MA206 Project #1 (200) **DUE: 1600, 9 February 1998**
Spring 1998 Expected time to complete: 3 hours/cadet
Issued Lesson 3

Each member of your group is expected to contribute up to 3 hours towards the completion of this project. This project effort is compensated by each group member devoting only 1 hour towards preparation for lessons 4, 5 and 7.

GENERAL:

1. The project will be worked in groups of two or three as determined by your instructor.
2. For project format, follow the guidance given in the Project and Technical Report Format and Writing Guide. This guide is available on the MA206 home page (http://www.dean.usma.edu/math/core/ma206/index.htm) by clicking on the "Project Template" link under "Administrative Links".
3. This project requires you to review material from your MA103 course. If you do not review this material, you may not be able to complete the entire project.
4. Follow the Department of Mathematical Sciences policy on reporting numerical answers. This policy is also available on the MA206 home page. Click on "Significant Digits Memorandum" under "Administrative Links" to access this.
5. **This project is due NLT 1600 on Monday, 9 February 1998. Turn in your project at your instructor's office.**
6. **This is a graded homework assignment. You are required to document any assistance that you received in accordance with *Documentation of Written Work*, July 1997.**

SITUATION:

Charles H. Dow introduced his industrial stock average in 1896 to an economy that held the stock market in low regard. In those days the stock market was ripe with insider trading and unscrupulous operators. Bonds were considered the safest and wisest investments.

Today, many stocks are considered to be conservative investment vehicles. Even in the late 1800's there was a need for some measure that would capture the general trend of the market at a given time. The same Dow Jones Industrial Average provides the public with such a measure – a benchmark for comparing individual stocks to the overall market or for comparing the market with other economic indicators.

The current Dow Jones Industrial Average has 30 stocks, all widely held by individuals and institutions alike – AT&T, Boeing, Walt Disney, McDonald's and Wal-Mart are just a few.

You are interested in developing a mathematical model to investigate long term trends in the stock market using the Dow Jones Average. You observe that at the close of trading on a given day, the Dow Jones average either goes up or down.

You decide to model the Dow Jones Average with a *Markov Chain*. Let U_i be the event that the Dow Jones Average goes up on day i. Let D_i be the event that the average goes down on day i. You can think of the average being in some "state" (either U or D) at the close of trading on day i. Based on your experience and advice you get from an economics major in your company you assume that

$$P(U_{i+1} \mid U_i) = _____ \text{ and } P(D_{i+1} \mid D_i) = _____.$$

REQUIREMENTS:

1. (15) Draw a classical probability tree diagram, which correctly portrays the state of the Dow Jones Average at the end of the next day based on its state at the end of the current day. Ensure that the branches of your diagram are properly labeled and that you have clearly defined all your events. Include any probabilities that you currently know. Label branches with "UNKNOWN" if you do not know its corresponding probability.

2. (5) What assumption(s) did you have to make to model the Dow Jones Average as a *Markov Chain*?

3. (5) Do you think a Markov Chain is a reasonable model for the Dow Jones Average? Why or why not?

4. (20) Find the unknown probabilities in the tree diagram. First, you must write the equations for the DDS that models the state of the Dow Jones Average from one day to the next. Then find $P(U_i)$ and $P(D_i)$. Clearly define all variables that you use in your model. **HINT**: Review sections 3.4 and 8.4 of Sandefur's *Discrete Dynamical Modeling*.

5. (30) Answer the following questions. Show your work.

   a. What is the probability that the Dow Jones Average will be in the same state tomorrow as it was today?

   b. What is the probability that the Dow Jones Average went up yesterday given that it went up today?

   c. What state is the Dow Jones Average most likely to be in the long term? Why? What is the numerical value of this long-term probability?

SITUATION (continued):

Now you want to improve your model by considering another state. You redefine your states as follow:

Let U_i be the event that the Dow Jones Average goes up more than one percent on day i
S_i be the event that the Dow Jones Average stays the same (within ± 1%) on day i
and D_i be the event that the Dow Jones Average goes down more that one percent on day i.

Now you revise your assumptions regarding the transition probabilities to the following:

Probability the state of the DJ Average the next day will be	Current state of the DJ Average		
	U_i	S_i	D_i
U_{i+1}			
S_{i+1}			
D_{i+1}			

REQUIREMENTS (continued):

6. (15) You notice that there are two blank spaces in your probability table. What values should be put in those spaces? Why?

7. (40) Draw a properly label a new classical probability tree diagram which includes the new readiness states. Write the system of equations representing the DDS that models this new situation. Clearly define all variables that you use in your model. Use your model to calculate any unknown probabilities that are on your tree diagram.

8. (40) Answer the following questions. Show your work.

   a. What is the probability that the Dow Jones Average will be in a different state tomorrow than it was today?

   b. How does your answer to part (a) change if you are given the fact that the Dow Jones Average went down today?

   c. What is the probability that the Dow Jones Average went up yesterday given that it went up today?

   d. In the long term, what is the most likely state of the Dow Jones Average? What is the numerical value of this long-term probability?

9. (10) Does this model seem reasonable to you? Why or why not? Validate your model by comparing your long-term probabilities to stock market data.

The remaining 20 points will be awarded by your instructor based upon how well you packaged and presented your analysis in your project submission.