MA 104 PROJECT TWO, SPRING 99
"Good Vibrations"

Since you did such a great job on the firefighting mission, the 14th Engineer Battlion S-3 attaches you to the 864th Engineer Battalion (CBT HVY), 555th Engineer Group, Ft. Lewis, Washington. The 864th Engineer Battalion (CBT HVY) has the mission to design and build a new water tower (Figure 1) which will be used to replace the one damaged during the fire. This water tower will be built just outside the town of Leavenworth, Washington. The 864th Engineer Battalion commander is concerned about earthquake survivability of the water tower. He has given you the mission to research and model what would happen to the water tower during an earthquake and make recommendations on design improvements.

Figure 1. Water Tower

Your goal is to analyze the effects of an earthquake on a water tower. Catastrophic failure occurs when the top of the water tower experiences excessive lateral displacement.

Introduction

The motion created by the earthquake will cause the top of the tower to move, thereby creating a displacement and velocity at the top of the tower as shown in Figure 2. The first step in our analysis is to investigate the effect of imposing an initial displacement and velocity on the top of the tower. We will model the tank as a spring-mass system with lateral displacement $u(t)$ as shown in Figure 3.

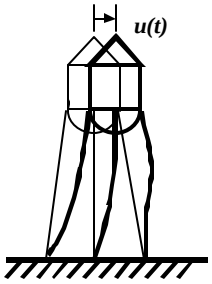

Figure 2. Lateral Displacement

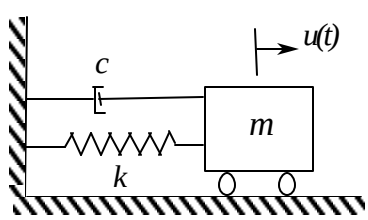

Figure 3. Spring-Mass System.

The tower supports, or the legs of the tower act as the spring. The spring constant of the tower supports depends on the material properties and dimensions of the supports. In particular, the spring constant, k, is a function of the Young's modulus of the tower, E, and the cross-sectional moment of inertia of the tower supports, I. Young's modulus is a material property that is physically the ratio of stress to strain under elastic deformations. In order to simplify our study, we shall assume that $k =$ _____ N/m. Additionally, the mass of the tank varies between $M_{\max} =$ _____ kg when the tank is full and $M_{\min} =$ _____ kg when the tank is at the minimum allowable water level.

A. Unforced Vibrations

I. Assuming that Hooke's Law holds for the spring-mass system model of the tower, use Newton's Second Law to derive the differential equation describing the lateral displacement of the top of the tower. Assume that the earthquake imposes only an initial displacement of $d_0 =$ _____ m and initial velocity $v_0 =$ _____ $m/\sec$. Provide a plot of the lateral displacement of the top of the tower for the first _____ seconds of motion.

II. In order to make our mathematical model more realistic, we need to account for internal damping of the tower. Damping causes the lateral displacements to diminish over time. The dashpot shown in Figure 3 is used to denote the presence of this phenomenon. There are many mechanisms within a structure which determine its damping characteristics. Viscous damping is a form of damping in which the natural energy dissipation of the structure is assumed to be proportional to the velocity of motion. The proportionality constant c, is typically referred to as the damping coefficient. Assuming that the damping coefficient is $c =$ _____ $N-\sec/m$, refine your differential equation model of the tower system to include viscous damping and plot the lateral displacement for the first ____ seconds. Describe the long-term effects of viscous damping when the tank is full. Investigate how decreasing the water level in the tank affects the long-term behavior of the water tower's motion in terms of the various types of damping. What type of damping occurs? What mass of the tank will cause critical damping?

III. We will now assume that the tower undergoes Coulomb damping. In this case while the tower is in motion, a constant force, which is **independent** of the velocity, creates damping. This constant force always **opposes** the motion. Use this concept to refine your differential equation model of the tower to include Coulomb damping. Given that the magnitude of the force is $F =$ _____ N, use this new model to plot the lateral displacement for the first ___seconds of motion.

IV. Based on your analysis of the models you developed in Parts I, II, and III, which model would you recommend the town use to design its tower? Why?

B. Forced Vibrations

Now we will make our model even more realistic. To do so requires that we include the movement of the earth $u_G(t)$ as an ongoing contributor to the total displacement of the tower. Under this condition, the total lateral displacement is given by $u_T(t) = u(t) + u_G(t)$ as shown in Figure 4. We will again use Newton's Second Law and Hooke's law to derive the differential equation models governing the lateral displacement of the top of the tower. Assuming the ground beneath the tower is imparting an acceleration on the tower of $A\sin(wt)\,m/\sec^2$, where w = _____ $radians/\sec$ and A = _____ $m/\sec^2$

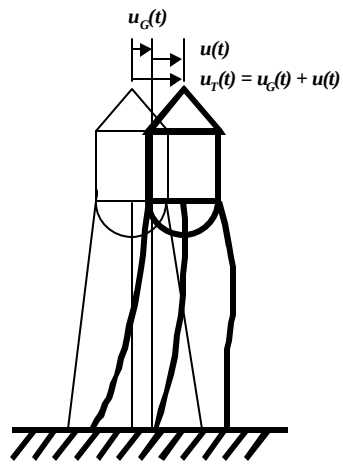

Figure 4.
Lateral Displacement with Ground Motion

I. Decide what initial conditions would be realistic to consider in this case and justify your decision. Ignoring all damping in the system, develop a differential equation model for the motion of the water tower and plot the displacement for the first ____ seconds and describe what you observe. Assume the tank is full.

II. Plot the lateral displacement for the first ____seconds with viscous damping included in your model. Use the same damping coefficient as in Part A. What effect does changing the value of the mass M have on the lateral displacement? Describe what type of long term behavior and classify the type of damping the tower displays when the tank is less than full.

III. If catastrophic failure occurs when the lateral displacement is more than ____meters, determine what values of M will cause catastrophic failure in the viscous damping model. How would you use this information in your tower design?

IV. Once the tower is in motion, its supports will fail if the distance that the top of the tower travels exceeds _____meters in _____seconds. Using the viscous damping model when the tank is full, determine if the structure will fail.

Prepare a one page executive summary, in memorandum format, for the 864th Engineer Battalion (CBT HVY) commander on your modeling and research efforts, in which you describe the situation and your major findings. Include in this summary any recommendations on how the tower design should be changed to prevent catastrophic failure if necessary (i.e.

structural changes, damping systems, etc). Attach this memorandum to a complete report of your modeling and research efforts, which support your findings.