MA103 Problem Set (200 Points) **DUE: 14 October 1999**
Fall 1999 Expected time to complete: 2 hours
Issued Lesson 24

GENERAL

1. This is an **INDIVIDUAL** graded homework set.
2. Follow the Department of Mathematical Sciences' policy on reporting numerical answers. This policy is available on the MA103 home page. To access it, click on "Significant Digits Memorandum" under "Administrative Links."
3. Indicate all answers on the provided answer sheet. Attach all supporting calculations and analysis. You will not receive credit for work that is not supported! When technology is used, include a printout or appropriate description of your procedure. Ensure all supporting work is logical, neat, organized and legible.
4. This problem set is due on **14 October 1999**, at the beginning of class.
5. This is a **graded homework assignment** worth 200 points. Document any assistance you receive in accordance with *Documentation of Written Work,* August 1999.

BACKGROUND

In Project One, we investigated the bioaccumulation of methylmercury in the average adult who ate fish from the New York City reservoir system. In this problem set, we will examine the potential impact of dumping mercury into the system.

A few weeks after excessive amounts of mercury began showing up in the smallmouth bass, the FBI began investigating an alleged cover-up of mercury spills from machinery on the grounds of two of the reservoirs in the New York City system. Although state and city officials do not believe that any of the spilled mercury is responsible for the high mercury counts in fish, they are continuing to monitor the mercury in the water. They are also taking steps to clean the Neversink, Rondout and Ashokan reservoirs. Their primary concern is whether the reservoirs can rebound from the mercury contamination.[1] We have been asked to estimate the length of time it would take for all three reservoirs (Neversink, Rondout and Ashokan) to be restored to safe levels of mercury if they should ever reach a toxic level.

Officials from the New York City Department of Environmental Protection (NYCDEP) have provided us with vital information about the reservoir system. The three reservoirs are connected through a series of aqueducts. Water flowing from the Neversink Reservoir flows into the Rondout reservoir, and water flowing from the Rondout reservoir flows into the Ashokan Reservoir. The two machinery sites are located at the Neversink and Ashokan Reservoirs.

[1] Wayne A. Hall, " Mercury in the reservoirs: Water's OK, but don't eat the fish," The Times Herald-Record [Middletown, NY], 11 Jul. 1999, p. 6, cols. 1-4.

Part 1

We have determined that, each year, the percentage of water that outflows from the Neversink, Rondout and Ashokan reservoirs is approximately ____%, ____% and ____%, respectively. For the purpose of our investigation, we will assume that currently there are _____kg of toxic mercury in the Neversink, _____kg in the Rondout and _____kg in the Ashokan and that the mercury is evenly distributed throughout the reservoirs. Additionally, assume that the only new mercury contamination is that introduced by the two machinery sites located at the Neversink and Ashokan reservoirs that dump ____ kilograms (kg) of toxic mercury into their respective reservoirs each year.

Requirements:

1. Given: $\begin{cases} a(n) = \text{the total amount of mercury (in kg) in the Neversink Reservoir afer } n \text{ years.} \\ b(n) = \text{the total amount of mercury (in kg) in the Rondout Reservoir afer } n \text{ years.} \\ c(n) = \text{the total amount of mercury (in kg) in the Ashokan Reservoir afer } n \text{ years.} \\ \quad n = 0, 1, 2, 3, \ldots \text{ years.} \end{cases}$

Write the system of Discrete Dynamical Systems (DDSs) that models the amount of mercury contamination in kg in each of the three reservoirs.

To simplify your model, assume that 100 percent of the water that flows out of the Neversink flows into the Rondout and 100 percent of the water that flows out of the Rondout flows into the Ashokan.

2. Write the system in matrix form ($A(n+1) = RA(n) + B$).

3. Write the particular solution of the system in matrix form ($A(k) = R^k C + D$).

4. Determine the eigenvalues of the coefficient matrix.

5. Determine the eigenvectors of the coefficient matrix.

6. Find the particular solution using eigenvector decomposition ($A(k) = x\, l_1^k A_1 + y\, l_2^k A_2 + D$).

7. If the volumes of the Neversink, Rondout and Ashokan Reservoirs are $135 \times 10^6 \text{ m}^3$, $200 \times 10^6 \text{ m}^3$ and $500 \times 10^6 \text{ m}^3$, respectively, determine how long it would take for the three reservoirs to be returned to the federal marketplace standard of 1 ppm or $\mu g/m^3$.

Part 2

City officials have removed all the mercury from the reservoir machinery. With the external contamination source eliminated, they have now asked us to revise our estimate of the length of time it would take for all three reservoirs to be returned to the federal marketplace standard. They have also asked us to take into account the effects of nature (evaporation and chemical decomposition) in reducing the amounts of mercury in the reservoirs.

The amount of mercury lost through evaporation is proportional to the amount of mercury in the reservoir. The amount of mercury that can be decomposed chemically is proportional to the square root of the amount of mercury present in the lake.

Given: $\begin{cases} a(n) = \text{the total amount of mercury (in kg) in the Neversink Reservoir afer } n \text{ years.} \\ b(n) = \text{the total amount of mercury (in kg) in the Rondout Reservoir afer } n \text{ years.} \\ c(n) = \text{the total amount of mercury (in kg) in the Ashokan Reservoir afer } n \text{ years.} \\ \quad n = 0, 1, 2, 3, \ldots \text{ years.} \end{cases}$

Our revised Dynamical System Model becomes:

$$a(0) = _____, \quad b(0) = _____, \quad c(0) = _____$$

$$a(n+1) = _____\, a(n) - _____\, \sqrt{a(n)}$$
$$b(n+1) = _____\, a(n) + _____\, b(n) - _____\, \sqrt{b(n)}$$
$$c(n+1) = _____\, b(n) + _____\, c(n) - _____\, \sqrt{c(n)}$$

Requirements:

1. Use Mathcad or your TI-89 handheld calculator to create a table of values* for the amounts of mercury in each of the three reservoirs predicted by the above system of DDSs for the next 20 years.

2. Use Mathcad or your TI-89 handheld calculator to produce graphs* of the **concentration** of mercury in each of the three reservoirs for the 30 years following the hypothetical mercury contamination.

3. According to this new model, how long will it take for the mercury in the three reservoirs to be reduced to the federal marketplace standard of 1 ppm?

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* If you use your TI-89 handheld calculator to create a table of data or graphs, you may either:
    1. see your instructor for instructions on how to download your table of data and graphs to your computer
    2. or include a description of your procedures along with an electronic or hand drawn copy of your screen.

PROBLEM SET ANSWER SHEET

Part 1 Requirements

1. Write the system of Discrete Dynamical Systems (DDSs) that models the amount of mercury contamination in kg in each of the three reservoirs.

2. Write the system in matrix form ($A(n+1) = RA(n) + B$).

3. Write the particular solution of the system in matrix form ($A(k) = R^k C + D$).

4. Determine the eigenvalues of the coefficient matrix.

5. Determine the eigenvectors of the coefficient matrix.

6. Find the particular solution using eigenvector decomposition ($A(k) = x\,l_1^k A_1 + y\,l_2^k A_2 + D$).

7. How long does it take for the three reservoirs to be returned to the federal marketplace standard of 1 ppm or $\mu g/m^3$?

PROBLEM SET ANSWER SHEET

Part 2 Requirements

1. Attach your printed Mathcad worksheet, your TI-89 printout, or a copy of your TI-89 screen.

2. Attach your printed Mathcad worksheet, your TI-89 printout, or a picture of your TI-89 screen.

3. How long will it take for the mercury in the three reservoirs to be reduced to the federal marketplace standard of 1 ppm?