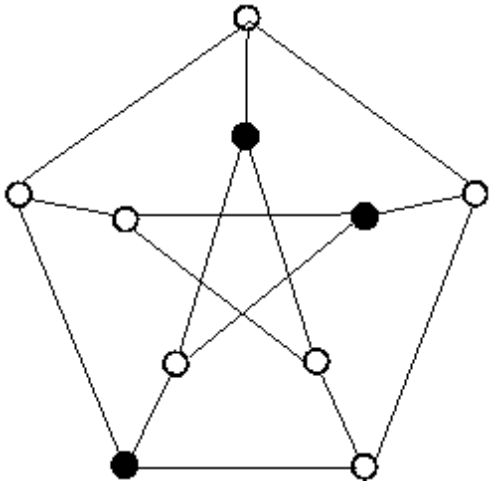

At most 3 vertices are needed in a dominating set. We show that at least 3 vertices are needed in a dominating set.

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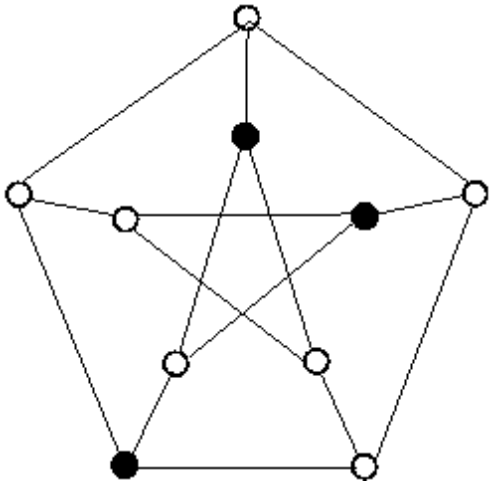

Petersen graph is 3-regular, and so 1 vertex can dominate at most 4 vertices.

Domination number

Definition

The domination number of G, denoted by $\gamma(G)$, is the smallest cardinality of a dominating set of G.

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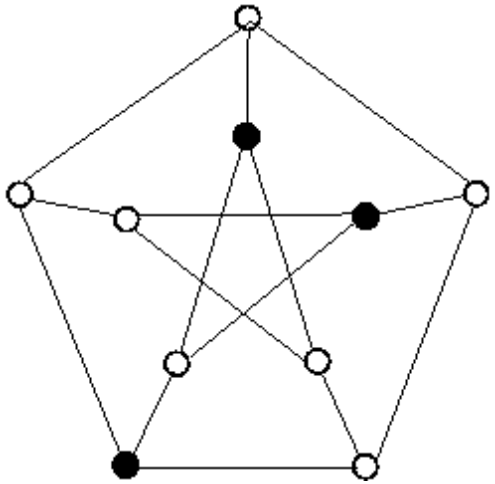

Since 2 vertices can dominate at most 8 vertices, it follows that $\gamma(G) \geq 3$. Thus $\gamma(G) = 3$.

Domination in graphs

Optimization problem: find a dominating set of minimum cardinality.

Numerous variants of this problem have been studied by Berge, Ore, Haynes, Hedetniemi, Slater, and others, and here we study one more.

Given a graph and an integer k, it is NP-complete to decide if the graph has a DOMINATING SET of order k or less. [Garey and Johnson].

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Colorings in graphs

Definition

A proper coloring of a graph G is a function from the vertices of the graph to a set of colors such that no two adjacent vertices have the same color.

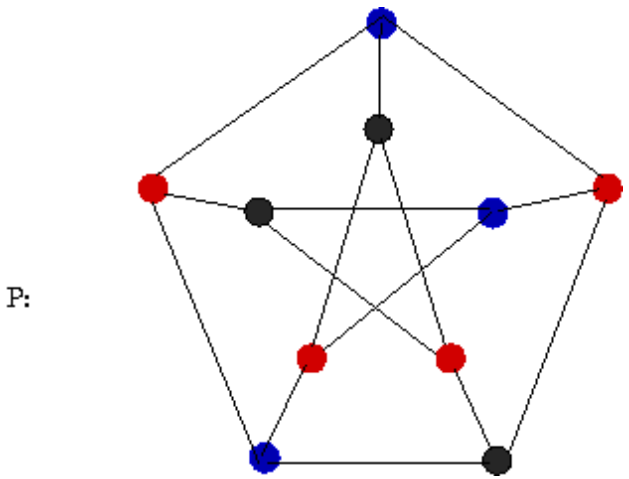

Figure: A minimum coloring of Petersen graph

At most 3 colors are needed to color Petersen Graph.

Colorings in graphs

Definition

The chromatic number $\chi(G)$ is the smallest number of colors used in proper coloring.

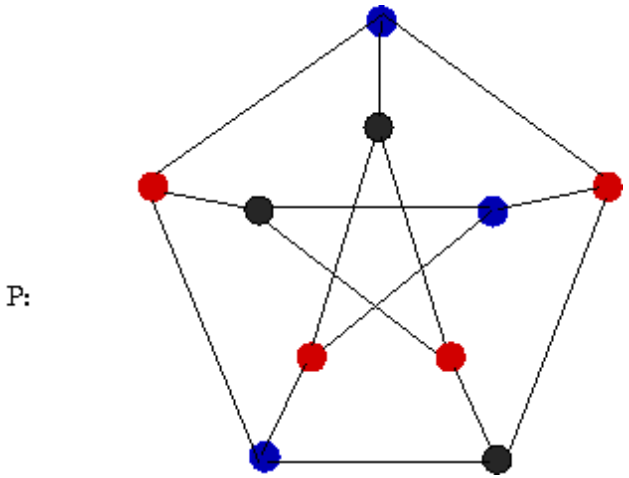

Figure: $\chi(P) = 3$.

At least 3 colors are needed to color an odd cycle.

Colorings in graphs

Graph coloring is used as a model for a vast number of practical problems involving allocation of scarce resources (e.g., scheduling problems), and has played a key role in the development of graph theory and, more generally, discrete mathematics and combinatorial optimization.

Graph k-colorability is NP-complete in the general case [Garey and Johnson].

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Dominator Colorings

Definition

A dominator coloring is a proper coloring in which each vertex of the graph dominates all the vertices of some color class.

Example:

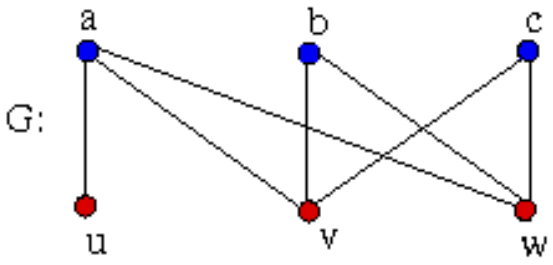

Figure: A proper coloring of G

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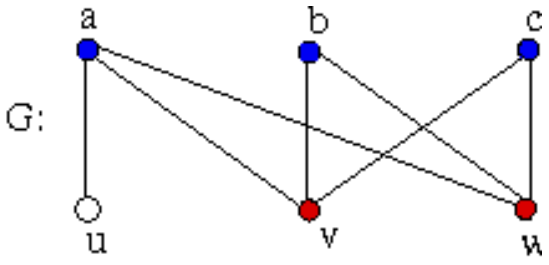

Figure: A dominator coloring of a graph

Dominator Chromatic Number

Definition

The dominator chromatic number $\chi_d(G)$ is the minimum number of color classes in a dominator coloring of a graph G.

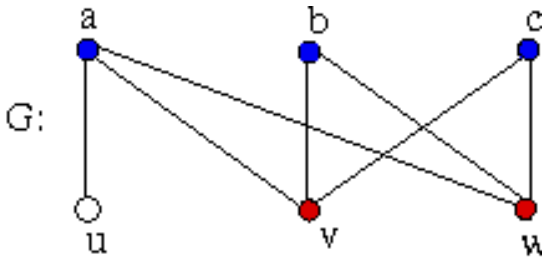

Figure: $\chi_d(G) = 3$

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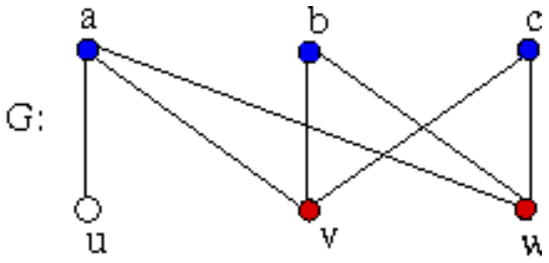

Figure: $\chi_d(G) = 3$

Dominator Chromatic Number

Our study of this problem is motivated by E. Cockayne, S. Hedetniemi, and S. Hedetniemi, Dominating partitions of graphs, tech. rep., 1979.

Given a graph and an integer k, it is NP-complete to decide if the graph has a DOMINATING COLORING with k colors. [Gera, Horton, Rasmussen].

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Dominator Coloring Examples

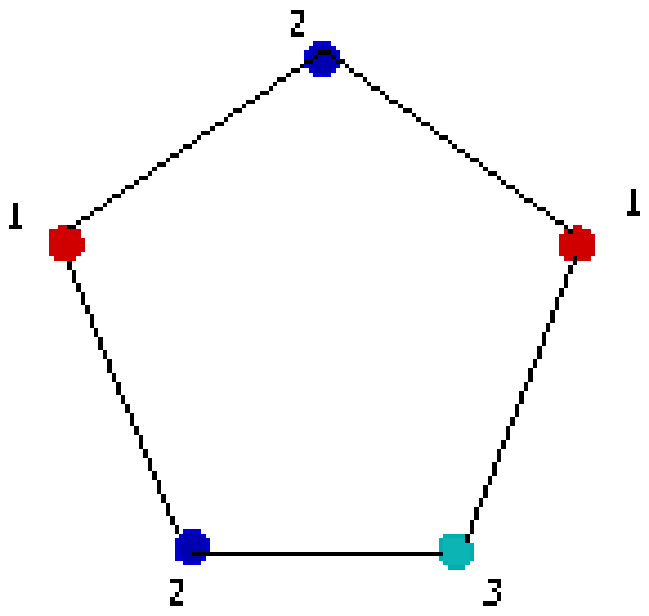

Figure: $\chi_d(C_5) = 3$

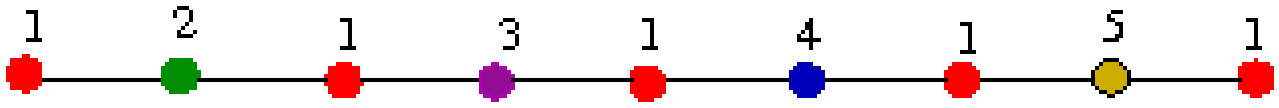

Figure: $\chi_d(P_9) \leq 5$

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Dominator Chromatic Number for Disconnected Graphs

Theorem

If G is a disconnected graph with components $G_1, G_2, \ldots, G_k$ $(k \geq 2)$, then

$$\max_{i \in \{1,2,\ldots,k\}} \chi_d(G_i) + k - 1 \leq \chi_d(G) \leq \sum_{i=1}^{k} \chi_d(G_i),$$

and these bounds are sharp.

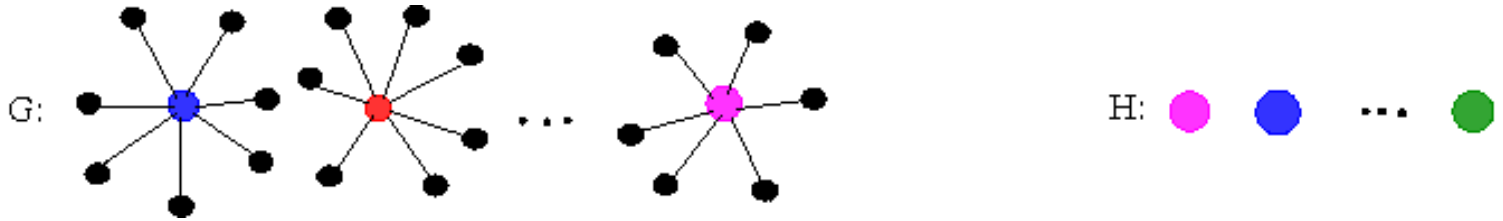


Figure: lower bound: each $G_i \cong K_{1,n}$; upper bound each $G_i \cong K_1$

We Now Study Dominator Chromatic Number for Connected Graphs Only

Bounds for Dominator Chromatic Number in Connected Graphs

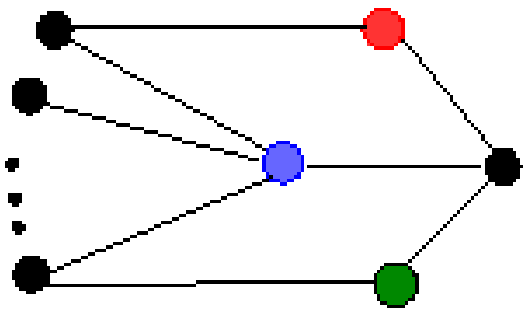
Theorem

For a connected graph G we have:

$$\chi_d(G) \leq n + 1 - \alpha(G),$$

where $\alpha(G)$ is the independence number.

Proof:



Bounds for Dominator Chromatic Number in Connected Graphs

Theorem

For a connected graph G we have:

$$\max\{\chi(G), \gamma(G)\} \leq \chi_d(G) \leq \gamma(G) + \chi(G),$$

where $\chi(G)$ is the chromatic number,
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Proof: A dominator coloring needs to be a proper coloring.

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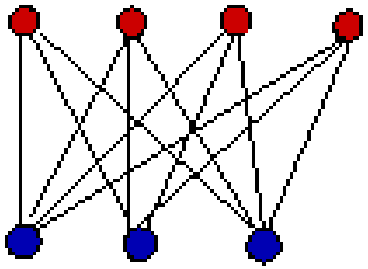

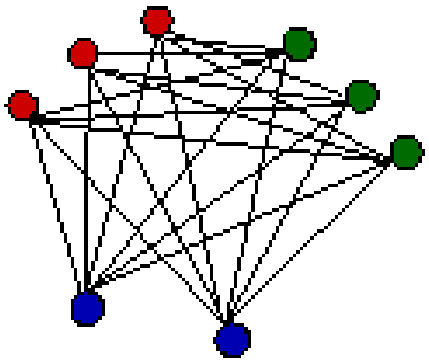

Let c be a minimum dominator coloring of G with colors $1, 2, \ldots, \chi_d(G)$.

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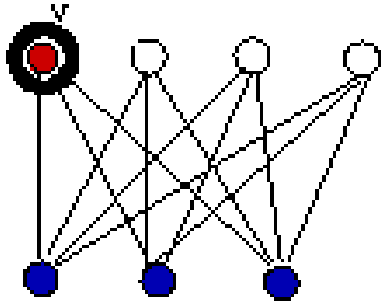
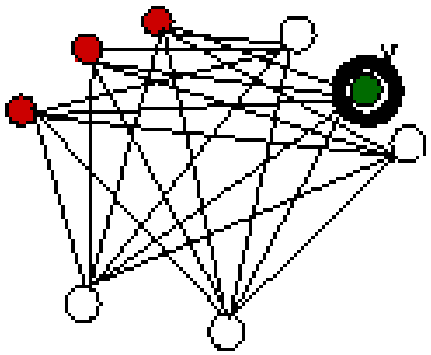
Proof (Continued):





Let $v \in V(G)$. Then v dominates a color class, say color class i.

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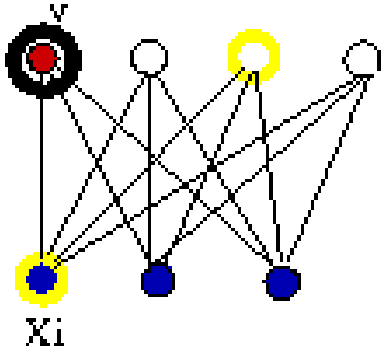

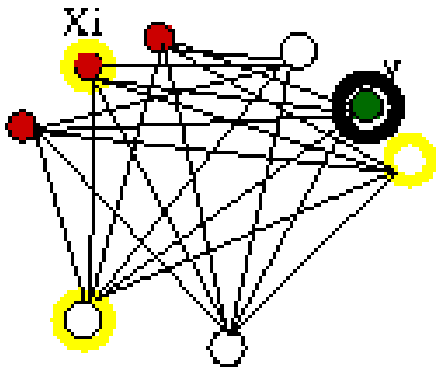

Let $S = \{x_i : 1 \leq i \leq \chi_d(G)\}$, where x_i is any vertex in color class i.

Bounds for Dominator Chromatic Number in Connected Graphs

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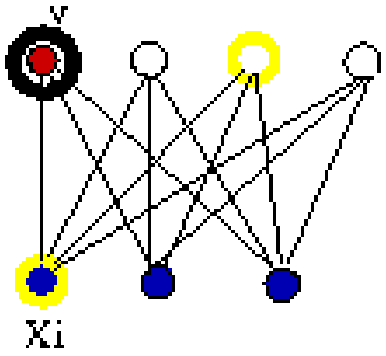
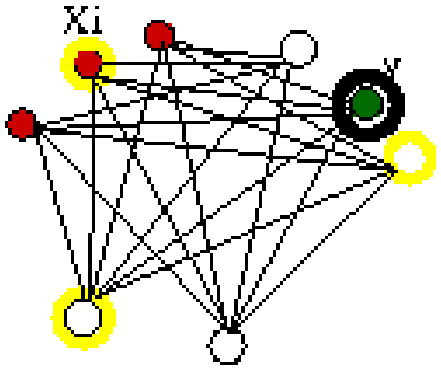
Proof (Continued):





We show that S is a dominating set: v dominates color class $i \Rightarrow v$ is dominated by the color class i (by x_i).

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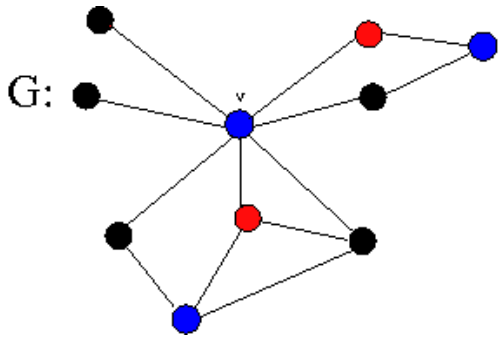

Find a proper coloring for G with $\chi(G)$ colors.

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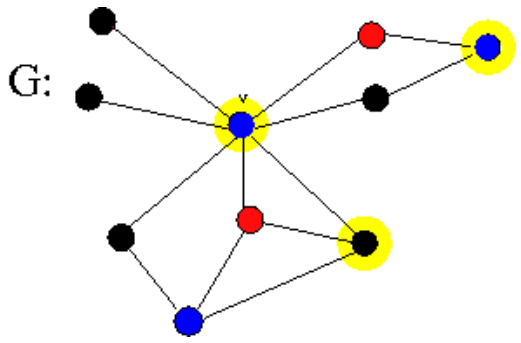

Identify a minimum dominating set.

Bounds for Dominator Chromatic Number in Connected Graphs

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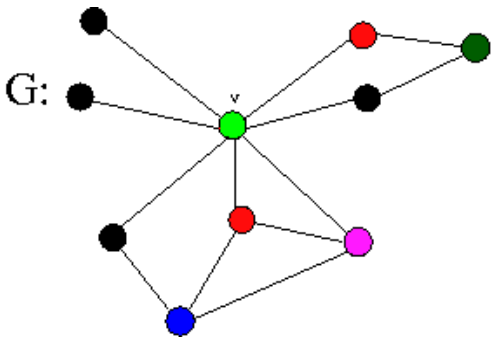

Assign color $\chi(G) + i$ to the vertices of a minimum dominating set $1 \leq i \leq \gamma(G)$.

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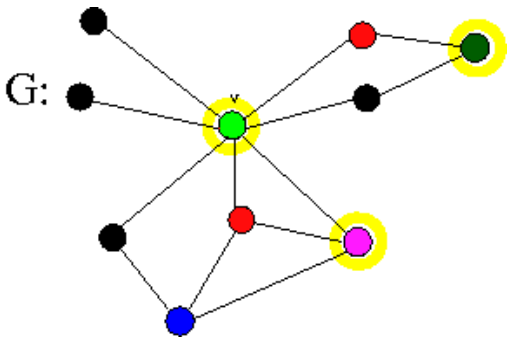

The dominating set provides the color class that every vertex dominates.

Bounds for Dominator Chromatic Number in Connected Graphs

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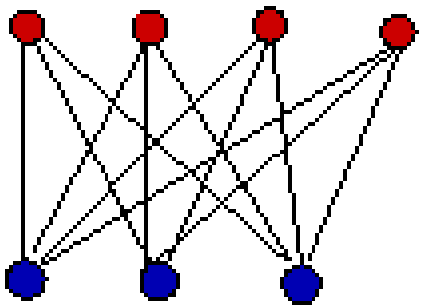
Sharpness of lower bound:



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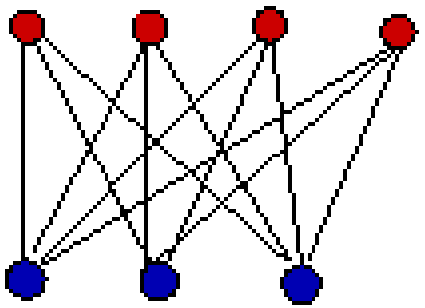

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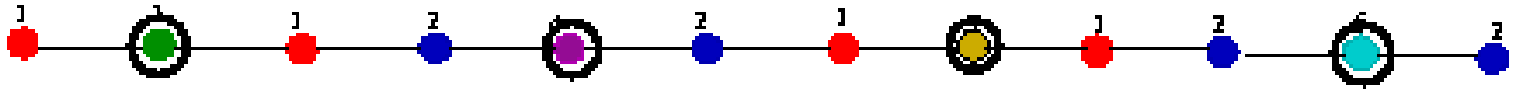

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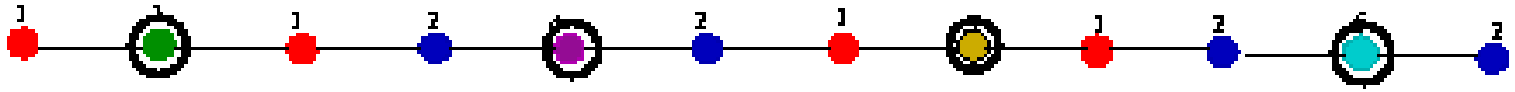
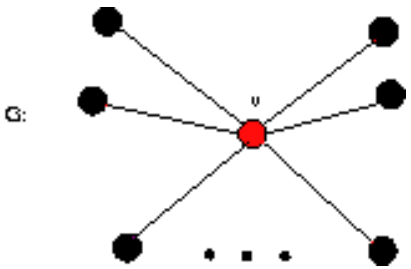


Dominator Chromatic Number for Classes of Graphs

* stars $\chi_d(K_{1,n}) = 2$



* the multistar $K_n(a_1, a_2, \ldots, a_n)$ has $\chi_d(K_n(a_1, a_2, \ldots, a_n)) = n + 1$

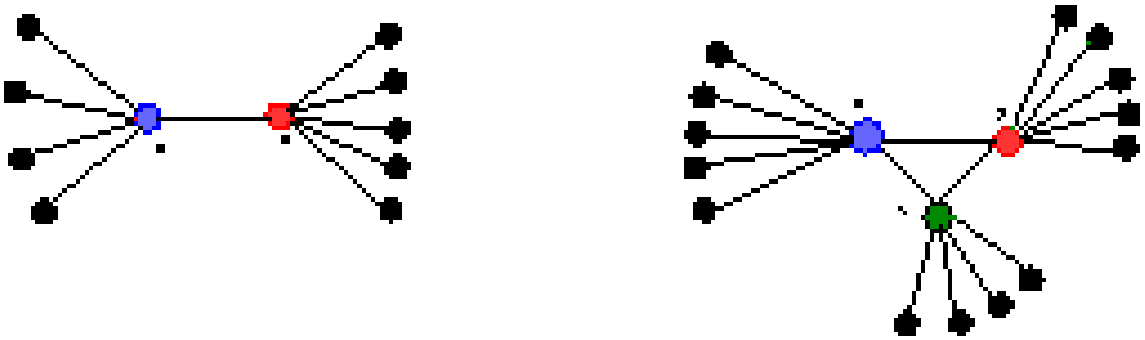


Figure: $K_2(4, 5)$ and $K_3(4, 5, 5)$

Dominator Chromatic Number for Classes of Graphs

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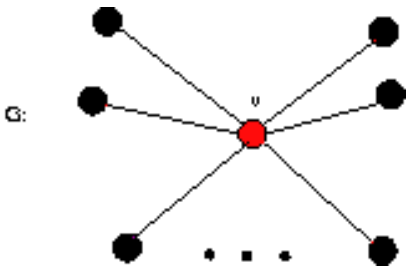

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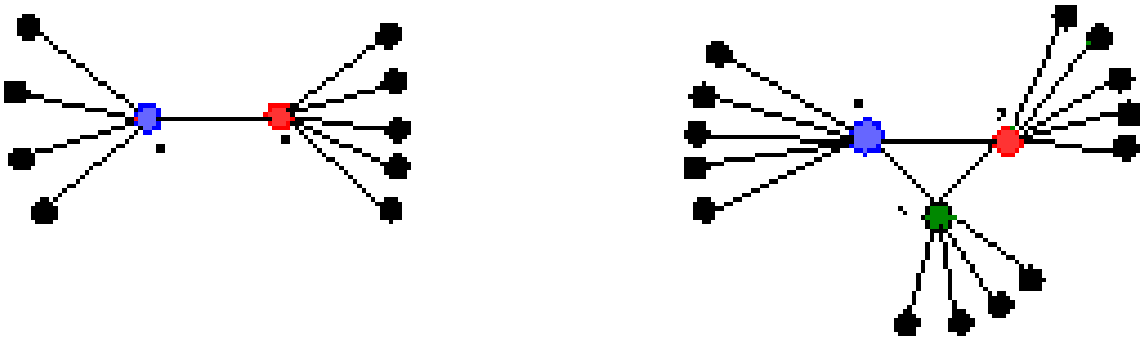
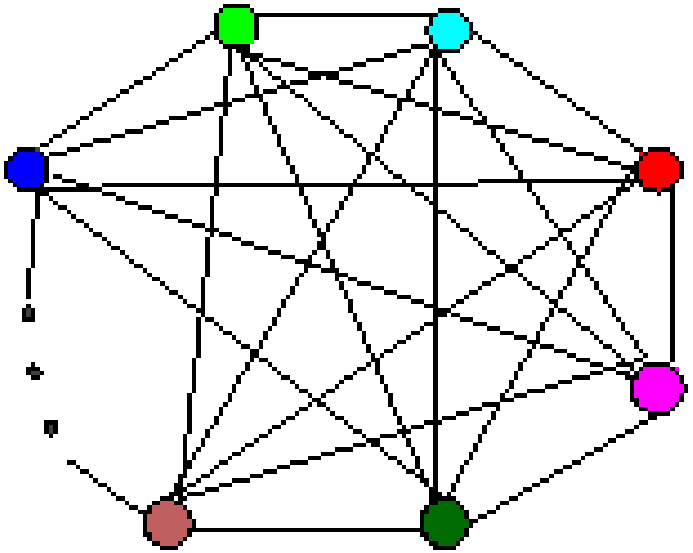


Figure: $K_2(4, 5)$ and $K_3(4, 5, 5)$

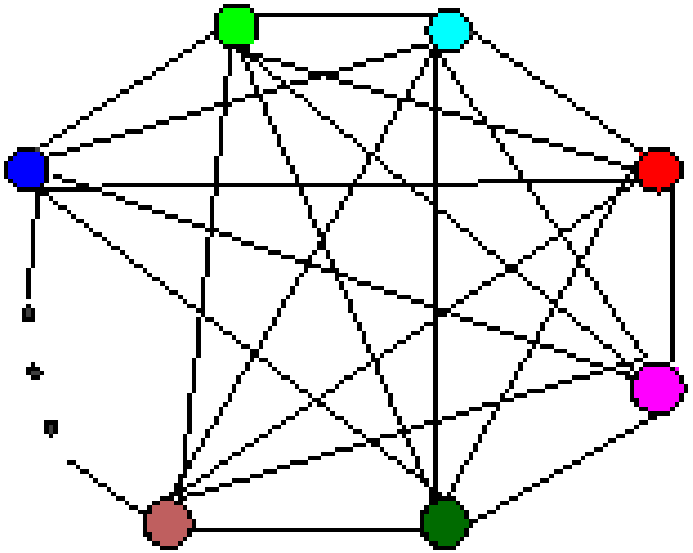
Dominator Chromatic Number for Complete Graphs

$$\chi_d(K_n) = n$$



Dominator Chromatic Number for Complete Graphs

$$\chi_d(K_n) = n$$



Dominator Chromatic Number for Paths

$$\chi_d(P_n) = \begin{cases} 1 + \lceil \frac{n}{3} \rceil; & n = 2, 3, 4, 5, 7 \\ 2 + \lceil \frac{n}{3} \rceil; & \textit{otherwise}. \end{cases}$$

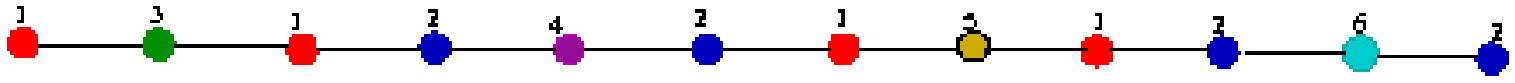


Figure: $\chi_d(P_{12}) = 6$

Dominator Chromatic Number for Paths

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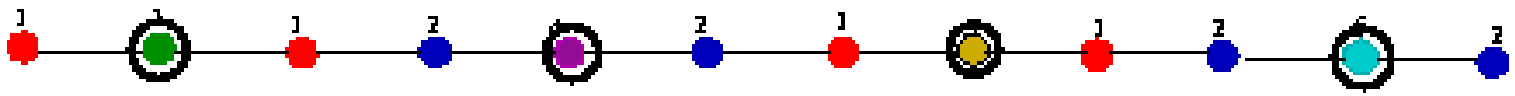

Figure: $\chi_d(P_{12}) = 6$

Dominator Chromatic Number for Cycles

$$\chi_d(C_n) = \begin{cases} \lceil \frac{n}{3} \rceil; & n = 4 \\ \lceil \frac{n}{3} \rceil + 1; & n = 5 \\ \lceil \frac{n}{3} \rceil + 2; & \textit{otherwise}. \end{cases}$$

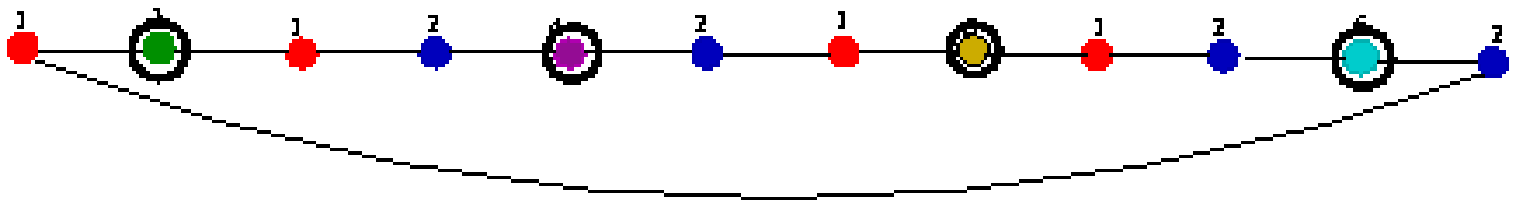

Figure: $\chi_d(P_{12}) = 6$

Dominator Chromatic Number for Classes for the Complete n-partite Graph

The complete n-partite graph $K_{a_1,a_2,\ldots,a_n}$ has $\chi_d(K_{a_1,a_2,\ldots,a_n}) = n$

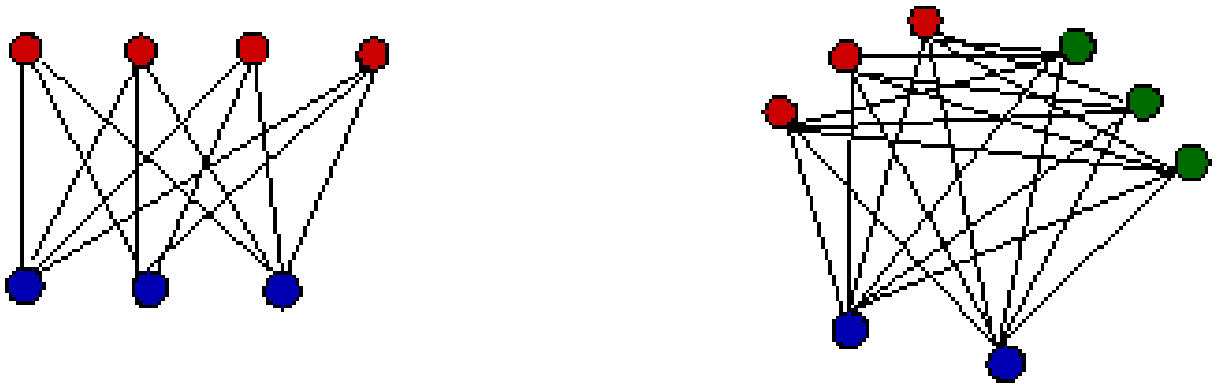

Figure: complete bipartite and 3-partite graphs

Dominator Chromatic Number for the Wheel

$$\chi_d(W_{1,n}) = \begin{cases} 3 & \text{if } n \text{ is even} \\ 4 & \text{if } n \text{ is odd.} \end{cases}$$

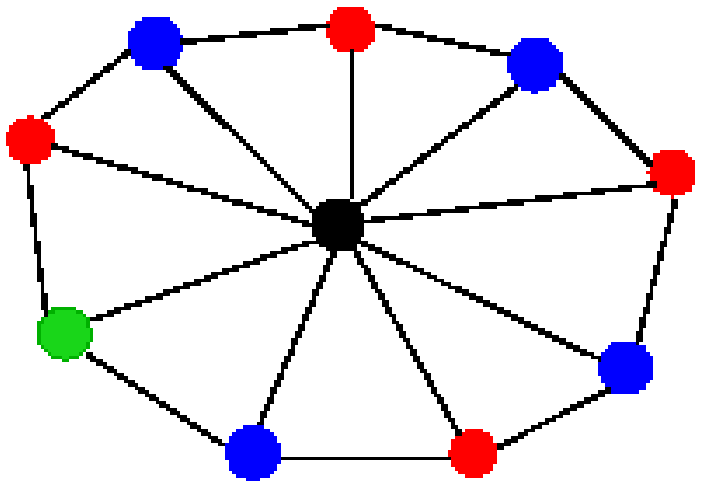

Figure: $\chi_d(W_{1,9}) = 4$

Dominator Chromatic Number for Classes for Caterpillars

Caterpillars in which the vertices of degree less than or equal to 2 are independent:

$$\chi_d(G) = r + 1,$$

where the spine has r vertices of degree at least 3.

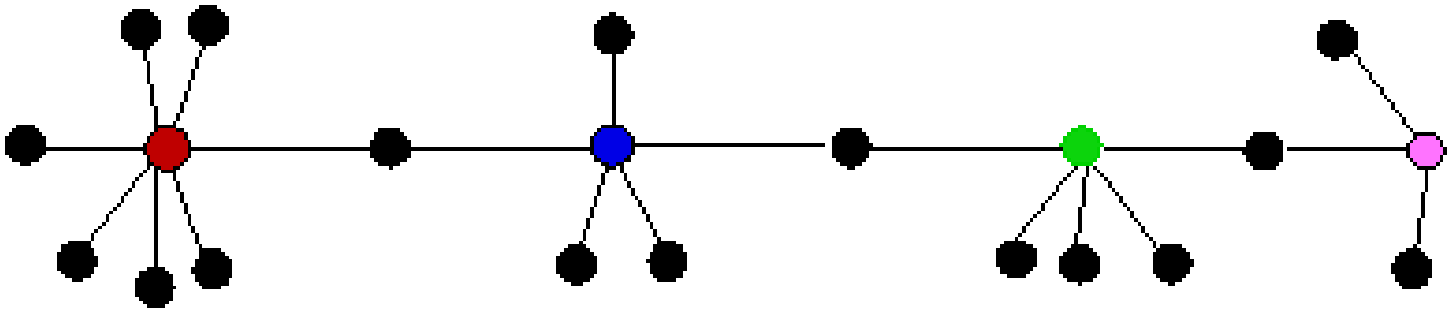

Figure: An example of such a caterpillar with $\chi_d(G) = 5$

Dominator Chromatic Number for Classes for Caterpillars

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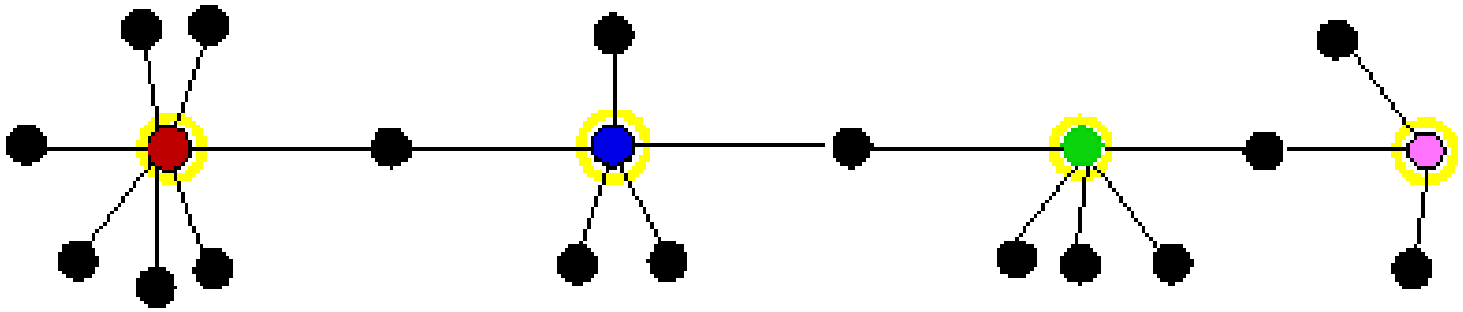

Figure: An example of such a caterpillar with $\chi_d(G) = 5$

Dominator Chromatic Number for Hypercubes

Let Q_n be the n-cube of dimension $n \geq 3$.
Then $\chi_d(Q_3) = 4$,

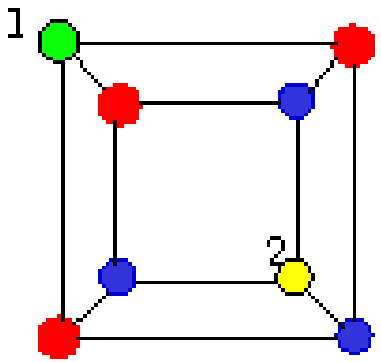

Figure: $\chi_d(Q_3) = 4$

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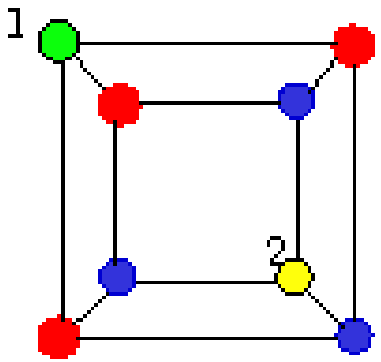

Figure: $\chi_d(Q_3) = 4$

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Dominator Chromatic Number for Hypercubes

Let Q_n be the n-cube of dimension $n \geq 3$. Then

$$\chi_d(Q_{n+1}) \leq \chi_d(Q_n) + \gamma(Q_n),$$

with $\chi_d(Q_3) = 4$.

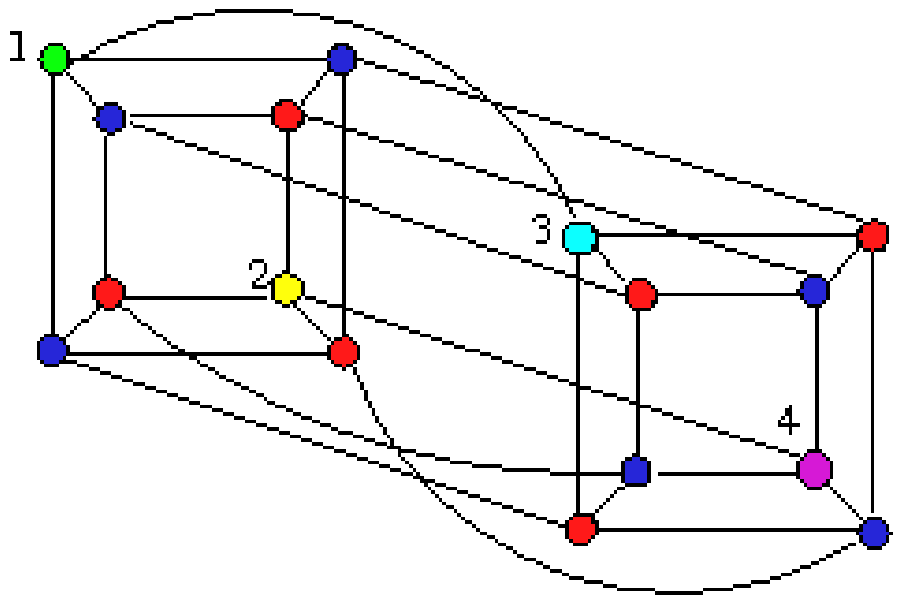

Figure: $\chi_d(Q_4) \leq 6 = \gamma(Q_3) + \chi_d(Q_3)$

Dominator Chromatic Number for Hypercubes

CONJECTURE:
Let Q_n be the n-cube of dimension $n \geq 3$. Then

$$\chi_d(Q_{n+1}) = \chi_d(Q_n) + \gamma(Q_n),$$

with $\chi_d(Q_3) = 4$.

Dominator Chromatic Number for Bipartite Graphs

Corollary

If G is a bipartite, then $\chi(G) = 2$, and so

$$\gamma(G) \leq \chi_d(G) \leq 2 + \gamma(G).$$

We define G to be of class i if $\chi_d(G) = \gamma(G) + i$ $(0 \leq i \leq 2)$.

Dominator Chromatic Number for Bipartite Graphs

A graph G of class 0 (i.e. if $\chi_d(G) = \gamma(G)$)

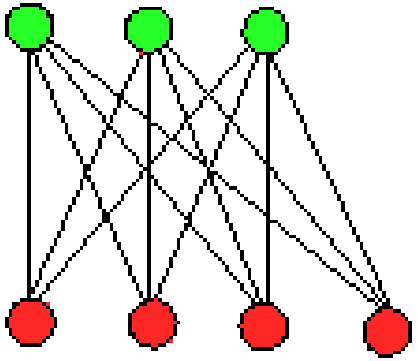

Figure: A graph of class 0.

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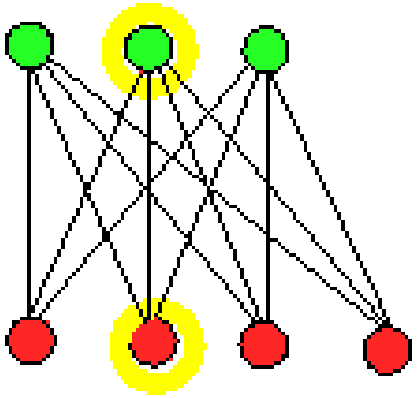

Figure: A graph of class 0.

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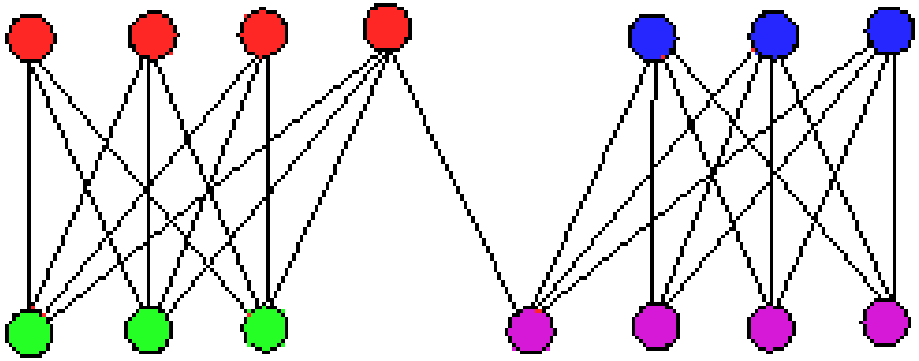

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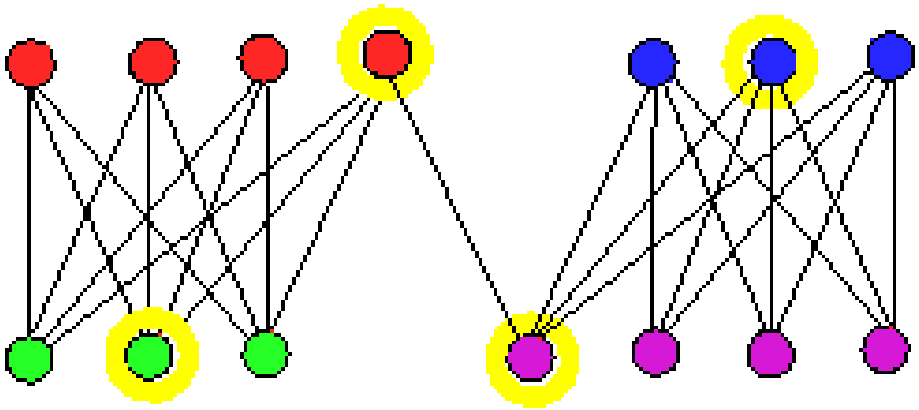


Figure: A graph of class 0.

Dominator Chromatic Number for Bipartite Graphs

A graph G of class 1 (i.e. if $\chi_d(G) = \gamma(G) + 1$)

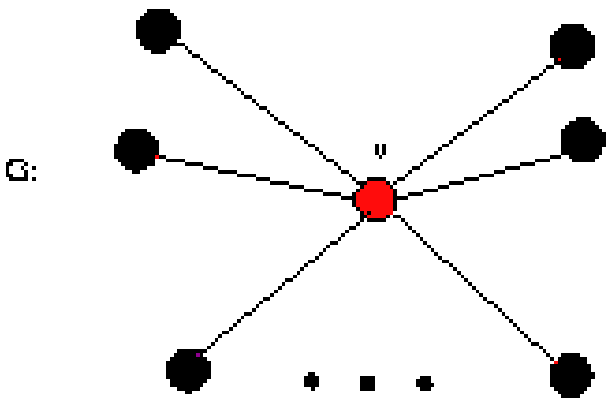
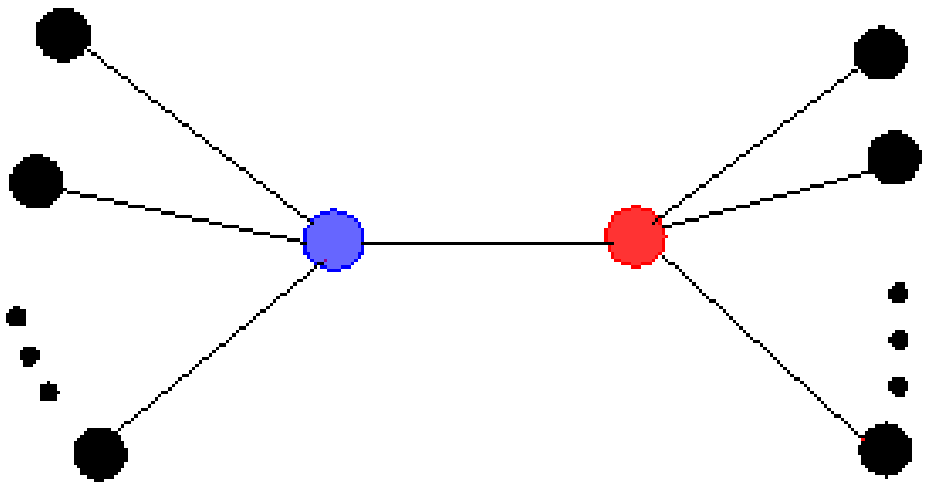


Figure: A graph of class 1.

Dominator Chromatic Number for Bipartite Graphs

A graph G of class 1 (i.e. if $\chi_d(G) = \gamma(G) + 1$)



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Dominator Chromatic Number for Bipartite Graphs

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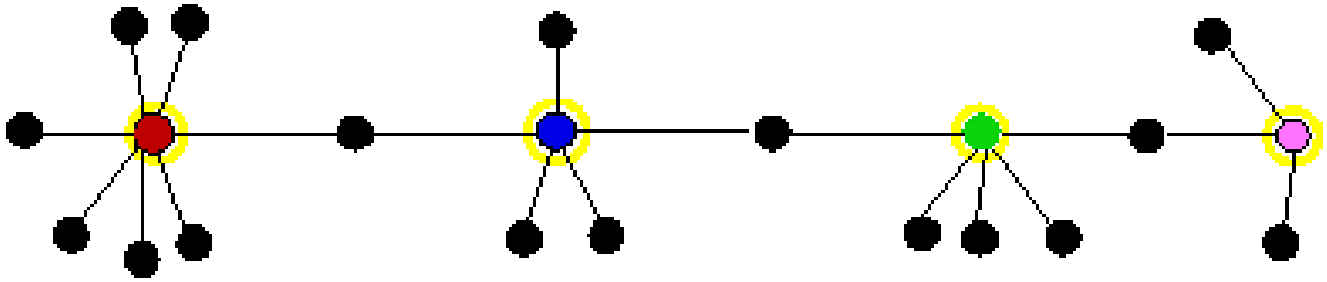

Figure: A graph of class 1.

Dominator Chromatic Number for Bipartite Graphs

A graph G of class 2 (i.e. if $\chi_d(G) = \gamma(G) + 2$)

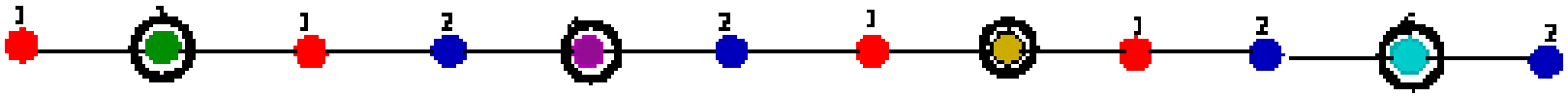

Figure: A graph of class 2, for $n \geq 8$.

Dominator Chromatic Number for Bipartite Graphs

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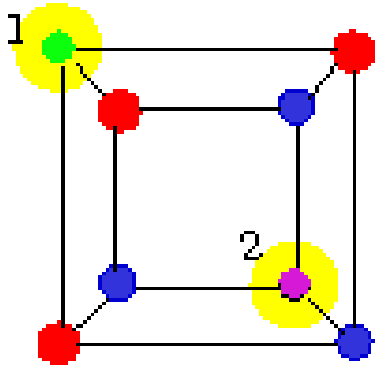

Figure: A graph of class 2.

Dominator Chromatic Number for Bipartite Graphs

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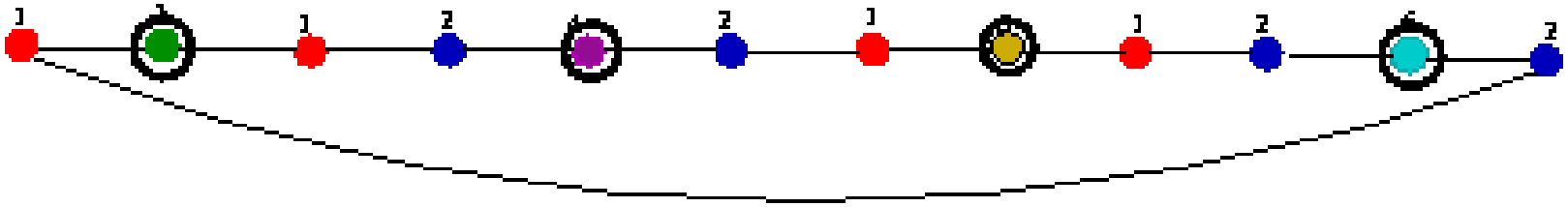

Figure: A graph of class 2, for $n \geq 6$.

Bounds for Dominator Chromatic Number in Connected Graphs

Theorem

For a nontrivial connected graph G: $2 \leq \chi_d(G) \leq n$.

Moreover, all pairs (k, n) with $2 \leq k \leq n$ are realizable as the dominator chromatic number and order of some connected graph G.

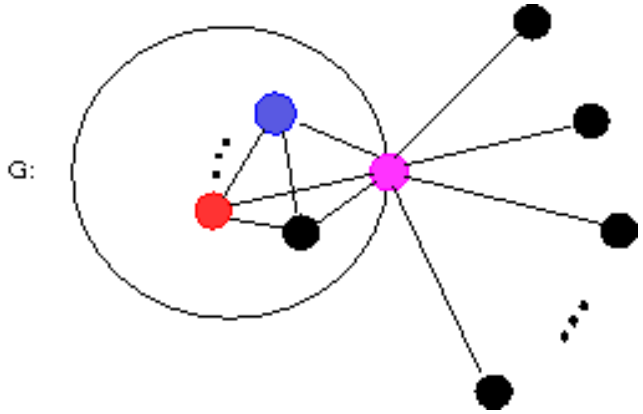

Figure: Realization graph

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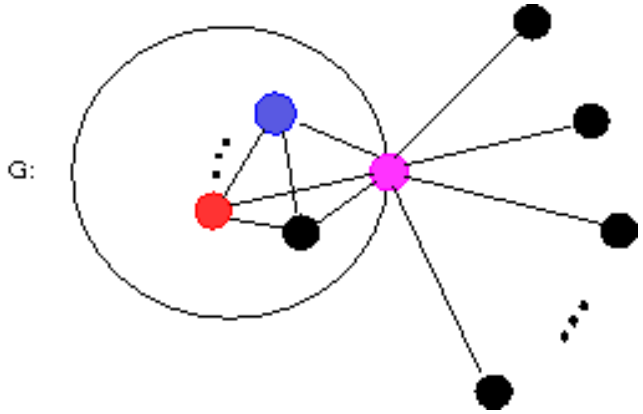
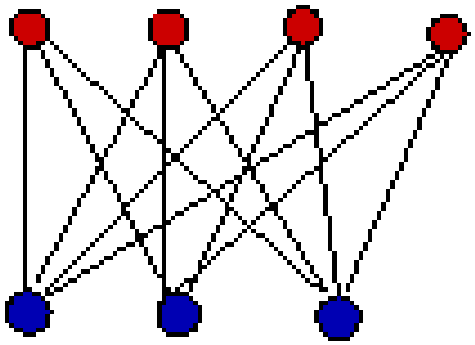


Figure: Realization graph

Characterization for the Bounds for in $2 \leq \chi_d(G) \leq n$.

Theorem

$\chi_d(G) = 2$ *if and only if* $G = K_{a,b}$ *for* $a, b \in \mathbb{N}$



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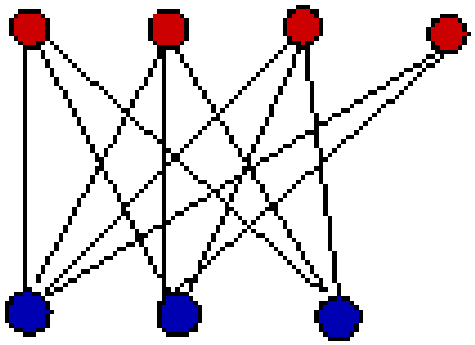

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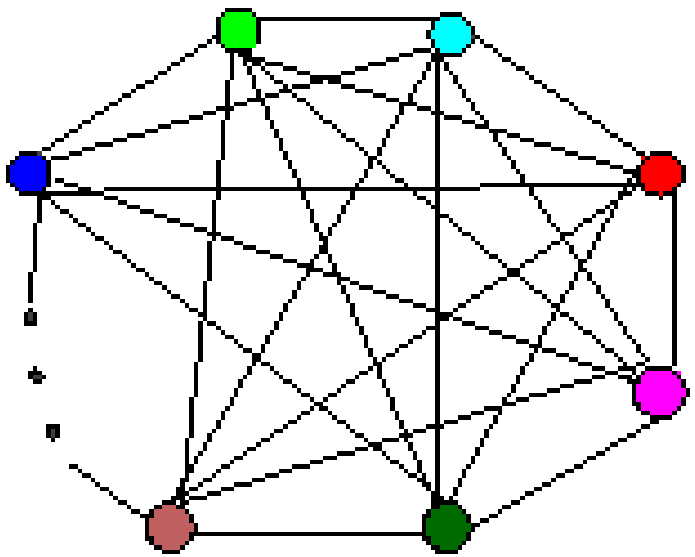

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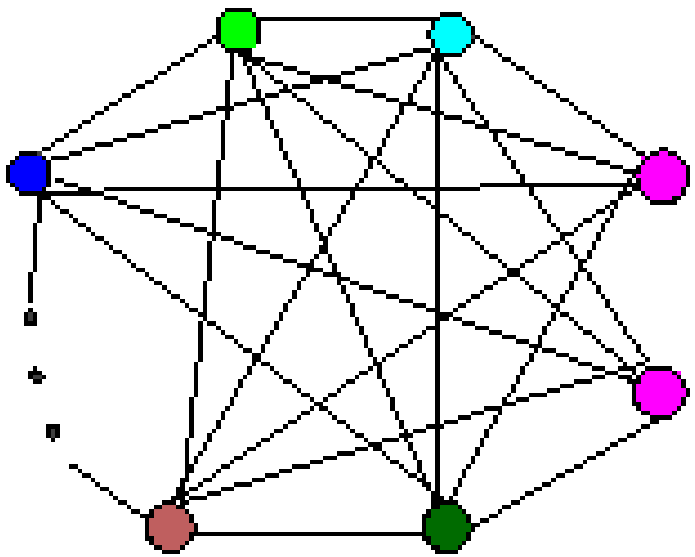

Theorem

Let G be a connected graph. If $\gamma(G) = 1$, then $\chi(G) = \chi_d(G)$, and every pair $(1, a)$ with $a \geq 1$ is realizable as $\gamma(G) = 1$ and $\chi(G) = \chi_d(G) = a$.

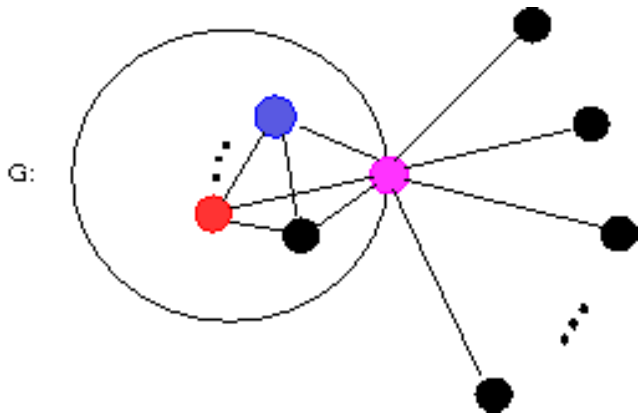

Figure: Realization graph

Triple realization

Theorem

For each ordered triple of integers (a, b, c) $(c \geq a \geq 1, c \geq b \geq 2, c \leq a + b)$ *there is a connected graph* G *with*

$$\gamma(G) = a, \chi(G) = b, \text{ and } \chi_d(G) = c.$$

Proof:

If $a = 1$, the result follows by the previous Proposition.

Thus assume that $c \geq a \geq 2$, and $c \geq b \geq 2$.

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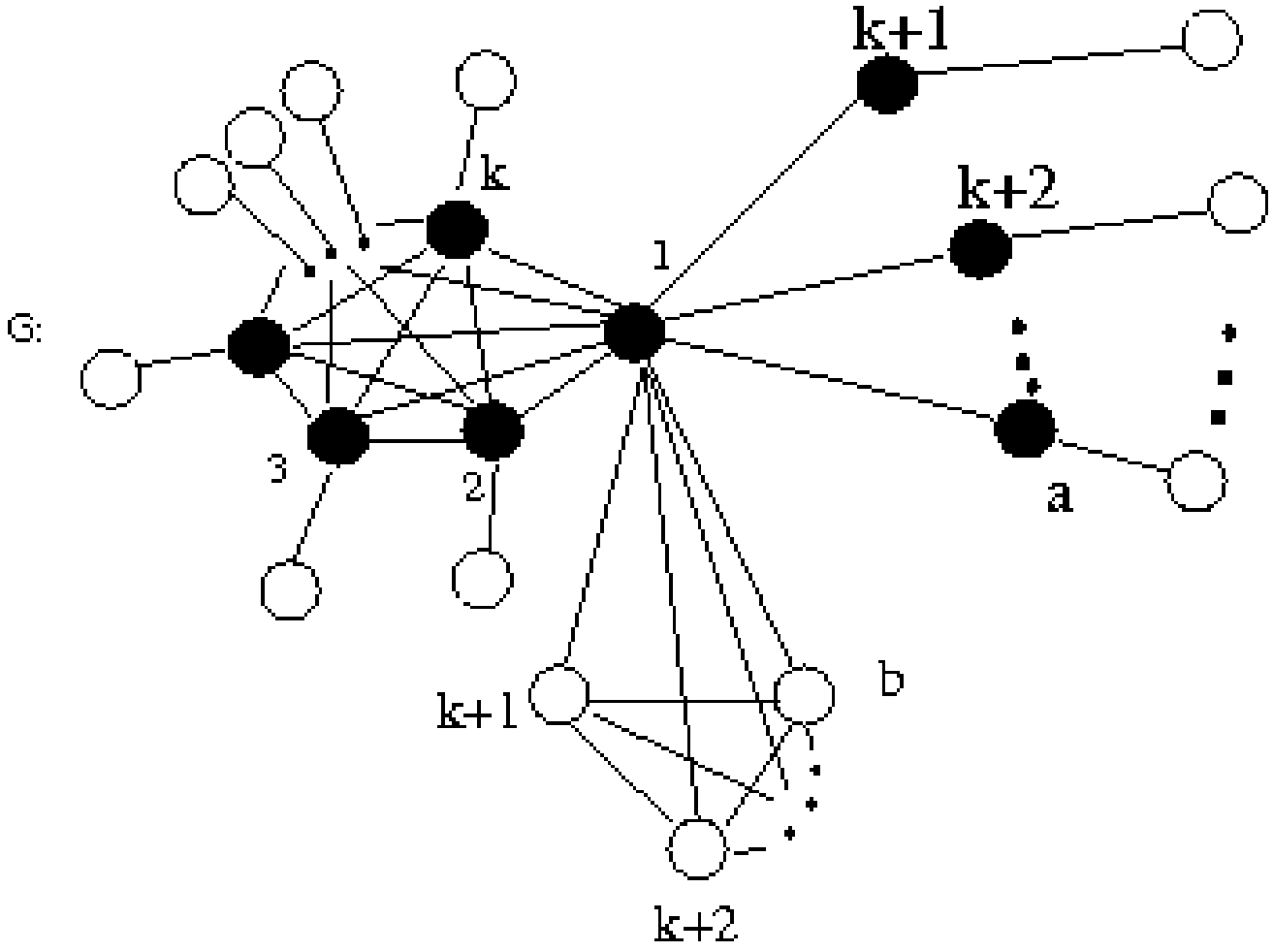

Figure: $\gamma(G) = a$

Triple realization

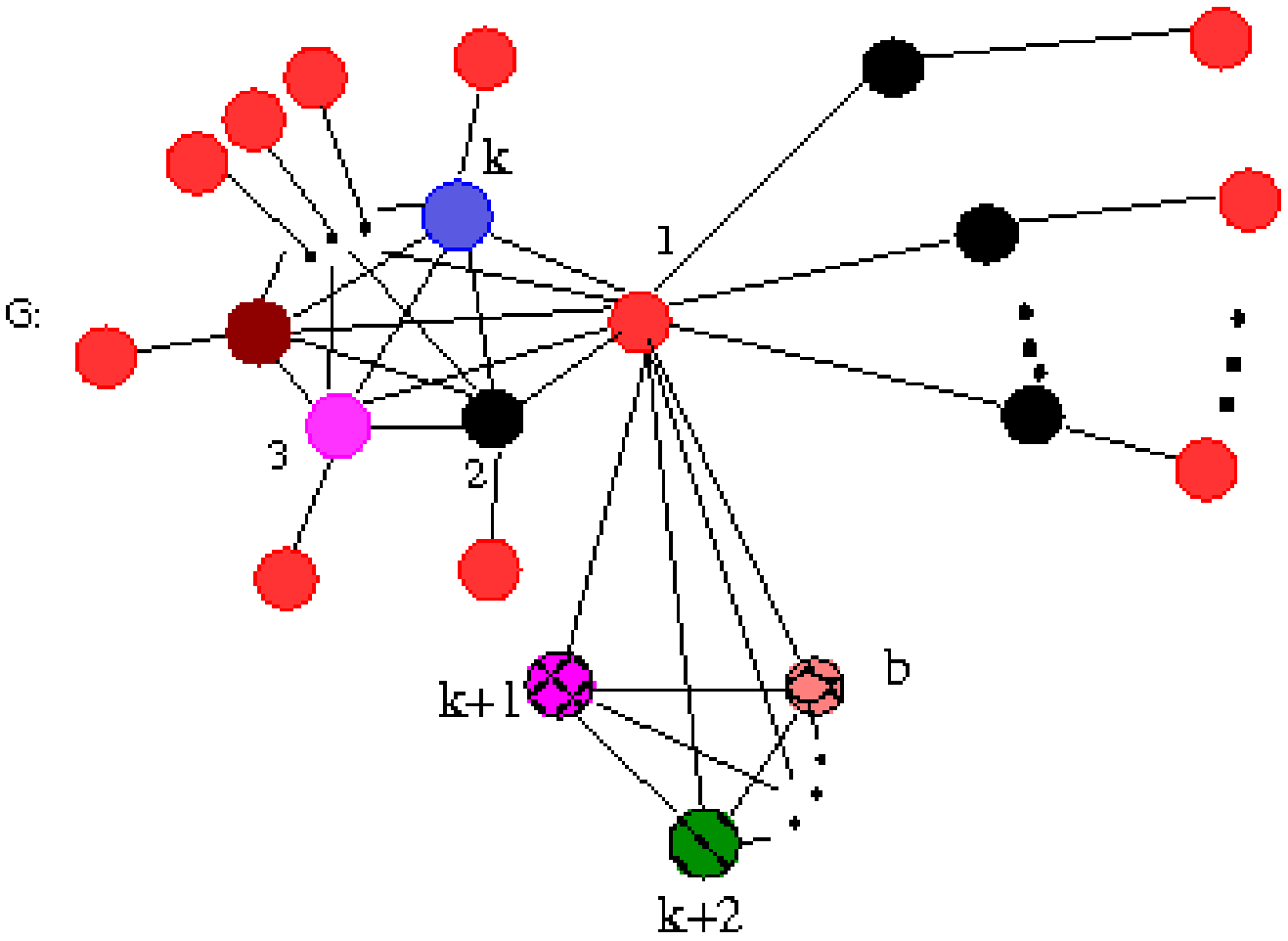

Figure: $\chi(G) = b$

Triple realization

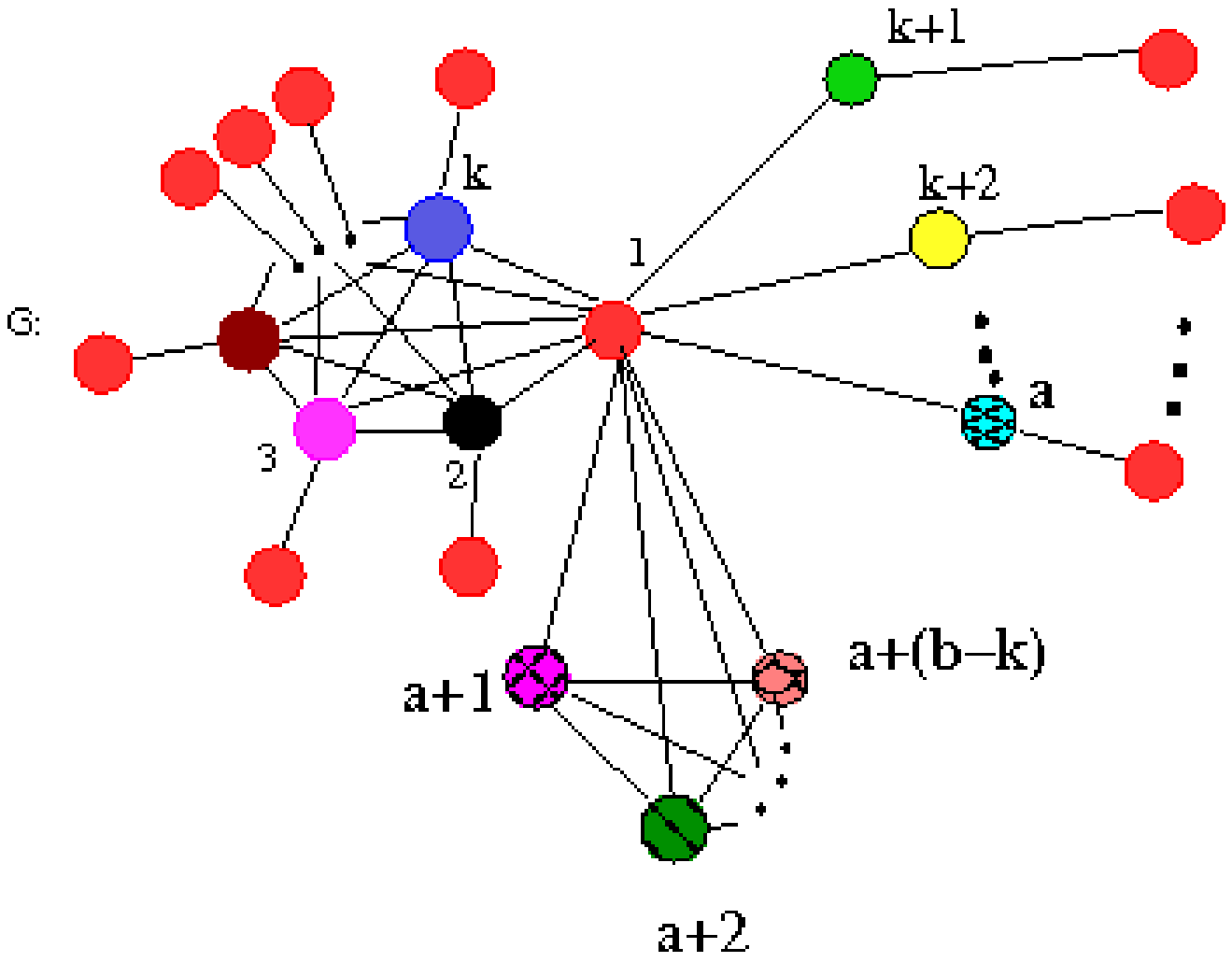

Figure: $\chi_d(G) = a + b - k$

Perhaps a Surprise...

If H is a subgraph of G it is clear that $\chi(H) \leq \chi(G)$ since any proper coloring of G will be proper for H.

Theorem

However, if H is a subgraph of some graph G, then $\chi_d(H) > \chi_d(G)$ is possible!

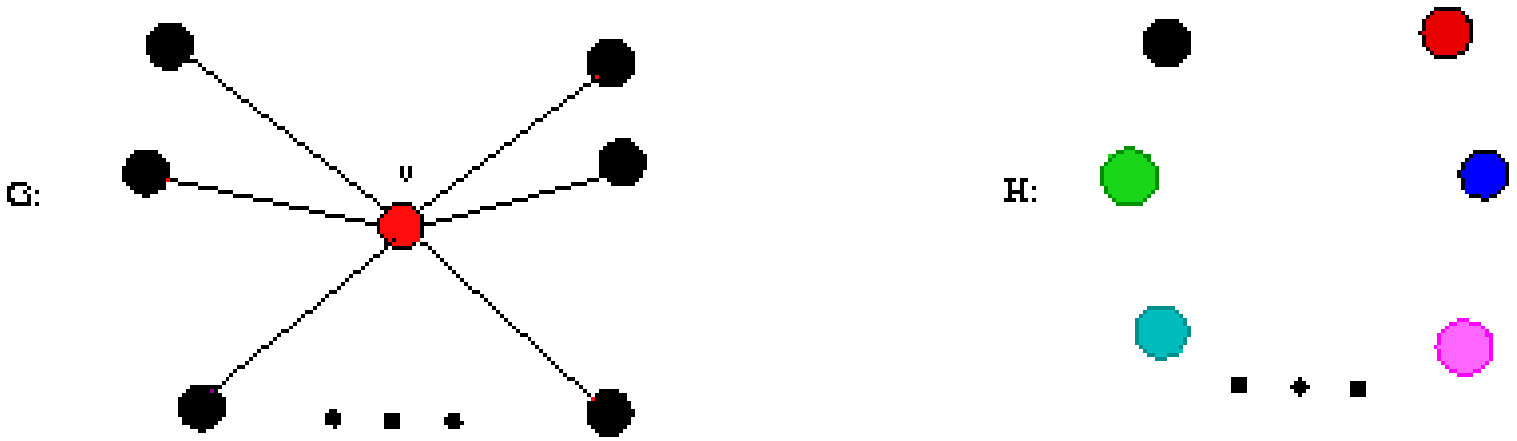

Figure: $\chi_d(G) = 2 < \chi_d(H) = n$

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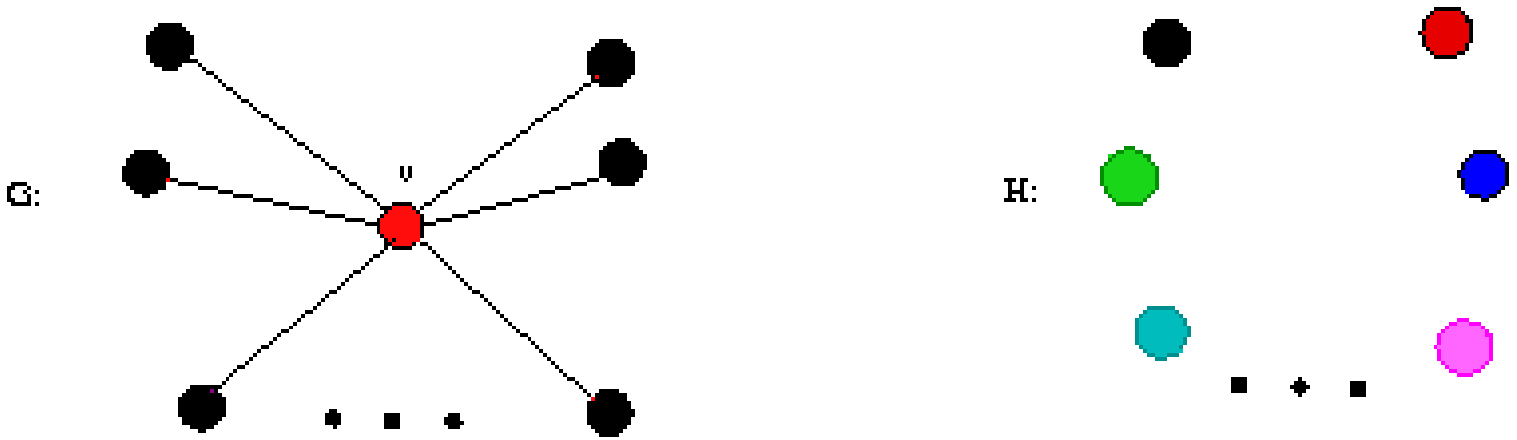

Figure: $\chi_d(G) = 2 < \chi_d(H) = n$

Outline

1 Intro
- Domination in graphs
- Colorings in graphs
- Dominator Colorings in graphs

2 Results
- Dominator Chromatic Number
- Safe Clique Partition

Proper Colorings and Partitions

Definition

For a graph G, the complement $\bar{G}$ of G is the graph that has the same vertex set as G, and all the edges not present in G. That is, an edge between two vertices is either in G or in $\bar{G}$.

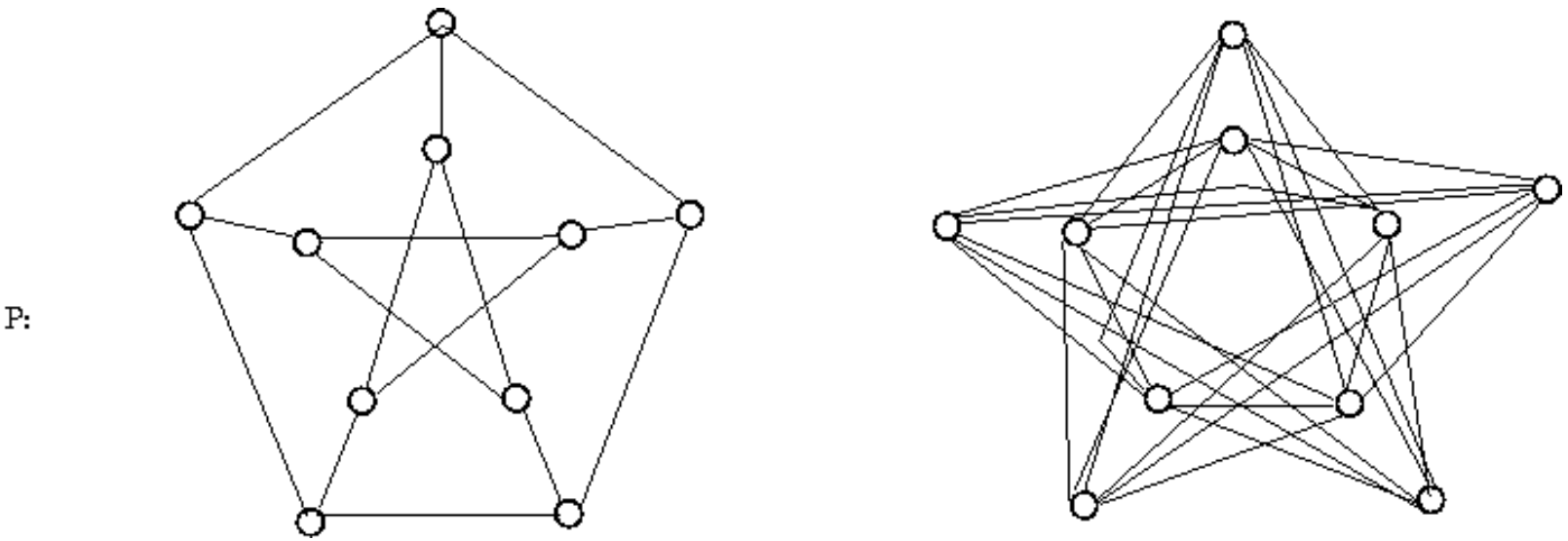

Figure: A graph and its complement

Colorings as Partitions

Definition

A coloring of G can also be thought of as a partition of V into color classes $V_1, V_2, \ldots, V_q$.

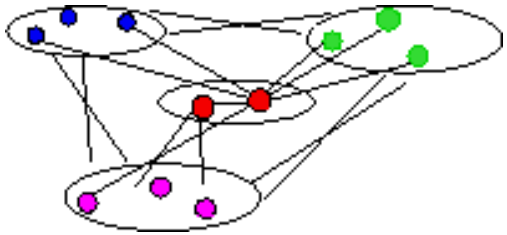

Figure: A coloring of G is a partition of $V(G)$ into classes

Definition

A proper coloring of G is then a coloring in which each V_i, $1 \leq i \leq q$ is an independent set of G.

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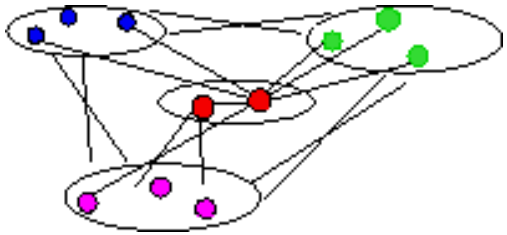

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Clique Partition

Definition

A clique partition is a partition of V into subsets $V_1, V_2, \ldots, V_q$ so that every V_i $(1 \leq i \leq q)$ is a clique in G.

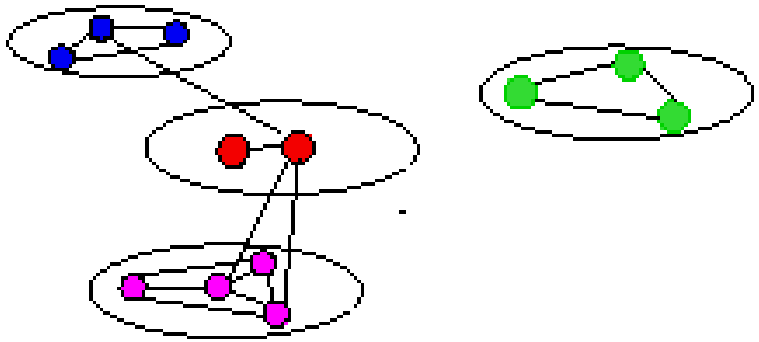

Figure: A partition of G into cliques

Definition

The clique partition number $\overline{\chi}(G)$ is the minimum number of subsets over all such partitions.

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Definition

A clique partition is a partition of V into subsets $V_1, V_2, \ldots, V_q$ so that every V_i $(1 \leq i \leq q)$ is a clique in G.

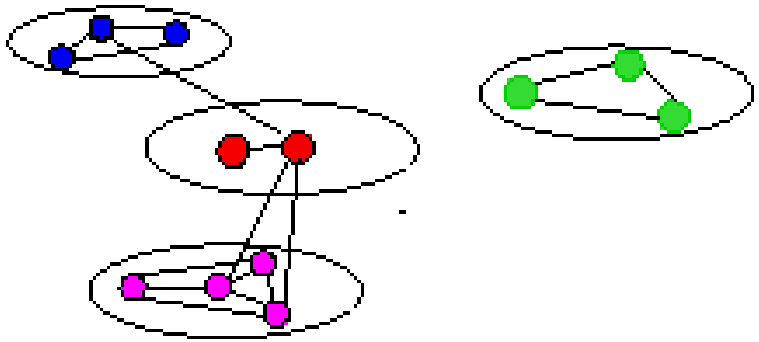

Figure: A partition of G into cliques

Definition

The clique partition number $\overline{\chi}(G)$ is the minimum number of subsets over all such partitions.

Colorings as Clique Partitions

A graph coloring on $\overline{G}$ is a clique partition on G, since the cliques of G are precisely the independent sets of $\overline{G}$

Figure: A proper coloring of G is a clique partition of $\overline{G}$

$$\overline{\chi}(G) = \chi(\overline{G})$$

Safe Clique Partition

The dominator coloring problem on a graph $\overline{G}$ can also be recast as a problem on G, called the *safe clique partition* problem:

Definition

For a clique partition $V_1, V_2, \ldots, V_q$ of a graph G, a vertex v of G is safe if there is some i $(1 \leq i \leq q)$ such that $N(v) \cap V_i = \emptyset$.

Figure: A safe vertex v in G dominates a color class in $\overline{G}$

Definition

A partition into cliques is safe if every vertex is safe.

Definition

The safe clique partition number of a graph G, $\overline{\chi_d}(G)$, is the smallest integer q for which $V_1, V_2, ..., V_q$ is a safe clique partition.

$$\overline{\chi_d}(G) = \chi_d(\overline{G}).$$

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Matchings

Definition

A matching is a set of mutually independent edges (no two edges share a common vertex).

Definition

A maximum matching M is a maximum set of independent edges.

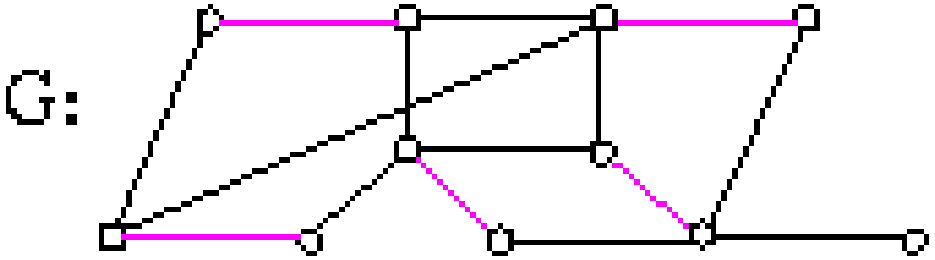

Figure: $|M| = 5$

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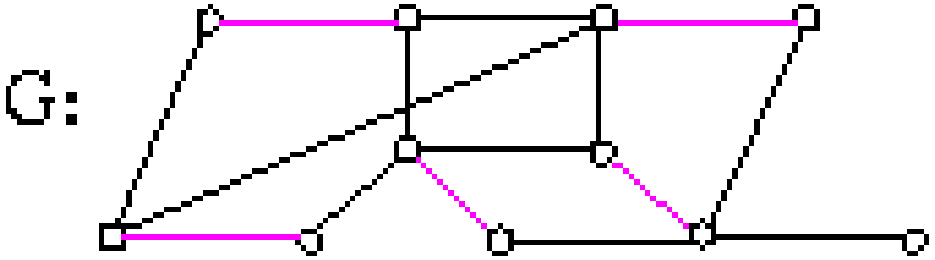

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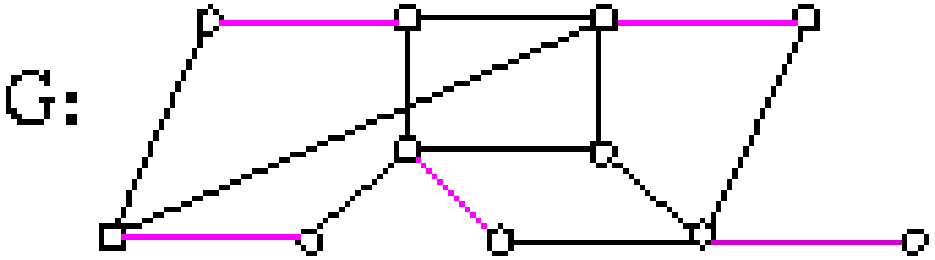

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Results on Safe Clique Partition for Connected Graphs G

Theorem

Given a triangle free graph G of order n, a maximum matching M^* corresponds to a minimum clique partition. If the latter happens to be safe, then $\overline{\chi_d}(G) = n - |M^*|$.

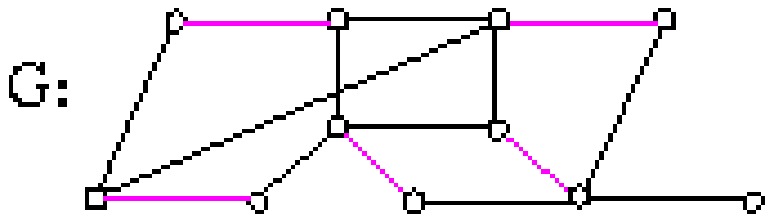


Figure: $\overline{\chi_d}(G) = 11 - 5 = 6$

Theorem

A clique partition of a tree with $n \geq 5$ can have at most one unsafe vertex. If a clique partition of a tree has an unsafe vertex, it is in a K_2, not in a K_1.

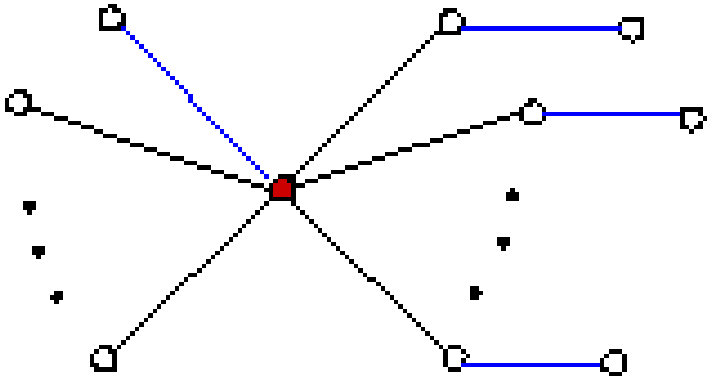
Safe Clique Partition for Trees of order $n \geq 5$

Definition

A maximum matching is safe if it corresponds to a safe clique partition; otherwise it is unsafe.

Theorem

A maximum matching M^* of a tree T is unsafe if and only if T is a wounded spider $W_{a,b}$ $(a \geq 1, b \geq 0)$ below:

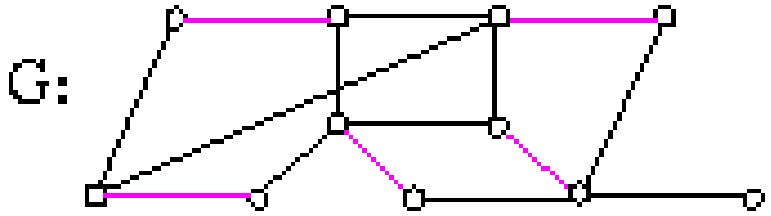


Theorem

Let T be a tree of order $n \geq 5$, and M^* be a maximum matching in G. If M^* is safe, then any other maximum matching of T is safe.

Corollary

The maximum matchings in a tree of order $n \geq 5$ are all safe or all unsafe.



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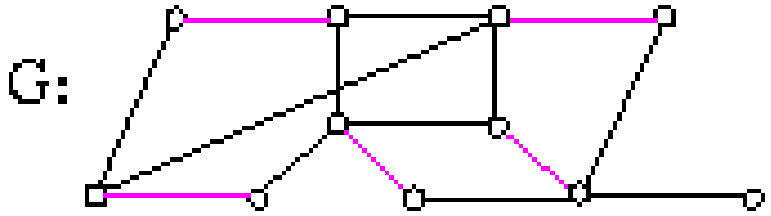

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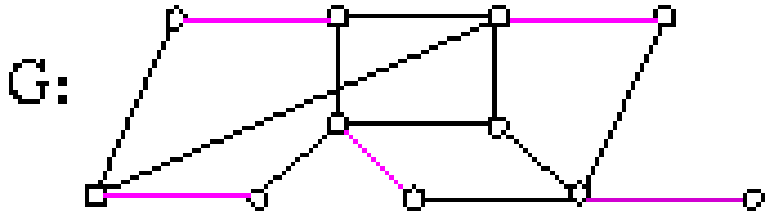

Figure: $|M| = 5$

Conclusion.

- Let G be a graph of order n whose complement $\bar{G}$ is a triangle free graph.
- Find a maximum matching of $\bar{G}$ of size k.
- The safe clique partition number of $\bar{G}$ is $n - k$.
- Dominator chromatic number of G is $n - k$.

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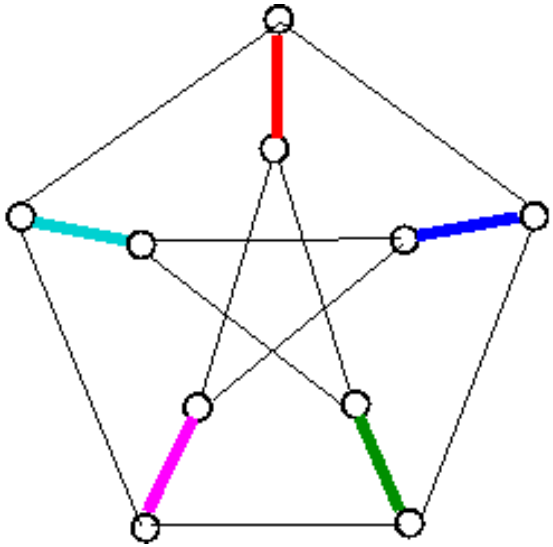
Petersen Graph and its complement.

Petersen Graph is triangle free, thus a maximum matching gives the clique partition number of the Petersen graph, which is also the dominator chromatic number of its complement.

P:



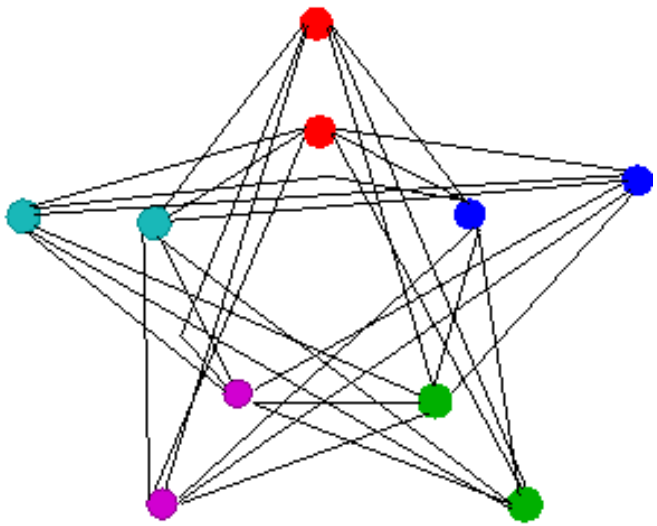


Figure: $\chi_d(\overline{P(5,2)}) = \overline{\chi_d}(P(5,2)) = n - |M^*| = 5$