BOW AND ARROW ANALYSIS

ILAP 1 - MA205P / PH251M
Project web page: http://a-math053m/ilap1.htm
Estimated time per cadet: 5 hours
Due in class 14 October 1999 (Part One only)
25 October 1999 (Parts Two through Five)
400 MA205P points; 50 PH251M points

Group Members: _____
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Group Name: _____
SSN last 4 of first member: __ __ __ __
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Your work will be graded for both MA205P and PH251M, so ensure that the physical and mathematical principles which you use are correct and clearly explained!

Background:

A relatively recent improvement over the centuries-old longbow is the *compound bow*. It contains a system of cables and cams that launches arrows at higher velocities than traditional bows. Located at opposite ends of the bow arms, the cams rotate to allow a decrease in the amount of force required to hold the drawstring at full draw.

Compound bow

In order to prepare for the upcoming hunting season, you decide to apply concepts from MA205P and PH251M courses to analyze your own compound bow and arrow system. Your bow has circular cams. Your arrows have interchangeable heads so that the same arrows can be used for target practice as well as for hunting. Target practice arrow heads have the same diameter as the arrows, while broadheads with three razor blades protruding at even intervals are used for hunting large game.

http://www.mansfieldpages.co.uk/Robin.htm

Longbow

Some data:

Mass of arrow (without arrow head)	29.20 g
Length of arrow (without arrow head)	31.0 in
Mass of target practice arrow head	9.15 g
Mass of 3-bladed broadhead	7.92 g
Draw length	75 cm
Diameter of sight pin	2 mm

Part One – Set up the experiment. **Individual work; due in class 14 October.**

(a) Describe at least three ways that you could **experimentally** determine the velocity of an arrow **in flight**. What are some advantages and disadvantages for each experimental method? What assumptions do you need to make in order to use each method?

(b) **Without actually shooting an arrow**, how could you analyze the bow and arrow system to theoretically estimate the arrow's initial velocity? What physical principles are required for this analysis? What assumptions need to be made to apply those principles?

Part Two – Arrow velocity.

(a) Calculate the velocity of a flying arrow using experimental data from the project web page. Determine the uncertainty of your calculated velocity.

(b) Use data from the Java applet on the project web page to analyze the bow to predict an arrow's theoretical velocity without firing it. How does the predicted velocity compare with experimental data? If there is a difference between predicted and experimental results, can you explain why? What are some sources of error that may account for the difference? **Use this theoretical velocity to answer the remaining questions.**

Part Three - Target practice.

(a) Based on the predicted results from Part Two (b), write a vector equation for the position of the arrow at time t seconds, given that it was launched at angle q.

(b) Determine the maximum range that an arrow launched from the bow could travel. Fully explain any assumptions and their implications.

(c) There is a limit to the angle that an arrow can be accurately launched from the bow. Based on the location of the sight pins and photos on the project web page, estimate the maximum launch angle and determine the maximum engagement range for shooting at a target with this bow.

(d) Your friend wants to try taking a few shots with your bow. His arms have different length than yours, and his sight picture changes the angle that the arrows are launched when he uses the same sight pin placement. Describe the mathematical effect of each of these factors on the maximum engagement range. If there is a 1% difference in either draw length or sighting angle, which would have the largest effect?

Part Four - Go hunting.

(a) The lethal area in which one can inflict a fatal wound on a deer is approximately 9 inches high. Failure to hit this region could result in a clean miss or a non-fatal wound to the deer. Use predicted velocity, as well as measurement error in draw length and launch angle, to determine the maximum effective range that you should take a shot. What mathematical principle is used in your calculation? What assumptions do you need to make?

(b) Some hunters choose to hunt from a deer stand attached to the trunk of a tree. If a hunter is 5 meters above his target, how should he estimate range? Should he estimate by distance on the ground (horizontal distance), or straight line distance to the target? Does hunting from an elevated stand affect the maximum effective range? Support your opinions with mathematical calculations.

(c) While sitting in the woods, you determine that the terrain elevation around your hunting site can be modeled using the function $h(x, y) = 0.1x - (.03x - 0.3)^3 - 0.002y^2 + (0.01y)^4$ where elevation and coordinates are measured in meters, the positive x direction is east, the positive y direction is north, and you are at the origin. After some time, you spot a deer at $(x, y) = (-20, 57)$ moving at 1.00 m/sec along a compass heading of $130 + a =$ ___ degrees. Trees are widely spaced, and you are not worried about accidentally hitting a tree trunk, but the overhead canopy of dense tree limbs is $4.00 + b/10 =$ ___ meters above the ground. To get the best shot, you want to hold your fire until the deer comes closest to you along its path. What is the closest that the deer will come to your position? Should you take a shot? (Remember to use the predicted arrow velocity from Part Two (b).)

Part Five - Advanced concepts.

(a) Until now, you have probably been ignoring the drag force, $\vec{D}$, which opposes motion and occurs as the arrow travels through air. If $\vec{r}(t)$ is a parameterization of the path of the arrow, assume that the force of drag is proportional to velocity, that is, $\vec{D} = k\vec{r}'(t)$ for some constant $k > 0$. This leads to an "icky" differential equation for the arrow's position which is somewhat difficult to solve. A reasonable approximation to $\vec{D}$ would consist of ignoring the vertical component of drag, since the velocity in the vertical direction is very small compared with the velocity in the horizontal direction. Further, if we ignore other aerodynamic forces acting on the arrow after it leaves the bow, then we can simplify the differential equation to $\vec{a} = \langle -kr_x', -g \rangle$. Using this approximation and $k = .01 + c/1000 =$ ___ kg/s, solve the differential equation and recalculate the maximum effective range for shooting at a deer.

(b) Your physics instructor has invented small rocket motors and wants to attach them to your arrows, and then call them "Patriot Arrows." He isn't sure whether he should adjust the rocket (which has a mass of only $10.0 + d =$ ___ grams, but provides 0.25 newtons of thrust) to always fire downward (thrust is perpendicular to level ground) or always fire straight back (thrust is parallel to level ground). A rocket firing downward would increase the arrow's flight time, increasing the arrow's range. A rocket firing straight back would increase the arrow's horizontal velocity, also increasing range. If those are his only options for rocket orientation, and assuming negligible mass change in the rocket, calculate which orientation provides the greatest increase in maximum range.