Project 3, MA104
A Bungee Jumping Problem

In bridge jumping, a participant attaches one end of a bungee cord to himself, attaches the other end to a bridge railing and then drops off of the bridge. In this problem, the jumper will be dropping off of the Royal Gorge Bridge, a suspension bridge that is 1053 feet above the floor of the Royal Gorge in Colorado. The jumper will use a **200** foot long bungee cord. It would be nice if the jumper had a safe jump, meaning that the jumper will not crash into the floor of the gorge or run into the bridge on the rebound, so you're going to do some analysis of the fall.

To begin with, the jumper weighs **160** lbs, and the jumper will free-fall until the bungee cord begins to exert a force that acts to restore the cord to its natural position. In order to determine the spring constant of the bungee cord, you found that a mass weighing **4** lbs stretches the cord **8** feet. This spring force, hopefully, will help to slow the descent sufficiently so that the jumper does not hit the floor of the gorge.

Throughout this problem, we will assume that down is the positive direction.

1. Before the bungee cord begins to retard the fall of the jumper, the only forces that act on the jumper are his weight and the force due to wind resistance.

a. If the force due to wind resistance is **.9** times velocity of the jumper, then use Newton's Second Law to write a differential equation that models the fall of the jumper.

b. Solve this differential equation and find:

1. a function that describes the jumper's velocity (as a function of time).
2. a function that describes the jumper's position (as a function of time).

c. What is the velocity of the jumper after he has fallen **200** feet?

d. What is the terminal velocity of the jumper, if any?

2. After a bit more research, you've found that the force due to wind resistance is not linear, as assumed above. Apparently, the force due to wind resistance is more closely modeled by $\mathbf{.9}v + \mathbf{.0009}v^2$.

a. Write a new differential equation governing the velocity of the jumper (prior to the bungee cord coming into effect).

b. You should notice that the new equation is no longer an easy one to solve. Nonetheless, you believe that you can determine the velocity of the jumper after 4 seconds by using a numerical solution technique. Choose a suitable *h* value, and estimate the velocity of the jumper after 4 seconds.

c. Find the terminal velocity, if any.

d. How do your results compare with those found under the first assumption?

e. What is the velocity of the jumper after he has fallen **200** feet? (Hint: what is the relationship between velocity and position?)

3. For the last part of our analysis, we'll consider what happens after the bungee cord comes into play. When the jumper reaches the point **200** feet below the bridge, the bungee cord begins to stretch. As we know, it takes some force to stretch the bungee cord, and the cord exerts an equal and opposite force upward. Hooke's law tells us that the force of the bungee cord will be directly proportional to the amount that it is stretched ($F = ks$, where F is the force exerted by the bungee cord, k is a constant of proportionality, and s is the distance that the cord is stretched beyond its normal length). To further retard the speed of the jumper, assume that there is a drag force due to air resistance. We'll assume that this force is as in requirement 1 (that is, assume that the force due to air resistance is equal to the **.9** times the velocity of the jumper).

a. Write a differential equation that models the jumper's motion after the bungee cord begins to stretch. What are the conditions that will allow you to solve the differential equation?

b. Solve the differential equation and find:

1. a function that describes the jumper's velocity (as a function of time).
2. a function that describes the jumper's position (as a function of time).

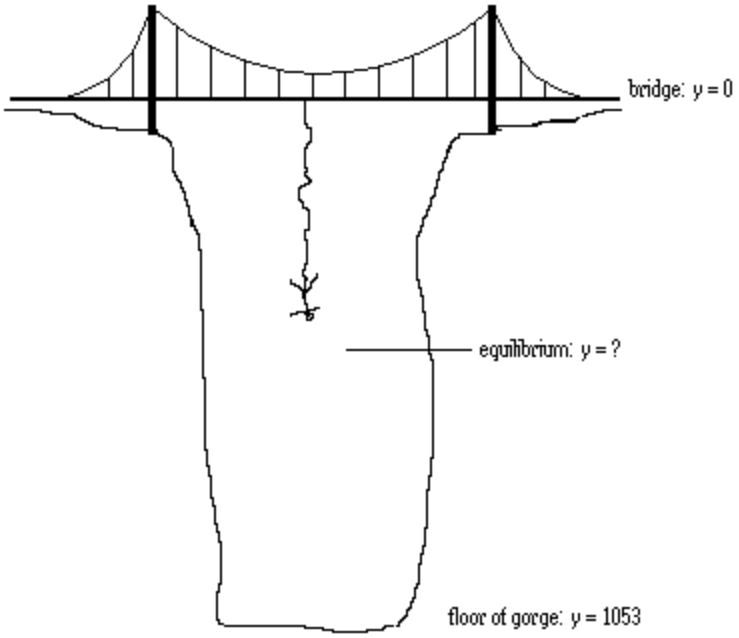
c. Does the jumper crash into the floor of the gorge? If not, then what is the distance from the floor of the gorge when the jumper begins to move upward?

d. How close to the bridge does the jumper come on the first rebound? Explain the assumptions that you make in solving this portion of the problem.

e. How far below the bridge will the jumper be once the bouncing stops?

4. What assumptions have you made in your model? What effect will these assumptions have on your results? How could you refine your model?

5. (Difficult.) How long should the bungee cord be if the jumper wants to touch the water on his jump? Make an attempt at finding the length of the cord that would make the jumper's velocity zero at the time when the jumper reaches a point 1053 feet below the bridge. You might wish to find reasonable bounds on the length and discuss how you might solve this problem.



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a. If the force due to wind resistance is ____ times velocity of the jumper, then use Newton's Second Law to write a differential equation that models the fall of the jumper.

b. Solve this differential equation and find:

1. a function that describes the jumper's velocity.
2. a function that describes the jumper's position.

c. What is the velocity of the jumper after he has fallen ____ feet?

d. What is the terminal velocity of the jumper, if any?

2. After a bit more research, you've found that the force due to wind resistance is not linear, as assumed above. Apparently, the force due to wind resistance is more closely modeled by ___v + _____v^2.

a. Use Newton's Second Law to write a differential equation governing the velocity of the jumper (prior to the bungee cord coming into effect).

b. You should notice that the new equation is no longer an easy one to solve. Nonetheless, you believe that you can determine the velocity of the jumper after 4 seconds by using a numerical solution technique. Choose a suitable *h* value, and estimate the velocity of the jumper after 4 seconds. Discuss why you think that your *h* value is suitable.

c. Find the terminal velocity of the jumper, if there is one.

d. How do your results compare with those found in part 1?

e. What is the velocity of the jumper after he has fallen ____ feet? (Hint: what is the relationship between velocity and position?)

3. For the last part of our analysis, we'll consider what happens after the bungee cord comes into play. When the jumper reaches the point _____ feet below the bridge, the bungee cord begins to stretch. As we know, it takes some force to stretch the bungee cord, and the cord exerts an equal and opposite force upward. Hooke's law tells us that the force of the bungee cord will be directly proportional to the amount that it is stretched (F = ks, where F is the force exerted by the bungee cord, k is a constant of proportionality, and s is the distance that the cord is stretched beyond its normal length). To further retard the speed of the jumper, assume that there is a drag force due to air resistance. We'll assume that this force is as in requirement 1 (that is, assume that the force due to air resistance is equal to ____ times the velocity of the jumper).

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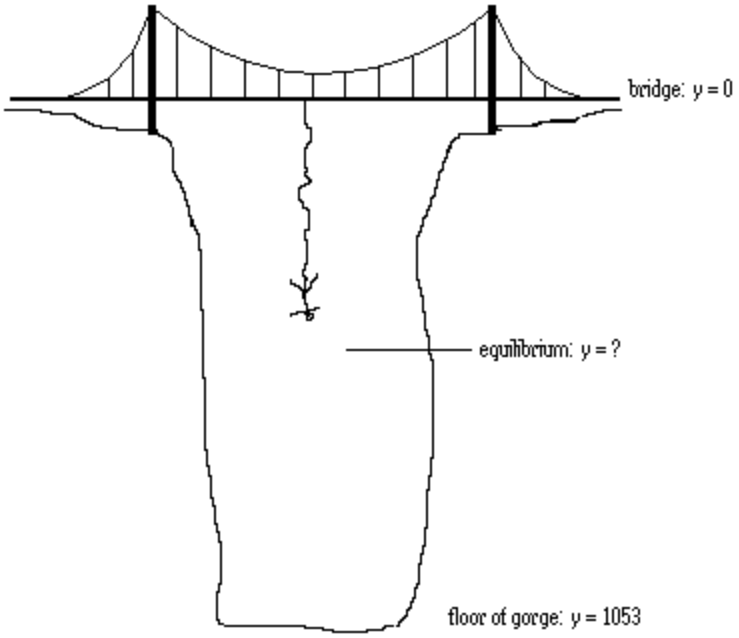
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