PENDULUM ANALYSIS

ILAP 2 - MA205P / PH251M
500 MA205P points / 100 PH251M points
Estimated time per cadet: 5 hours
Due in class: 29 November or 1 December 1999

Group Members: _____
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This project is combined with the PH251M design lab. It is composed of two requirements. The first requirement consists of the mathematical background to the problem, and the second requirement is the lab. Ensure that the physical and mathematical principles which you use are correct and clearly explained!

Your PH251M Laboratory Outline requires you to design your own lab, conduct the lab, and prepare a group written lab report for laboratory seven and eight. This ILAP is your PH251M Design lab. Parts one and two below constitute the Theory paragraph of your laboratory report. If you prepare them with the lab report format in mind, you may be able to resubmit parts one and two as your Theory paragraph with little modification. The PH251M laboratory Outline for Laboratory seven and eight explains the requirements for the physics portion of this ILAP/Design Lab. The information that is provided below in parts 3-5 provides specific guidance as to how this ILAP fits into the Design Lab requirements.

Background:

You don't trust all the naysayers who predict that nothing significant will happen as the clock strikes midnight on 31 December 1999. In fact, you think there is a good chance that electrical service will cease. In order to prepare for the Y2K bug, you decide that it would be a good idea to develop mathematical formulas to determine the period of a pendulum. That way, in case the power does not come back on for a long time, you can construct an accurate clock and begin to rebuild civilization.

Having taken MA205P and PH251M, you decide that the small angle approximation is for scientists of the 20th century. Being a 21st century thinker, you feel that you could determine the period of a "large angle" pendulum to a high degree of accuracy. To show your understanding of the mathematics and physics involved in such a device, you decide to team with a group of fellow cadets, derive the appropriate equations of motion, and then test your derivations in the lab.

Part One – Derive the equations.

(a) Sketch and describe a free-body diagram for a swinging pendulum. Describe relevant forces, and state and describe assumptions and their impact on the motion of the pendulum.

(b) Beginning with Newton's Second Law, derive the differential equation that governs the motion of a pendulum. The variables you use should be the angle (q) and time (t).

(c) Let v be the tangential velocity of the pendulum's bob. As the pendulum moves, v will vary and therefore the bob is in non-uniform circular motion. Describe the relationship between v, q, and t. Rewrite the differential equation you found in part b and solve for $\mathrm{d}v/\mathrm{d}t$.

(d) Use the chain rule in the form $\mathrm{d}v/\mathrm{d}t = \mathrm{d}v/\mathrm{d}q \cdot \mathrm{d}q/\mathrm{d}t$ to get a differential equation relating v and q. The variable t should disappear. The resulting equation should be separable. Separate variables and use an appropriate initial condition to solve for v explicitly as a function of q. (Note: You can also find this equation by using conservation of energy and appropriate initial conditions.)

(e) Use symmetry and the results from part d to find an integral relationship for the period of a pendulum. (Hint: The time required for the pendulum to swing from its maximum angle to vertical is one-fourth the total time required for it to return to its starting position.)

Part Two – Approximate the solution.

A portion of your equation from Part One (e) should look like

$$\int_0^{q_{\max}} \frac{1}{\sqrt{\cos q - \cos q_{\max}}}\, dq \ .$$

Using a trigonometric identity, along with making the substitution $\sin\left(\frac{q}{2}\right) = \sin\left(\frac{q_{\max}}{2}\right)\sin f$, this integral can be transformed into

$$\sqrt{2}\int_0^{p/2} \frac{1}{\sqrt{1-k^2\sin^2 f}}\, df \ ,$$

where $k = \sin\left(\frac{q_{\max}}{2}\right)$. This definite integral is a complete elliptic integra1 of the first kind. This type of integral can be found in mathematical reference books along with a formula that allows the solution to be represented by a Taylor series. (You could derive this Taylor series using course techniques, but we figured "Hey, it's Navy week!") Use this result to find a series solution for the period of a pendulum as a function of maximum angle $q_{\max}$. You should justify the number of terms of the series you used to get a reasonable approximation.

Part Three – Design the experiment.

The scientific objective of the laboratory portion of this ILAP is to measure the acceleration due to gravity. Design a laboratory apparatus and procedure based on the equations from parts one and two above in order to accomplish this objective. The specific requirements of the design are listed in the Laboratory 7 Advance Sheet. Additionally, you must vary the angle and period of oscillation for the purpose of plotting and regressing the data. You may choose to measure and/or vary other quantities if it supports your objective. Prepare a list of equipment that you will need to conduct the lab.

Part Four – Briefing.

(a) **200 MA205P points.** Conduct a briefing for your math and physics instructors on 29 November or 1 December during your regularly scheduled class period. The briefing should cover the first three parts of this project. All members of your group should participate in the briefing. All members of the group will receive the same grade for the briefing.

(b) **300 MA205P points.** Turn in a write-up for the first three requirements.

Part Five – Conduct the experiment.

(a) Conduct the laboratory in accordance with the your approved plan and the guidance provided in the PH251M laboratory outline. You will conduct the lab during the time scheduled for Lab 7 (1 Dec) or Lab 8 (3 Dec) on the course schedule.

(b) **100 PH251M points.** As a group, write and submit a formal lab report in accordance with the PH251M Laboratory Outline. The lab report is due prior to the beginning of class on 7 December. Scheduled Laboratory time not used for your briefing or to conduct the experiment constitute a drop for the purpose of preparing your written lab report.