Building Creativity Through Mathematics, Interdisciplinary Projects, and Teaching with Technology

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Introduction

One of the primary academic program goals at the United States Military Academy (USMA) is to have our graduates think and act creatively. This is a worthwhile goal for any educational program. There are many reasons to have such a goal. The accelerating rate of change in the world is producing an environment that is more unpredictable than ever before with a wider range of options. The creative talents of college graduates are crucial to their success. Our modern world requires people with increased flexibility and adaptability. Successful professional service necessitates college graduates to possess the essential ingredients of creativity: creative thinking, critical thinking, innovative problem solving, intellectual versatility, curiosity, and the ability to deal with ambiguity. The West Point mathematics program, along with its interdisciplinary partners, strives to make contributions to this goal by maintaining a creative environment, implementing an innovative curriculum, and using pedagogy to encourage the development of creativity. This paper outlines the contributions of a core mathematics program and its interdisciplinary activities in these endeavors.

Definitions and Terminology

Imagination is more important than knowledge. -- Einstein

Creativity is fundamental to the development of productive graduates. At West Point, the creativity goal supports the overall academic goal "to enable graduates to anticipate and respond effectively to the uncertainties of a changing technological, social, political, and economic world." [3] Creativity is the premier attribute needed to empower college graduates to respond effectively to the uncertainties in our changing world. [6] In the article in this volume by Froyd [4], creativity (and life-long learning) take center stage as course and program goals. Froyd's proposition is that "In the long run, your only sustainable source of competitive advantage is your ability to learn and create faster than others."

Our conception of creativity includes an interrelated set of intellectual skills, personal characteristics, and values. [3] The skills include: creative thinking, critical thinking, and innovative problem solving. We define creative thinking as the consideration of a broad range of new, sometimes abstract, ideas and the establishment of new connections and relationships among these ideas. Critical thinking is the performance of careful and exact analysis, ultimately leading to a deeper understanding of an issue. Innovative problem solving is defined as combining knowledge with imagination to produce solutions to problems. The personal characteristics linked with creativity include: versatility, tolerance for ambiguity, willingness to take risks, open-

mindedness, confidence, and curiosity. The values connected with creativity include: discipline, perseverance, and responsibility.

Graduates who achieve our creativity goal can confidently confront ambiguous situations. They apply their thinking skills and innovation to solve challenging problems. They are active, independent, and self-directed thinkers and learners. They are able to transfer what they know in one context or discipline to another. They respond successfully to new challenges and situations that require inventiveness. When faced with complex problems, they are able to go beyond traditional approaches to devise more useful and more favorable solutions. They are able to work in collaborative teams, as well as individually, to confront ill-defined situations, to generate new ideas, and to function successfully in different settings.

The USMA core mathematics program was designed with this creativity goal in mind. The program attempts to develop ingredients of creativity in the students. While the essence of mathematical theory is accuracy and precision, mathematics can be applied creatively to solve problems (especially those requiring an interdisciplinary perspective) and used to build the thinking skills, personal characteristics, and values associated with creativity. This paper provides a brief description of the core mathematics program at USMA and its contributions to the development of creativity.

Core Mathematics Program

An expert problem solver must be endowed with two incompatible qualities, a restless imagination and patient pertinacity. -- Howard H. Eves.

Core mathematics includes both the acquisition of a body of knowledge and the development of thought processes. [1, 2] Creativity plays a major role in the thought process component of this program. This intellectual foundation affords opportunities for students to progress as life-long learners, who are able to formulate intelligent questions, research answers, and reach logical conclusions. During their required core mathematics program at USMA, students learn to blend theory and practice. To enhance creative and critical thinking, students study concepts from several perspectives. For example, the program strives for understanding of mathematics from different perspectives: discrete and continuous, linear and nonlinear, deterministic and stochastic, disciplinary and interdisciplinary.

The development of critical thinking skills facilitates the presentation of higher-level concepts. Within the core mathematics classes, concepts are constantly interconnected and applied to problems from various disciplines. The requisite problems, especially the interdisciplinary, open-ended projects assigned in each course, develop student experience in employing new technology, applying the art of mathematical modeling, and writing in creative ways. In addition, students spend time engaged in experimentation, discovery, and reflection. Through this experience, students develop a curious and experimental disposition and a creative mindset.

Because the core mathematics program at USMA is taken by all students to prepare them for required science and engineering courses, there are seven essential subjects, which are integrated into the four courses. [1] These seven subjects are: differential calculus, integral calculus, vector and multivariable calculus, differential equations, matrix algebra, discrete mathematics, and probability and statistics. West Point implemented this “7-into-4” curriculum in 1990.

Descriptions of the content and the creative component of the four core mathematics courses are as follows:

- First semester (Discrete Mathematical Modeling and an Introduction to Calculus) -- discrete mathematics in the form of difference equations, matrix algebra through eigenvalues and eigenvectors, and differential calculus using a sequential approach. This innovative course establishes the need for and value of creativity in mathematics. Students are required to analyze data, look for patterns, conjecture solutions, and verify their conjectures using discrete mathematics. The art of mathematical modeling is introduced as students encounter open-ended, realistic problems from several disciplines.
- Second semester (Calculus I) -- integral calculus, differential equations through higher order and systems, and a consolidation of matrix algebra. Once again, conjecturing solutions (undetermined coefficients in differential equations) and using modeling to solve complex, sometimes open-ended problems are major parts of this course.
- Third Semester (Calculus II) -- multivariable and vector calculus and discrete mathematics of sequences and series. This course requires students to innovate in order to provide examples of functions meeting certain properties or counter-examples of functions with certain properties. These "inverse problems" require creative thinking and are often initially difficult for students. More open-ended problems are encountered and experience is gained in developing innovative mathematical modeling skills and an interdisciplinary perspective.
- Fourth semester (Probability and Statistics) -- descriptive statistics, classical probability, random variables, and hypothesis testing. This course provides students with a new way of thinking and problem solving. Stochastic modeling is introduced and developed. Students confront situations where creativity is needed to collect and display data, to analyze realistic scenarios, and to present their results. A large-scale capstone interdisciplinary project is used to consolidate and refine the mathematical, scientific, and technological knowledge and skill of the students.

Examples

We present several examples of activities (homework, exam questions, and project components) that we use to build the ingredients of creativity in our students. The first two examples are questions that show how creativity can be essential in solving open-ended problems and performing mathematical reasoning.

Example 1 (Homework in 1st semester): Given the following difference equation, $a(n+1) = 5a(n) - \frac{n\,2^n}{n+3}$, iterate a few values to reveal a pattern, conjecture a solution, and test your conjecture. Explain why your conjecture was correct or not.

Example 2 (Exam in 2nd semester): Give an example or explain why this can not happen: a function that is differentiable at $x=3$, but not continuous at $x=3$.

The next example involves solving an "inverse" problem. Once students learn to find derivatives of functions of two variables, can they reverse the process? There are infinite correct solutions, but this is often difficult for students.

Example 3 (Exam in 3rd semester): Find two different functions of 2 variables with a derivative vector of [4,-2] at the point (3,7).

The next example looks for innovation in the analysis of a conceptual result. Once again, there are many "correct" answers.

Example 4 (Homework in 4th semester): Given a distinctly bimodal set of data, what are the advantages and disadvantages of using the mean as a measure of location of the data? Can you suggest any (traditional or nontraditional) alternatives?

The following two examples explain two application problems where the art of mathematical modeling was needed to solve and analyze the problems. This is often difficult for first-year students, but they did remarkably well on these two problems.

Example 5 (Project in 1st semester): Using an interdisciplinary perspective, students were required to model the motion of a bridge subject to external forces using a difference equation. Then they were asked to conjecture or test the effects of applying different external forces. Students were required to explain what they discovered and the implications of their results on designing bridges and similar structures. We required and received creative solutions that showed the connections between different external forces and design criteria. Many students discovered the phenomenon of resonance while investigating this problem.

Example 6 (Project in 2nd semester): Students were asked to determine how much grass seed was needed to re-seed an area of campus where no scaled maps were available. As a follow up to this investigation, budget constraints were added and students were asked to determine ways to re-seed the area with a limited amount of seed. Both questions required and produced ingenious solutions.

The final two examples are questions from application projects where creativity helps in the solution.

Example 7 (Project in 3rd semester): Given a contour map of a mountain (like Pikes' Peak), design a path of a hike to traverse from a base camp to the summit. What criteria did you use? Justify that your path meets the criteria? Based on the criteria is your path optimal?

Example 8 (Project in 4th semester): A vital component of a machine has a failure rate of 1 per 20 hours of operation. How many spare components are needed to obtain a 99% probability that the machine can operate for 96 hours? What assumptions did you make to solve this problem?

Student Growth

Because student growth is important in a four-course program, we have developed specific educational threads for all four courses. These threads utilize objectives in each course to insure student development. In many ways, creativity plays an important role in the development of these threads. Brief descriptions of the five educational growth threads and their major component in the development of creativity are as follows [1]:

- Mathematical reasoning: multiple representations (analytic, graphic, numeric), conjecturing, inferences in situations of uncertainty, generalizing concepts from specific examples, providing examples and counterexamples. Creative and critical thinking experiences are provided continually with increasing complexity.
- Mathematical modeling: the art of problem solving, making and identifying assumptions, testing conclusions and sensitivity of assumptions, and solving interdisciplinary problems through collaborative teamwork. These experiences help develop innovative problem solving skills.
- Scientific computing: manipulating and analyzing data, experimenting with parameters, discovering relationships and structures, recognizing capabilities and limitations of computing, understanding information technologies, and performing simple programming. This growth component develops students' imaginations and gives them the opportunity to create their own problem solving tools. New programming languages and nonlinear communication modes, such as html, web-based material, and linked and multi-media textual materials, are perfect tools for learning connections, concepts, and structure in subjects like mathematics.
- Communicating: expressing ideas clearly and effectively, synthesizing concepts, displaying results graphically, and using and understanding mathematical notation and visualization. These activities develop the students' intellectual versatility.
- History of mathematics: appreciating the human endeavor of the development of mathematics, understanding the service role of mathematics, and motivating the further study of the growing and lively discipline of mathematics. These ideas give students the proper attitude for studying mathematics and the importance of developing their creativity.

Through recent surveys of sample groups, students report confidence and growth in their creative abilities. The question "I am confident in my ability to demonstrate creative and critical thinking skill in my scientific reasoning" receives 69% agreement from students after their first semester of mathematics [2]. This grows to 72% or greater after completion of the core. The disagreement percentage holds at 6% in these surveys, and the remainder of the responses are neutral to this question. These data seem to indicate we are achieving some success in elements of our goal. However, since this question does not capture all the elements of creativity that we desire to measure, new, more detailed, questions will be added to future surveys.

Learning Model

The mind uses its faculty for creativity only when experience forces it to do so. -- Jules Henri Poincaré

Our creativity learning model involves environmental, curricular, and pedagogical dimensions. [3, 5] The foundation for the development of creativity is established through a supportive, open-minded learning environment. The mathematics program tries to foster this kind of environment. As discussed, our curricular program provides numerous opportunities for creative development. In addition, our pedagogy of interactive and engaging teaching develops the students' thinking processes.

Role of the Environment: West Point educates students in a mathematics program that supports and values curiosity, imagining, exploring, questioning, and risk taking. The program takes advantage of opportunities to present, discuss, and debate ideas in class and to use innovative methods to perform problem solving. The advent and use of the internet is a large leap in building

and using a creative environment. In any case, factors like section sizes, student population and time, and technologies available have an impact on the environmental climate for developing creativity. As in any institution, the overall academic environment at USMA has elements that foster and at times hinder the development of creativity.

Role of Curriculum: We summarize some of the curricular elements of the USMA mathematics program that make contributions to this goal:

- The four-course program sparks the students' imaginations and develops their curiosity by exposing them to a wide variety of challenging subjects (7 major topics integrated in the 4 courses).
- The mathematics courses incorporate reasoning, writing, and problem solving that require students to use imagination and innovation.
- Many of the academic projects and computer laboratories are exploratory and require students to cope with ambiguity, to investigate alternatives, and to discover new results.
- The art of technical problem-solving through mathematical modeling is taught throughout the program. Interdisciplinary projects (approximately 3 projects in each of the 4 courses) demand the application of intellectual versatility to transfer students' learning from one context or discipline to another. Collaborative group work is the mode for solving these projects. Some of these projects are open-ended and require significant innovation to solve. USMA has been the lead institution in an NSF-sponsored consortium called Project INTERMATH, where faculty write and students solve interdisciplinary lively application projects (ILAPs). These projects are co-authored with faculty in partner disciplines and presented to students as challenging, relevant problems to solve in mathematics course. Examples of recent projects for the four core courses at USMA are given in Table 1.

Table 1: Titles of example ILAPs recently used in the four core courses at USMA

MA103 - Discrete Dynamical Systems and Introduction to Calculus
1D Heat Transfer
Pollution along a River
Chemical Chain Reaction
Great Lakes Pollution
SMOG in LA Basin
Car Financing.
Analysis of Military Retirement Pay
Making Water in Space
Water Treatment
Bridge Vibrations
Viral Infection

MA104 -- Calculus I -- Single Variable Calculus and Differential Equations
Flying Strategies
Bungee Cord / Parachute Jumping
Aerobic Capacity
Vibration of an Airplane Wing
Air Traffic Control

Clinic Profit Management
Wheel Suspension Design
Bass Population
Terrain Analysis
Telemetry Data Interpretation
Real Estate Taxation
Road Construction
Airport Construction
Forest Fire Fighting
Water Reservoir Management
Cut/fill and Bridge Abutment/Span Computations
Railway Headwall Design
Earthquake Tower Problem

MA205 Calculus II -- Multivariable Calculus
Missile Trajectory
Laser Guided Munitions
Vehicle Collision
The Health Management Organization (HMO)
The Oil Refinery
Chemistry ABC's
Rocket Control
The Satellite Problem
Trajectories in 3-space & Least Squares Analysis of Motion Lab Data

MA206 Probability & Statistics
Great Lakes Pollution
Vehicle Accident Analysis
Remotely Piloted Vehicle
Hudson River Pollution Data
Model of Dow Jones Industrial Average
Elementary Circuits

Role of Pedagogy: Creativity is an indispensable element of the active classroom. One way that faculty establish an interactive classroom is to encourage questioning and discussion. Question-based pedagogy recognizes that questions, not answers, are the driving force in thinking. When answers generate further questions, important thinking and problem solving occur. Students who develop curiosity and ask questions during their academic pursuits are thinking and learning. The mathematics faculty at West Point make use of classroom and homework activities (active learning and group interaction), technology in the form of computer/calculator discovery and experimentation laboratories, and student-relevant reading and discussions, to establish this interactive environment and to promote active learning. Small class sizes (18 or less students per class) help instructors insure students are participating fully in all classroom activities. The classroom layout of blackboards all around the room provides opportunities for all the students in the class to perform active work and to show and explain their work to others. In a typical class, mathematics instructors ask many questions, facilitate lively discussions, and require participation

by all students. Student involvement is critical to success in classrooms, computer laboratories, homework, and the development of creativity.

Conclusion

Discovery consists of seeing what everybody has seen and thinking what nobody has thought. -- Albert Szent-Gyorgyi

Mathematics courses can provide opportunities for the development of creativity. The environment, curriculum, and pedagogy of the USMA core mathematics program strive to provide these opportunities. While this is not a traditional role for mathematics education, programs like the one at West Point have enabled undergraduates develop their creativity. Students gain opportunities to develop their creativity and to grow in their capabilities, when mathematics is applied inventively to solve problems and to build the thinking skills, personal characteristics, and values associated with creative endeavors.

References

A college education should equip one to entertain three things: a friend, an idea, and oneself. -- Thomas Ehrlich

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