ILAP PROBLEM:

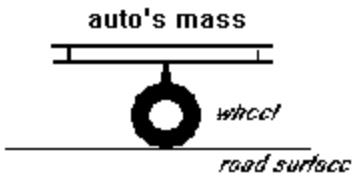
Automobile Design and Safety

The mathematical modeling process is an important element in analyzing real world problems. Often we begin with a simple model and refine it as necessary. Spring-mass systems play a vital role in modeling problems of motion and vibrations.

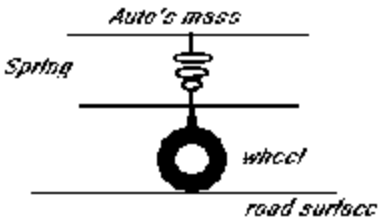
Assume that an automobile is resting on a hydraulic lift and that the wheels are hanging freely. An automobile's suspension might be thought of as a spring-mass system with damping (accomplished by the shock absorbers). The automobile provides the mass, the coiled springs suspend the mass above the wheels, and the shock absorbers damp the motion proportionally to the velocity of the vertical motion of the car. The road surface provides the forcing function, f(t).

Often we accept that automobiles have coil springs and shock absorbers, but is there really a need or is it another way to drive up the sticker price?

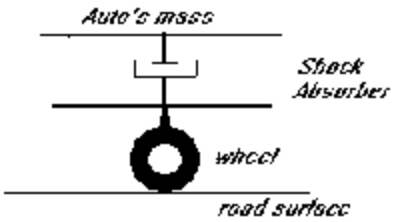
CASE I -- Assume that the car does not need coils or shocks.



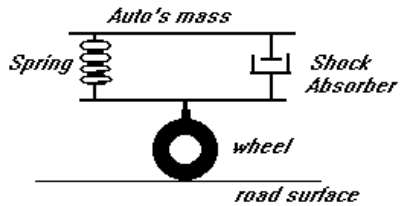
CASE II -- Assume that the car needs the coil spring.



CASE III -- Assume that the car needs only the shock absorber.



CASE IV -- Assume that the car needs both the coil springs and the shock absorbers.



Assume that the car weighs 3840 lbs and that the **weight is distributed over all four wheels**. Assume that the spring constant for the coil spring is 270 lbs/ft, and assume that the shock absorbers provide a resistance proportional to the instantaneous velocity where the damping coefficient equals 60 lb-sec/ft. Finally, assume that the road provides a forcing function, f(t) = 96 sin(5t).

Requirements:

- Model each case, developing a differential equation.
- Analytically solve each model.
- For Case II, if there is no forcing function, and the car is pushed down 0.25 feet and set in motion with a downward velocity of 1.5 feet per second, solve the differential equation. Write the solution in the alternate form $y = A\cos(wt - f)$, where $w = 3$.

  Hint: Use the trig identity $\cos(a - b) = \cos(b)\cos(a) + \sin(b)\sin(a)$ and let $a = wt$ and $b = f$.
- In Case II, if the forcing function is modeled by **f(t) = 96 sin(3t),** solve the differential equation.
- Graph each equation of motion.